Using Order in Distributed Computing

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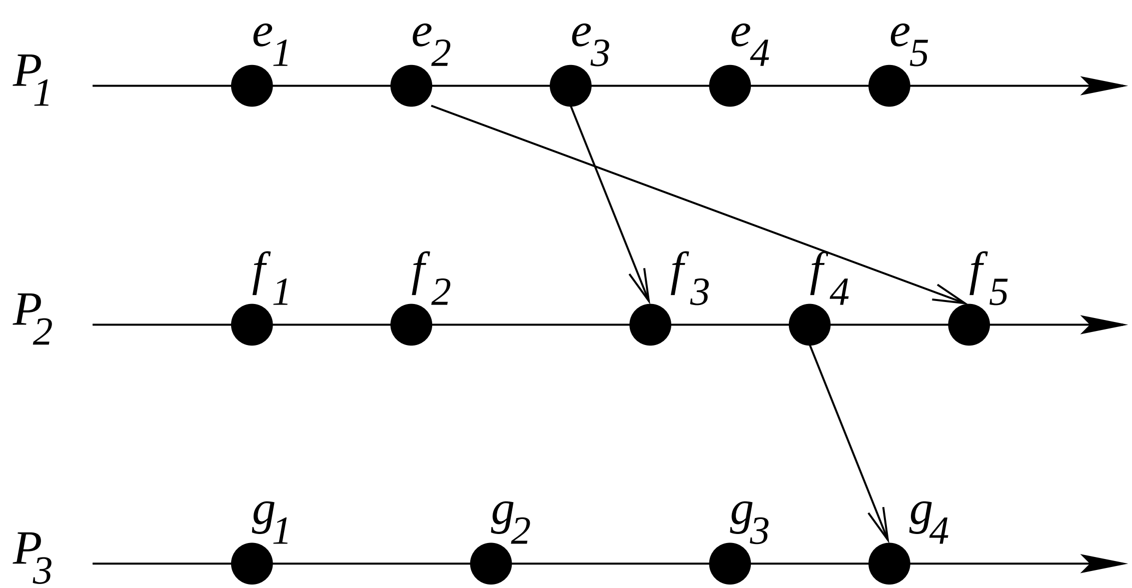
Happened-Before Relation in Distributed Computing

A computation is $(E, \rightarrow)$ where E is the set of events and $\rightarrow$ (happened-before) is the smallest transitive relation that includes:

(1) order within a process

(2) e is a send event and f is the receive implies $e \rightarrow f$.



[Lamport 78]

Talk Outline

- Happened-Before Relation
- Applications
  - Tracking Dependency: Chain decomposition, Dimension Theory
  - Detecting Global Predicates: Meet-closure, Chain merging, ideal enumeration
  - Computation Slicing: Birkhoff's representation theorem

Tracking Dependency

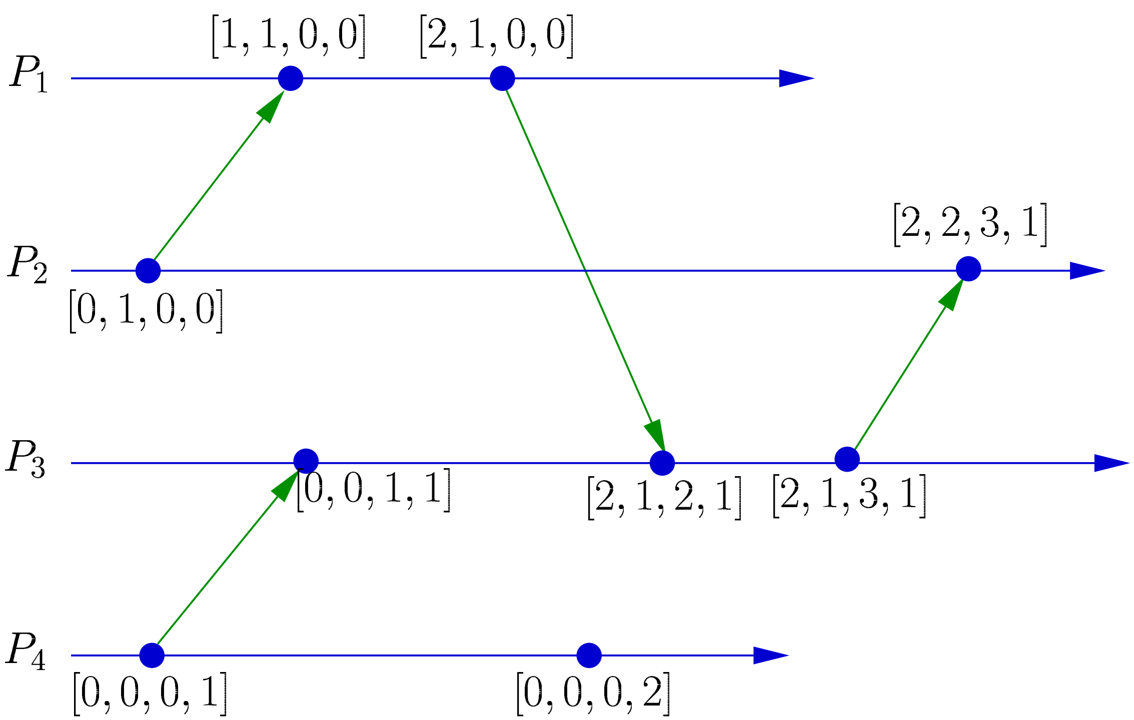
Motivation: Determine whether e happened before f.

