CHAPTER 17

Solutions for Exercises

E17.1 From Equation 17.5, we have

$$B_{gap} = Ki_a(t)\cos(\theta) + Ki_b(t)\cos(\theta - 120^\circ) + Ki_c(t)\cos(\theta - 240^\circ)$$

Using the expressions given in the Exercise statement for the currents, we have

$$B_{gap} = KI_m\cos(\omega t)\cos(\theta) + KI_m\cos(\omega t - 240^\circ)\cos(\theta - 120^\circ)$$
$$+ KI_m\cos(\omega t - 120^\circ)\cos(\theta - 240^\circ)$$

Then using the identity for the products of cosines, we obtain

$$B_{gap} = \tfrac{1}{2}KI_m[\cos(\omega t - \theta) + \cos(\omega t + \theta) + \cos(\omega t - \theta - 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ) + \cos(\omega t - \theta + 120^\circ)$$
$$+ \cos(\omega t + \theta - 360^\circ)]$$

However we can write

$$\cos(\omega t - \theta) + \cos(\omega t - \theta - 120^\circ) + \cos(\omega t - \theta + 120^\circ) = 0$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

$$\cos(\omega t + \theta - 360^\circ) = \cos(\omega t + \theta)$$

Thus we have

$$B_{gap} = \tfrac{3}{2}KI_m\cos(\omega t + \theta)$$

which can be recognized as flux pattern that rotates clockwise.

E17.2 At 60 Hz, synchronous speed for a four-pole machine is:

$$n_s = \frac{120f}{P} = \frac{120(60)}{4} = 1800\text{ rpm}$$

The slip is given by:

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1750}{1800} = 2.778\%$$

The frequency of the rotor currents is the slip frequency. From Equation

17.17, we have $\omega_{\text{slip}} = s\omega$. For frequencies in the Hz, this becomes:

$$f_{\text{slip}} = sf = 0.02778 \times 60 = 1.667 \text{ Hz}$$

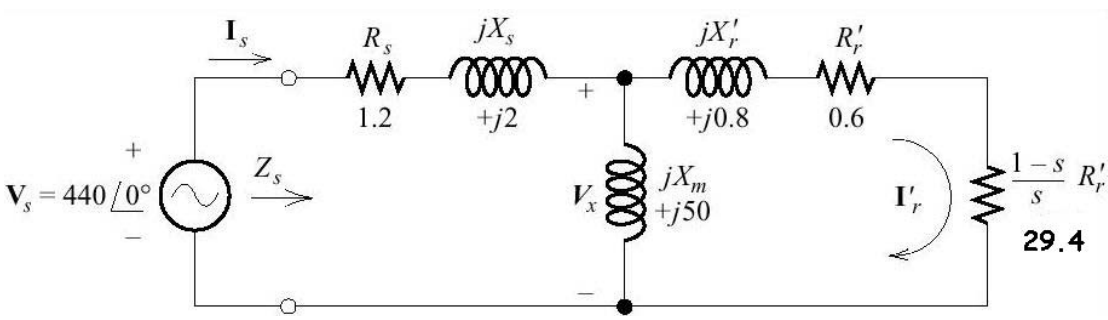
In the normal range of operation, slip is approximately proportional to output power and torque. Thus at half power, we estimate that $s = 2.778/2 = 1.389\%$. This corresponds to a speed of 1775 rpm.

E17.3 Following the solution to Example 17.1, we have:

$$n_s = 1800 \text{ rpm}$$

$$s = \frac{n_s - n_m}{n_s} = \frac{1800 - 1764}{1800} = 0.02$$

The per phase equivalent circuit is:



$$Z_s = 1.2 + j2 + \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$

$$= 22.75 + j15.51$$

$$= 27.53\angle 34.29^\circ$$

$$\text{power factor} = \cos(34.29^\circ) = 82.62\% \text{ lagging}$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{27.53\angle 34.29^\circ} = 15.98\angle -34.29^\circ \text{ A rms}$$

For a delta-connected machine, the magnitude of the line current is

$$I_{line} = I_s\sqrt{3} = 15.98\sqrt{3} = 27.68 \text{ A rms}$$

and the input power is

$$P_{\text{in}} = 3I_sV_s\cos\theta = 17.43 \text{ kW}$$

Next, we compute $\mathbf{V}_x$ and $\mathbf{I}'_r$.

