Using Order in Distributed Computing

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Happened-Before Relation in Distributed Computing

A computation is \((E, \rightarrow)\) where \(E\) is the set of events and \(\rightarrow\) (happened-before) is the smallest transitive relation that includes:

1. order within a process
2. \(e\) is a send event and \(f\) is the receive implies \(e \rightarrow f\).

[Lamport 78]
Talk Outline

- Happened-Before Relation
- **Applications**
  - Tracking Dependency: *Chain decomposition, Dimension Theory*
  - Detecting Global Predicates: *Meet-closure, Chain merging, ideal enumeration*
  - Computation Slicing: *Birkhoff’s representation theorem*
Tracking Dependency

**Motivation:** Determine whether \( e \) happened before \( f \).

**Problem:** Given \((E, \rightarrow)\), assign timestamps \( v \) to events in \( E \) such that

\[
\forall e, f \in E : e \rightarrow f \equiv v(e) < v(f)
\]

**Online Timestamps:** Vector Clocks [Fidge 89, Mattern 89]:
Every process maintains a vector \( v \) of size \( N \), the number of processes. 
\( v[k] \) at \( P_i \) = the number of events executed by \( P_k \) as known to \( P_i \).
Vector Clocks in a Distributed System

all events: increment $v[i]$
send events: piggyback $v$
receive events: combine timestamps

Theorem:

$$e \rightarrow f \equiv v(e) < v(f)$$
Dynamic Chain Clocks

Problem with vector clocks: scalability, dynamic process structure

Idea: Computing the “chains” in an online fashion [Aggarwal and Garg 05] for relevant events

Figure 1: (a) A computation with 4 processes (b) The relevant subcomputation
Online Chain Decomposition

- Elements presented in a total order consistent with the poset
- Assign elements to chains as they arrive
- Game: Bob presents elements, Alice assigns them to chains
- For a poset of width $k$, Bob can force Alice to use $k(k + 1)/2$ chains. [Felsner 97].
- An online algorithm that uses $O(k^2)$ chains with $O(k^2)$ comparisons per event. [Aggarwal and Garg 05]
Global Predicate Detection

**Predicate:** A global condition expressed using variables on processes (a boolean function on the set of ideals of the poset) e.g., more than one process is in critical section.

**Problem:** find an ideal (a consistent cut) that satisfies the given predicate
The Main Difficulty

Algorithm for general predicate

NP-complete
As many as $O(k^n)$ consistent cuts

$k$: number of events/process, $n$: number of processes
Detecting Linear Predicates

(Linearity): If $B$ is false in $G$ then there exists an event $e$, such that all “true” cuts greater than $G$ include $e$.

$$\neg B(G) \Rightarrow (\exists e \in E - G : \forall H \supseteq G : B(H) \Rightarrow (e \in H))$$

(Advancement Property) can determine the “crucial” event in polynomial time.

Theorem: Any linear predicate that satisfies advancement property can be detected efficiently.

Theorem: [Chase and Garg 95] $B$ is linear iff it is meet-closed.
Relational Predicates

Let $x_i \geq 0$ be variable at $P_i$. Predicates of the form [Groselj 93, Chase and Garg 95]

$$\sum x_i \geq k$$

Algorithm: Consistent cut with minimum value = min cut in the flow graph

![Diagram of flow graph with nodes and edges labeled with values.](image)
**Relational Predicates: Binary Variables**

**Restriction:** \( x_i \in \{0, 1\} \)

**Theorem** Exists an algorithm that merges \( N \) queues into \( N - 1 \) queues in an online fashion. [Tomlinson and Garg 96]
Detecting General Predicates

BFS: 00, 01, 10, 11, 20, 12, 21, 13, 22, 23, 33
DFS: 00, 10, 20, 21, 22, 23, 33, 11, 12, 13, 01
Lexical: 00, 01, 10, 11, 12, 13, 20, 21, 22, 23, 33

Enumerate all consistent cuts (ideals) of the poset
breadth first manner [Cooper and Marzullo 91], depth first manner [Alagar and Venkatesan 94], lexical order [Garg 03].
Talk Outline

- Happened-Before Relation
- Tracking Dependency
- Detecting Global Predicates
- Computation Slicing: Using Birkhoff’s Theorem
Motivation for Computation Slicing

- **computation**
- detect $b_1 \land b_2$
- slicing
- slice for $b_1$
- detect $b_2$
- retain all consistent cuts that satisfy $b_1$
- satisfy $b_1$
Example

Detect predicate \((x_1 * x_2 + x_3 < 5) \land (x_1 \geq 1) \land (x_3 \leq 3)\)

\[(a)\]
\[(b)\]

Slice with respect to \((x_1 \geq 1) \land (x_3 \leq 3)\)
Problem Given \((E, \rightarrow)\), and a global predicate \(B\), give the smallest sublattice containing \(B\).

Application of Birkhoff’s Theorem: The sublattice is distributive and therefore can be represented using its join-irreducible elements.
Results

**Theorem:** Let $L$ be a FDL generated by the graph $P$. For every sublattice $L'$, there exists a graph $P'$ obtained by adding edges to $P$ that generates $L'$.

Efficient algorithms for

- **general predicate:**
  **Theorem:** Given a computation, if a predicate $b$ can be detected efficiently then the slice for $b$ can also be computed efficiently. [Mittal, Sen and Garg 03]

- **linear predicates:** Direct computation of join-irreducibles [Garg and Mittal 01]

- **Combining slices:** Boolean operators

- **Temporal Logic Operators:** EF, AG, EG
Conclusions and Ongoing Work

Applications:

• Tracking Dependency: Chain decomposition, Dimension Theory
• Detecting Global Predicates: Meet-closure, Chain merging, ideal enumeration
• Computation Slicing: Birkhoff’s theorem

Ongoing Work

• Checking Temporal Logic Formulas on Infinite Posets
• Multislice Representation of Predicates
Online Chain Decomposition

- Use $k$ sets of queues $B_1, B_2, ..., B_k$. The set $B_i$ has $i$ queues with the invariant that no head of any queue is comparable to the head of any other queue.

- For a new element $z$, insert it into the first queue $q$ in $B_i$ with its head less than $z$.

- Swap remaining queues in $B_i$ with queues in $B_{i-1}$.
Add the synchronization necessary to maintain safety property

e.g., mutual exclusion

Efficient algorithms for computing the synchronization for:

- **Locks** [Tarafdar, Garg DISC'98]
  - *time-complexity:* $O(nm)$
• **disjunctive predicate** [Mittal, Garg 00]
  e.g., \((n - 1)\)-mutual exclusion
  – *time-complexity*: \(O(m^2)\)
  – minimizes the number of synchronization arrows

• **region predicate** [Mittal, Garg 00]
  e.g., virtual clocks of processes are “approximately” synchronized
  – *time-complexity*: \(O(nm^2)\)
  – maximizes the concurrency in the controlled computation

\(n\): number of processes, \(m\): number of events