Problem: Given $(E, \rightarrow)$, assign timestamps v to events in E such that

$$\forall e, f \in E : e \rightarrow f \equiv v(e) < v(f)$$

Online Timestamps: Vector Clocks [Fidge 89, Mattern 89]:
Every process maintains a vector v of size N, the number of processes.
($v[k]$ at P_i = the number of events executed by P_k as known to P_i).

Vector Clocks in a Distributed System



all events: increment $v[i]$
send events: piggyback v
receive events: combine timestamps
Theorem:

$$e \rightarrow f \equiv v(e) < v(f)$$

Dynamic Chain Clocks

Problem with vector clocks: scalability, dynamic process structure

Idea: Computing the "chains" in an online fashion [Aggarwal and Garg 05] for relevant events

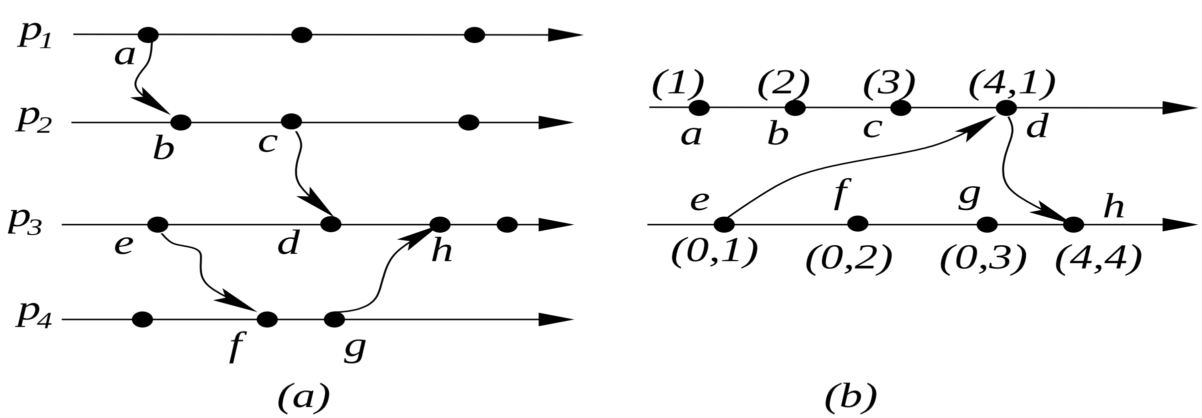
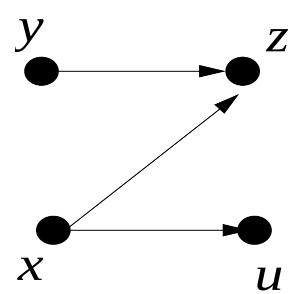