$$\mathbf{V}_x = \mathbf{I}_s \frac{j50(0.6 + 29.4 + j0.8)}{j50 + 0.6 + 29.4 + j0.8}$$
$$= 406.2 - j15.6$$
$$= 406.4\angle -2.2^\circ \quad \text{V rms}$$

$$\mathbf{I}'_r = \frac{\mathbf{V}_x}{j0.8 + 0.6 + 29.4}$$
$$= 13.54\angle -3.727^\circ \quad \text{A rms}$$

The copper losses in the stator and rotor are:

$$P_s = 3R_s I_s^2$$
$$= 3(1.2)(15.98)^2$$
$$= 919.3 \text{ W}$$

and

$$P_r = 3R'_r (I'_r)^2$$
$$= 3(0.6)(13.54)^2$$
$$= 330.0 \text{ W}$$

Finally, the developed power is:

$$P_{\text{dev}} = 3 \times \frac{1-s}{s} R'_r (I'_r)^2$$
$$= 3(29.4)(13.54)^2$$
$$= 16.17 \text{ kW}$$
$$P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 15.27 \text{ kW}$$

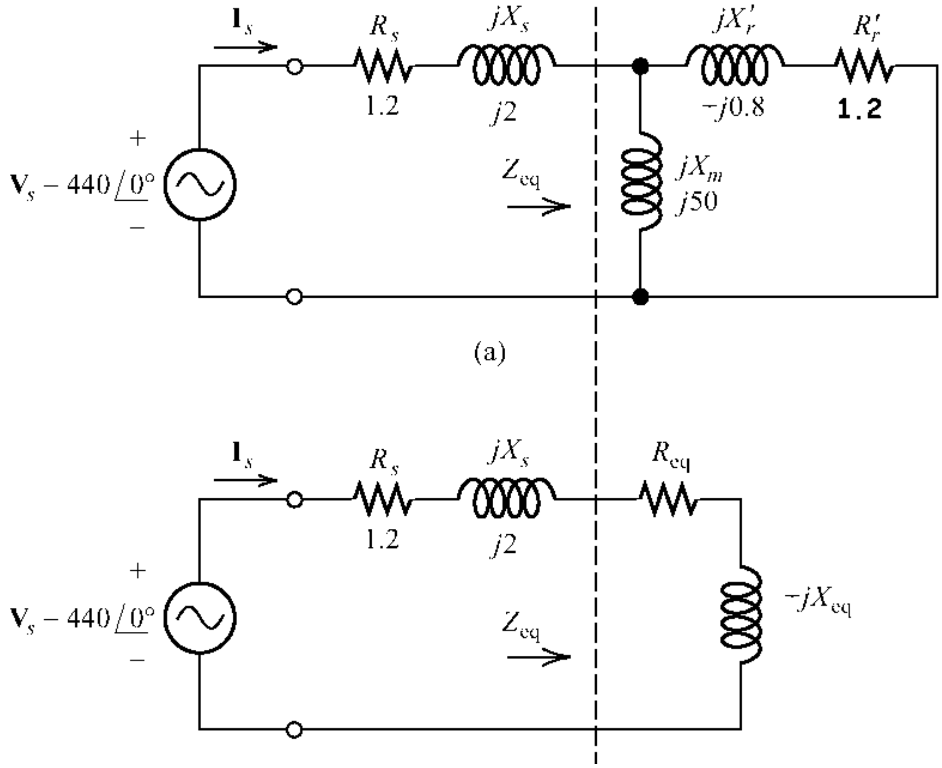
The output torque is:

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_m} = 82.66 \text{ newton meters}$$

The efficiency is:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 87.61\%$$

E17.4 The equivalent circuit is:



$$Z_{eq} = R_{eq} + jX_{eq} = \frac{j50(1.2 + j0.8)}{j50 + 1.2 + j0.8} = 1.162 + j0.8148$$

The impedance seen by the source is:

$$\begin{aligned} Z_s &= 1.2 + j2 + Z_{eq} \\ &= 1.2 + j2 + 1.162 + j0.8148 \\ &= 3.675\angle 50.00^\circ \end{aligned}$$

Thus, the starting phase current is

$$\mathbf{I}_{s,\,\text{starting}} = \frac{\mathbf{V}_s}{Z_s} = \frac{440\angle 0^\circ}{3.675\angle 50.00^\circ}$$

$$\mathbf{I}_{s,\,\text{starting}} = 119.7\angle -50.00^\circ \text{ A rms}$$

and for a delta connection, the line current is

$$I_{line,\text{starting}} = I_{s,\text{starting}}\sqrt{3} = 119.7\sqrt{3} = 207.3 \text{ A rms}$$

The power crossing the air gap is (three times) the power delivered to the right of the dashed line in the equivalent circuit shown earlier.

