The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381V: Convex Optimization—Fall 2007

Midterm Exam

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I. This is a closed-book, closed-notes exam. You do not need a calculator, and you will not need any special formulas.

II. Please be clear about any assumptions you make. Make it easy for me to give you partial credit.

III. Do all your work in the pages provided. If you need extra paper, ask for it. Make sure to put your name on all loose pages.

IV. There are 4 problems, for a total of 100 points. The breakdown of the point values are as follows (so divide your effort accordingly, i.e., as you see fit):

1. 15%
2. 30%
3. 30%
4. 25%

Problem 1 (15 points) Show that for $x_1, \ldots, x_m > 0$,

$$\Pi_{i=1}^{m}(x_i)^{1/m} \leq \frac{1}{m}\sum_{i=1}^{m} x_i.$$

Problem 2 (30 points) Let C be a closed convex set in $\mathbb{R}^n$, and $A : \mathbb{R}^n \to \mathbb{R}^m$ be a linear mapping.

(a) (5 points) Is $A(C)$ convex? (Prove or provide couterexample).

(b) (10 points) Show that $A(C)$ need not be closed in general.

(c) (15 points) Show that $A(C)$ is closed whenever $C_\infty \cap \ker A = \{0\}$.

Problem 3 (30 points) For A an $m \times n$ matrix, and $b \in \mathbb{R}^m$, show that exactly one of the two following statements can hold:

(i) There exists $x \geq 0$ such that $Ax = b$.

(ii) There exists a vector s such that $s^\top A \geq 0$, and $s^\top b < 0$.

(Hint: One direction is easy. For the other, think about separation arguments.)

Problem 4 (25 points) Consider the set

$$C \stackrel{\triangle}{=} \{M \in \mathbb{S}^n_+ \,:\, \mathrm{tr}(M) = 1\}.$$

(a) (5 points) Show that M is a closed convex set.

(b) (20 points) Compute the support function $\sigma_C : \mathbb{S}^n \to \mathbb{R}$.