Figure 1: (a) A computation with 4 processes (b) The relevant subcomputation

Online Chain Decomposition

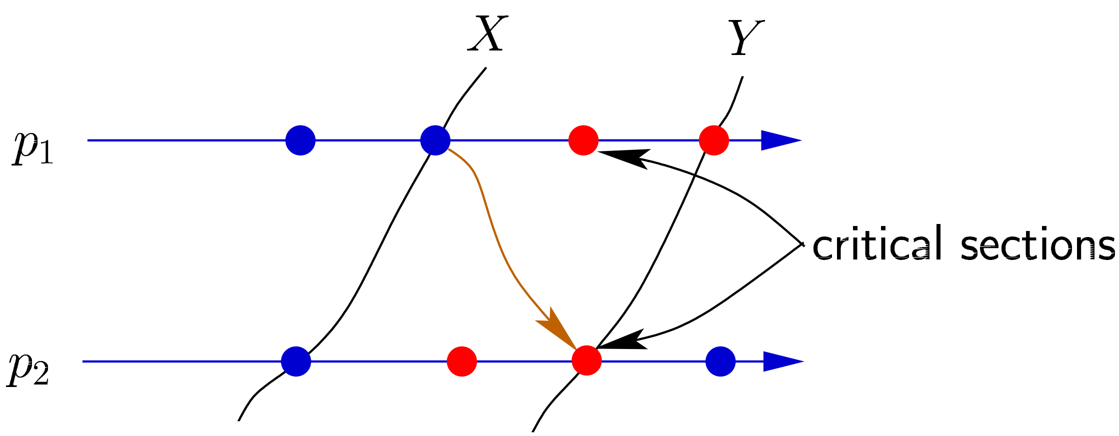
- Elements presented in a total order consistent with the poset
- Assign elements to chains as they arrive
- Game: Bob presents elements, Alice assigns them to chains
- For a poset of width k, Bob can force Alice to use $k(k+1)/2$ chains. [Felsner 97].
- An online algorithm that uses $O(k^2)$ chains with $O(k^2)$ comparisons per event. [Aggarwal and Garg 05]



Global Predicate Detection

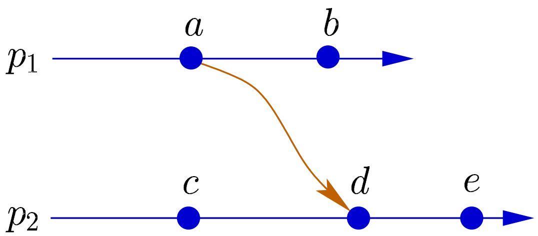
Predicate: A global condition expressed using variables on processes (a boolean function on the set of ideals of the poset) e.g., more than one process is in critical section.

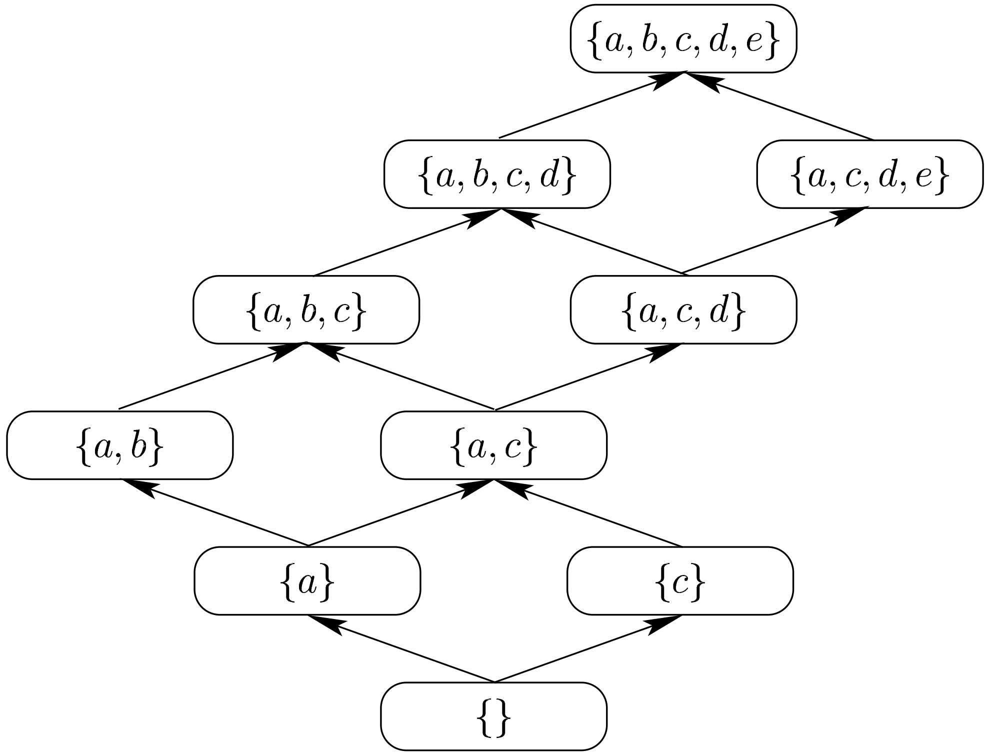
Problem: find an ideal (a consistent cut) that satisfies the given predicate