$$P_{ag} = 3R_{eq}\left(I_{s,\,\text{starting}}\right)^2 = 49.95 \text{ kW}$$

Finally, the starting torque is found using Equation 17.34.

$$T_{\text{dev, starting}} = \frac{P_{ag}}{\omega_s}$$

$$= \frac{49950}{2\pi 60/2}$$

$$= 265.0 \text{ newton meters}$$

E17.5 This exercise is similar to part (c) of Example 17.4. Thus, we have

$$\frac{\sin\delta_3}{\sin\delta_1} = \frac{P_3}{P_1}$$

$$\frac{\sin\delta_3}{\sin 4.168^\circ} = \frac{200}{50}$$

which yields the new torque angle $\delta_3 = 16.90^\circ$. E_r remains constant in magnitude, thus we have

$$\mathbf{E}_{r3} = 498.9\angle -16.90^\circ \quad \text{V rms}$$

$$\mathbf{I}_{a3} = \frac{\mathbf{V}_a - \mathbf{E}_{r3}}{jX_s} = \frac{480 - 498.9\angle -16.90^\circ}{j1.4} = 103.6\angle -1.045^\circ \quad \text{A rms}$$

The power factor is $\cos(-1.045^\circ) = 99.98\%$ lagging.

E17.6 We follow the approach of Example 17.5. Thus as in the example, we have

$$I_{a1} = \frac{P_{dev}}{3V_a \cos\theta_1} = \frac{74600}{3(240)0.85} = 121.9 \text{ A}$$

$$\theta_1 = \cos^{-1}(0.85) = 31.79^\circ$$

$$\mathbf{I}_{a1} = 121.9\angle -31.79^\circ \quad \text{A rms}$$

$$\mathbf{E}_{r1} = \mathbf{V}_a - jX_s\mathbf{I}_{a1} = 416.2\angle -20.39^\circ \quad \text{V rms}$$

The phasor diagram is shown in Figure 17.24a

For 90% leading power factor, the power angle is $\theta_3 = \cos^{-1}(0.9) = 25.84^\circ$. The new value of the current magnitude is

$$I_{a3} = \frac{P_{dev}}{3V_{a3}\cos(\theta_3)} = 115.1 \quad \text{A rms}$$

and the phasor current is

$$\mathbf{I}_{a3} = 115.1\angle 25.84^\circ \quad \text{A rms}$$

Thus we have

$$\mathbf{E}_{r3} = \mathbf{V}_a - jX_s\mathbf{I}_{a3} = 569.0\angle -14.77^\circ \quad \text{V rms}$$

The magnitude of E_r is proportional to the field current, so we have:

$$I_{f3} = I_{f1}\frac{E_{r3}}{E_{r1}} = 10 \times \frac{569.0}{416.2} = 13.67 \text{ A dc}$$

E17.7 The phasor diagram for $\delta = 90^{\circ}$ is shown in Figure 17.27. The developed power is given by

$$P_{\max} = 3V_a I_a \cos(\theta)$$

However from the phasor diagram, we see that

$$\cos(\theta) = \frac{E_r}{X_s I_a}$$

Substituting, we have

$$P_{\max} = \frac{3V_a E_r}{X_s}$$

The torque is

$$T_{\max} = \frac{P_{\max}}{\omega_m} = \frac{3V_a E_r}{\omega_m X_s}$$

Answers for Selected Problems

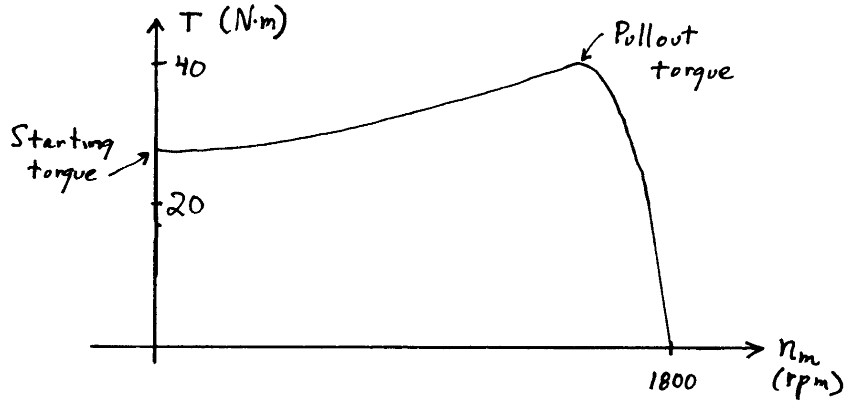
P17.1* $P = 8$ pole motor
$s = 5.55\%$

P17.7* $f = 86.8\,\text{Hz}$
$I = 5.298\,\text{A}$

P17.10* As frequency is reduced, the reactances X_s, X_m, and X_r' of the machine become smaller. (Recall that $X = \omega L$.) Thus the applied voltage must be reduced to keep the currents from becoming too large, resulting in magnetic saturation and overheating.