The Main Difficulty

Algorithm for general predicate

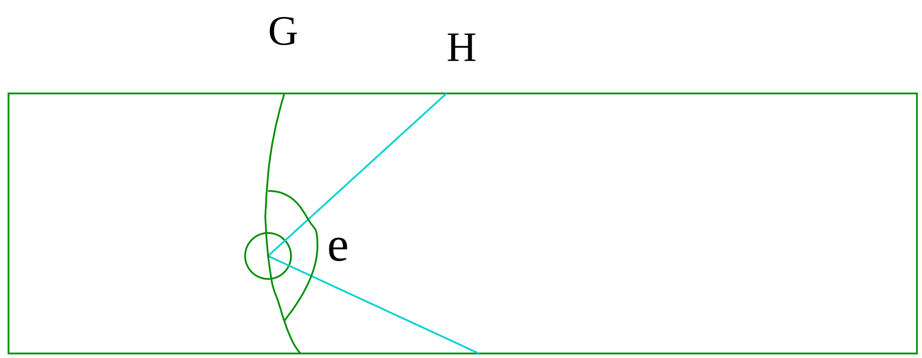




NP-complete
As many as $O(k^n)$ consistent cuts

k: number of events/process, n: number of processes

Detecting Linear Predicates



(Linearity): If B is false in G then there exists an event e, such that all "true" cuts greater than G include e.

$$\neg B(G) \ \Rightarrow \ (\exists e \in E - G : \forall H \supseteq G : B(H) \Rightarrow (e \in H))$$

(Advancement Property) can determine the "crucial" event in polynomial time

Theorem: Any linear predicate that satisfies advancement property can be detected efficiently.

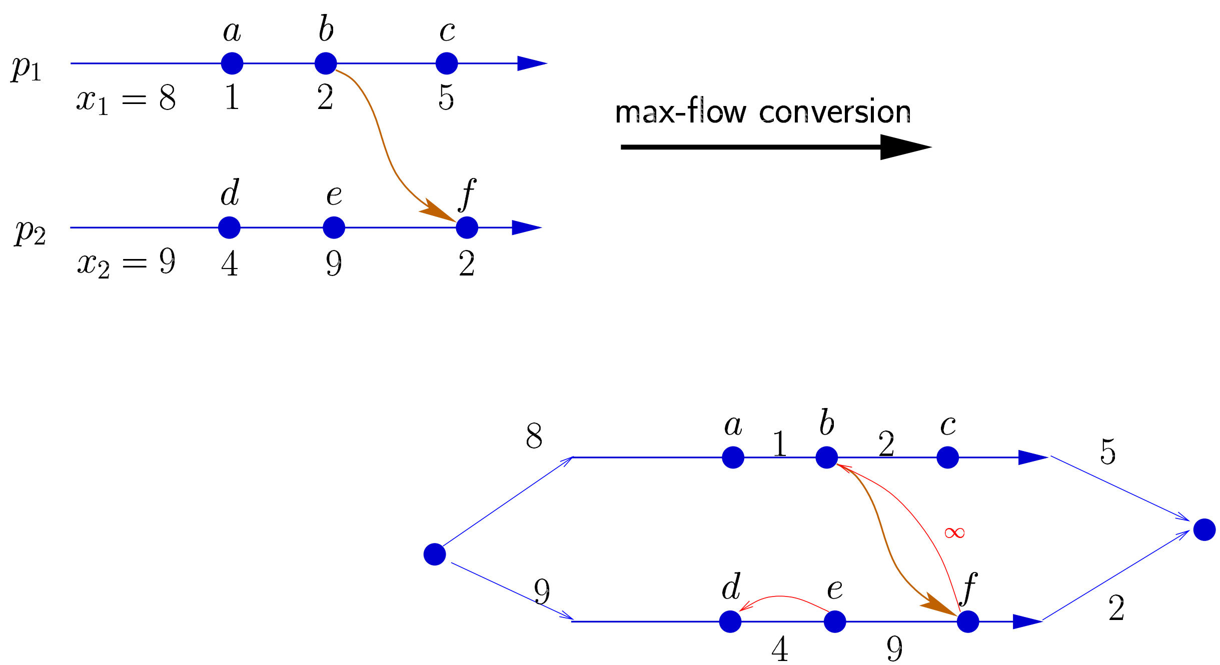
Theorem: [Chase and Garg 95] B is linear iff it is meet-closed.

Relational Predicates

Let $x_i \geq 0$ be variable at P_i. Predicates of the form [Groselj 93, Chase and Garg 95]

$$\Sigma x_i \geq k$$

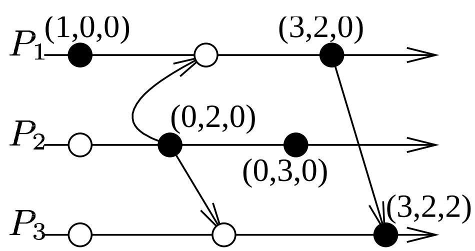
Algorithm: Consistent cut with minimum value = min cut in the flow graph



Relational Predicates: Binary Variables

Restriction: $x_i \in \{0, 1\}$

Theorem Exists an algorithm that merges N queues into $N-1$ queues in an online fashion. [Tomlinson and Garg 96]



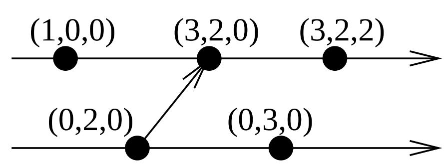
(a)

C_1	C_2	C_3
(1,0,0)	(0,2,0)	(3,2,2)
(3,2,0)	(0,3,0)	

(b)

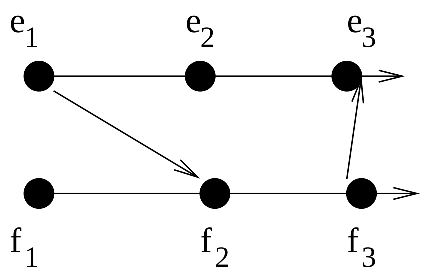
C_1	C_2
(1,0,0)	(0,2,0)
(3,2,0)	(0,3,0)
(3,2,2)	

(c)

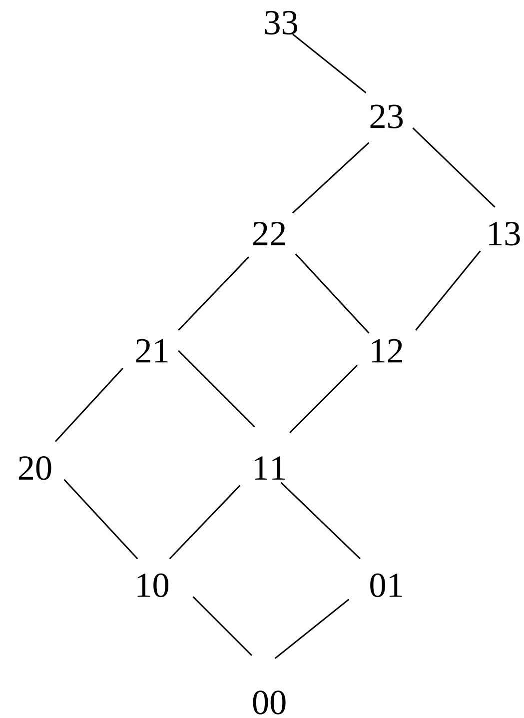


(d)

Detecting General Predicates



(a)



(b)

BFS: 00, 01, 10, 11, 20, 12, 21, 13, 22, 23, 33

DFS: 00, 10, 20, 21, 22, 23, 33, 11, 12, 13, 01

Lexical: 00, 01, 10, 11, 12, 13, 20, 21, 22, 23, 33

(c)

Enumerate all consistent cuts (ideals) of the poset *breadth first manner* [Cooper and Marzullo 91], *depth first manner* [Alagar and Venkatesan 94], lexical order [Garg 03].