P17.13* $B_{\text{four-pole}} = B_m \cos(\omega t - 2\theta)$
$B_{\text{six-pole}} = B_m \cos(\omega t - 3\theta)$

P17.16*



$I_{line} = 16.3$ A rms

Typically the starting current is 5 to 7 times the full-load current.

P17.20* $I_{line,\text{starting}} = 115.0$ A rms

$T_{\text{dev, starting}} = 40.8$ newton meters

Comparing these results to those of the example, we see that the starting current is reduced by a factor of 2 and the starting torque is reduced by a factor of 4.

P17.23* Neglecting rotational losses, the slip is zero with no load, and the motor runs at synchronous speed which is 1800 rpm.

The power factor is 2.409%.

$I_{line} = 10$ A rms

P17.26* The motor runs at synchronous speed which is 1200 rpm.

The power factor is 1.04%.

$I_{line} = 98.97$ A rms

P17.29* $P_{ag} = 9.893$ kW

$P_{dev} = 9.773$ kW

$P_{out} = 9.373$ kW

$\eta = 91.51\%$

P17.32* $P_{rot} = 76.12$ W

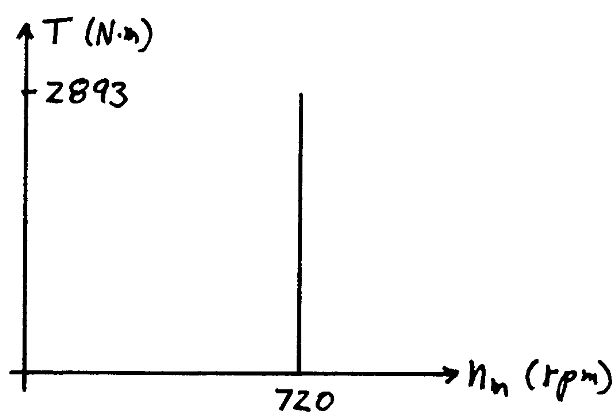
P17.35* 1. Use an electronic system to convert 60-Hz power into three-phase ac of variable frequency. Start with a frequency of one hertz or less and then gradually increase the frequency.

2. Use a prime mover to bring the motor up to synchronous speed before connecting the source.

3. Start the motor as an induction motor relying on the amortisseur conductors to produce torque.

P17.38* (a) Field current remains constant. The field circuit is independent of the ac source and the load.
(b) Mechanical speed remains constant assuming that the pull-out torque has not been exceeded.
(c) Output torque increases by a factor of 1/0.75 = 1.333.
(d) Armature current increases in magnitude.
(e) Power factor decreases and becomes lagging.
(f) Torque angle increases.

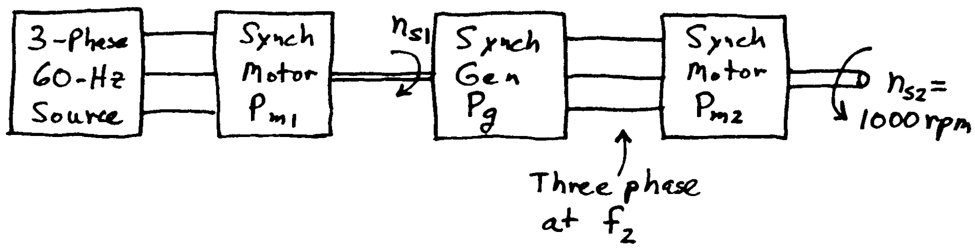
P17.41*



P17.44* $I_{f2} = 5.93\text{ A}$

P17.47* (a) $f_{gen} = 50\text{ Hz}$

(b)



One solution is:

$P_g = 10 \qquad P_{m1} = 12 \qquad$ and $\qquad P_{m2} = 6$

for which $f_2 = 50\text{ Hz}$.

Another solution is:

$P_g = 10 \qquad P_{m1} = 6 \qquad$ and $\qquad P_{m2} = 12$

for which $f_2 = 100\text{ Hz}$.

P17.50* (a) power factor $= 76.2\%$ lagging

(b) $Z = 11.76\angle 40.36^\circ\ \Omega$

(c) Since the motor runs just under 1800 rpm, evidently we have a four-pole motor.

P17.53* The percentage drop in voltage is 7.33%.