Talk Outline

- Happened-Before Relation
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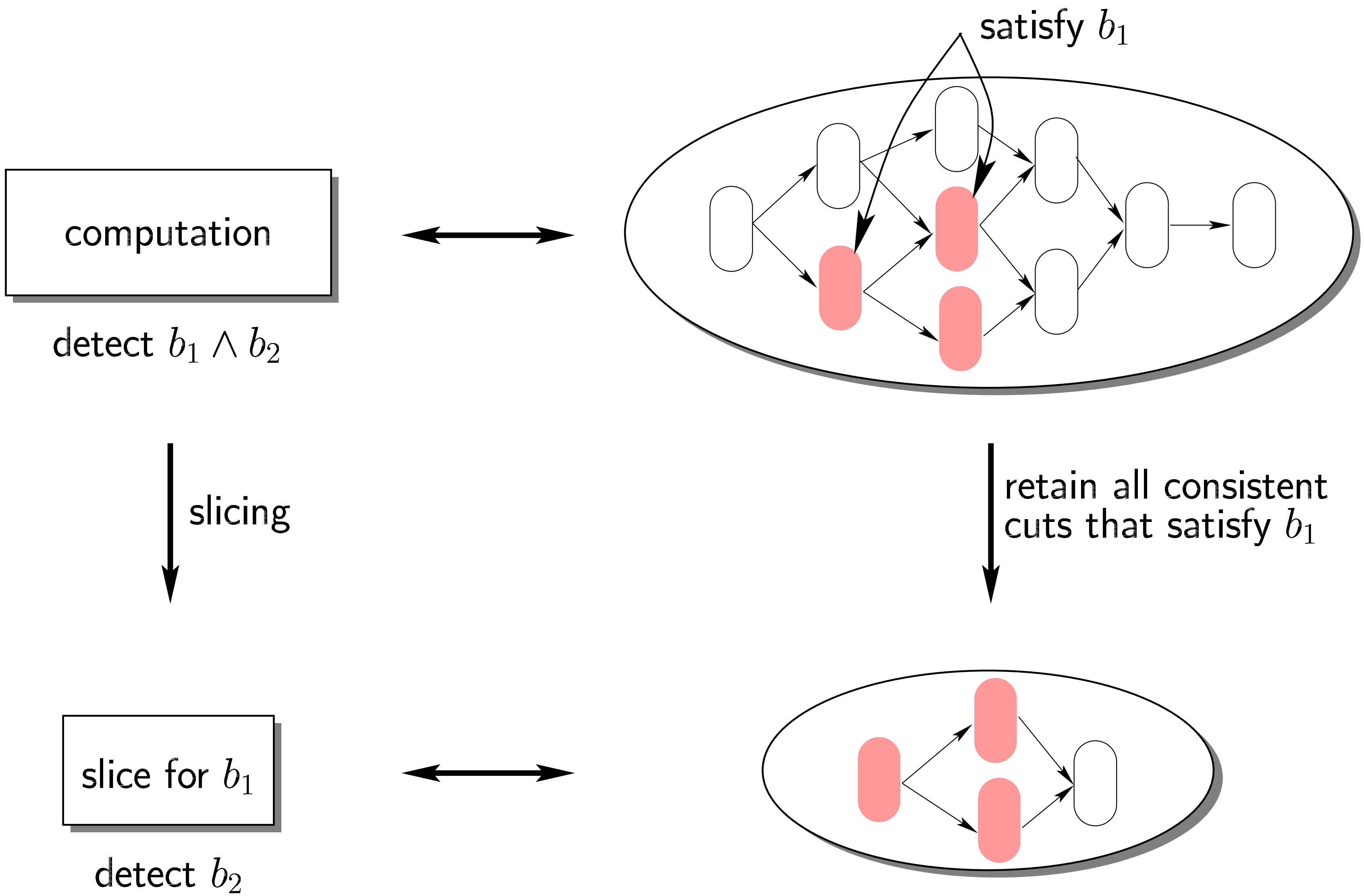
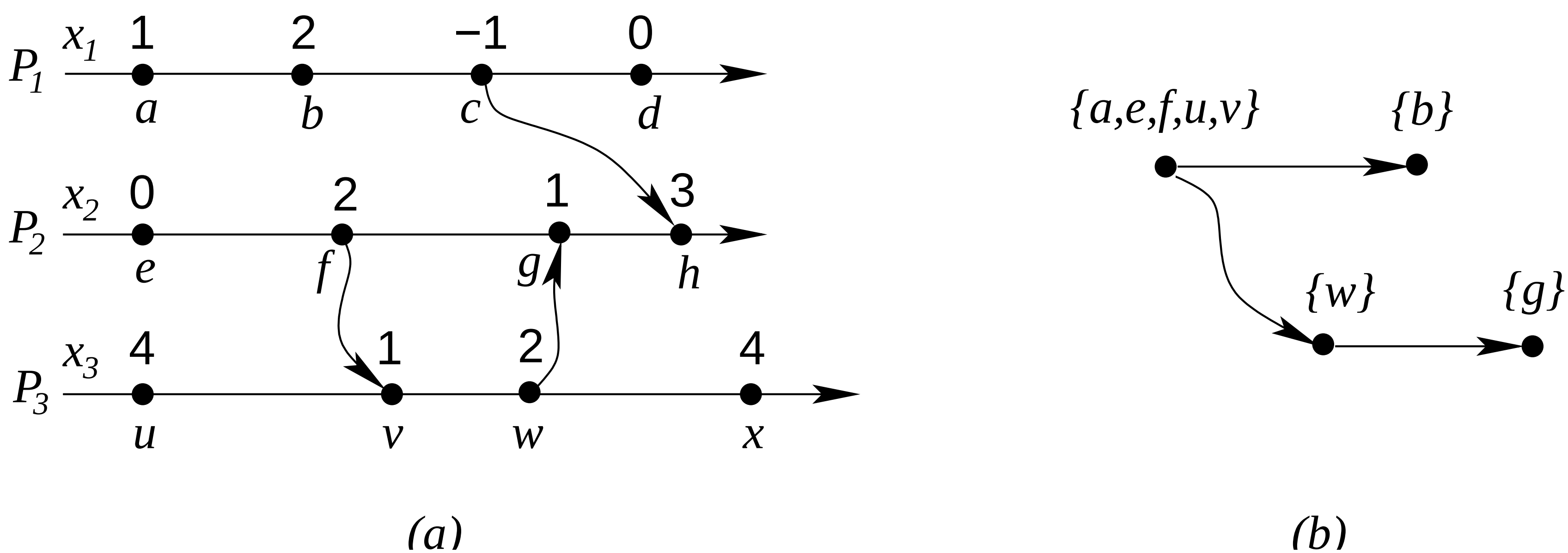
- Detecting Global Predicates
- Computation Slicing: Using Birkhoff's Theorem

Motivation for Computation Slicing



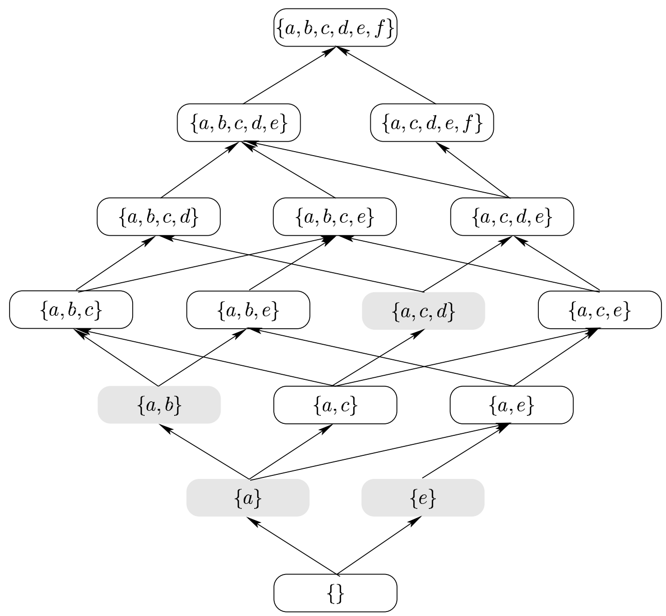
Example

Detect predicate $(x_1 * x_2 + x_3 < 5) \wedge (x_1 \geq 1) \wedge (x_3 \leq 3)$



Slice with respect to $(x_1 \geq 1) \wedge (x_3 \leq 3)$

Computation Slice



Problem Given $(E, \rightarrow)$, and a global predicate B, give the **smallest** sublattice containing B.

Application of Birkhoff's Theorem: The sublattice is distributive and therefore can be represented using its join-irreducible elements.

Results

Theorem: Let L be a FDL generated by the graph P. For every sublattice L', there exists a graph P' obtained by adding edges to P that generates L'.

Efficient algorithms for

- **general predicate:**
  Theorem: Given a computation, if a predicate b can be detected efficiently then the slice for b can also be computed efficiently. [Mittal, Sen and Garg 03]
- **linear predicates:** Direct computation of join-irreducibles [Garg and Mittal 01]
- **Combining slices**: Boolean operators
- **Temporal Logic Operators**: EF, AG, EG

Conclusions and Ongoing Work

Applications:

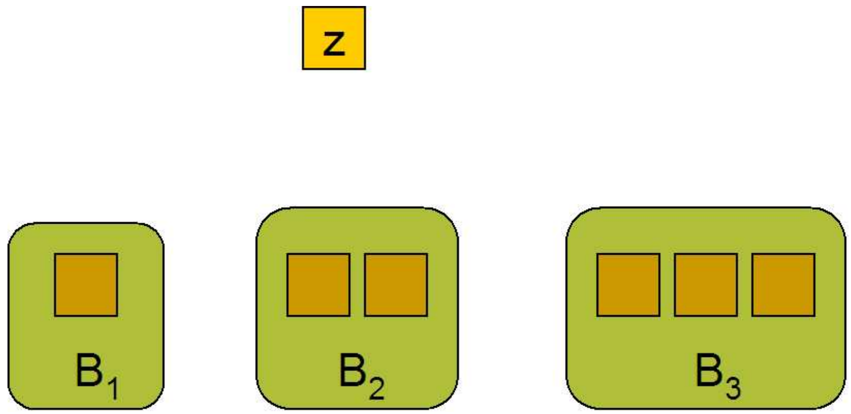
- Tracking Dependency: Chain decomposition, Dimension Theory
- Detecting Global Predicates: Meet-closure, Chain merging, ideal enumeration
- Computation Slicing: Birkhoff's theorem

Ongoing Work

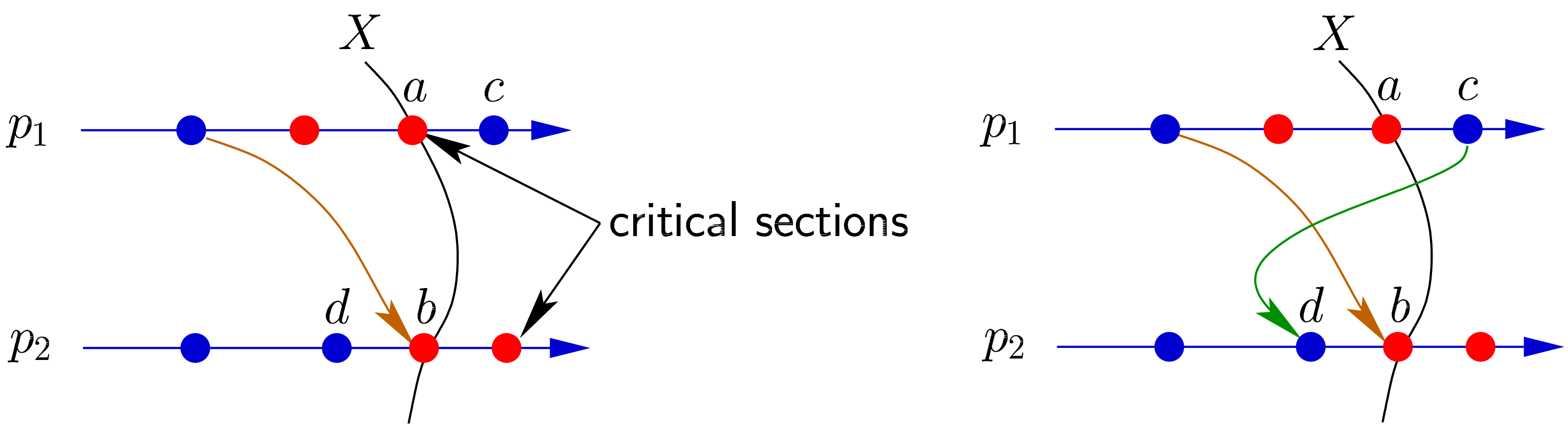
- Checking Temporal Logic Formulas on Infinite Posets
- Multislice Representation of Predicates

Online Chain Decomposition

- Use k sets of queues $B_1, B_2, ..., B_k$. The set B_i has i queues with the invariant that no head of any queue is comparable to the head of any other queue.
- For a new element z, insert it into the first queue q in B_i with its head less than z.
- Swap remaining queues in B_i with queues in B_{i-1}.


z
B1
B2
B3

Controlled Re-execution



Add the synchronization necessary to maintain safety property

e.g., mutual exclusion

Efficient algorithms for computing the synchronization for:

- **Locks** [Tarafdar, Garg DISC98]
  - *time-complexity:* $O(nm)$

- **disjunctive predicate** [Mittal, Garg 00]

  e.g., $(n-1)$-mutual exclusion

  - *time-complexity:* $O(m^2)$
  - minimizes the number of synchronization arrows

- **region predicate** [Mittal, Garg 00]

  e.g., virtual clocks of processes are "approximately" synchronized

  - *time-complexity:* $O(nm^2)$
  - maximizes the concurrency in the controlled computation

n: number of processes, m: number of events