A Lattice-Theoretic Approach to Monitoring Distributed Computations

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Motivation

Debugging and Testing Distributed Programs:

- Global Breakpoints: stop the program when $x_1 + x_2 > x_3$
- Traces need to be analyzed to locate bugs.

Software Fault-Tolerance:

- Distributed programs are prone to errors.
  - Concurrency, nondeterminism, process and channel failures
- Software faults are dominant reasons for system outages
- Need to take corrective action when the current computation violates a *safety* invariant

Software Quality Assurance:

- Can I trust the results of the computation? Does it satisfy all required properties?

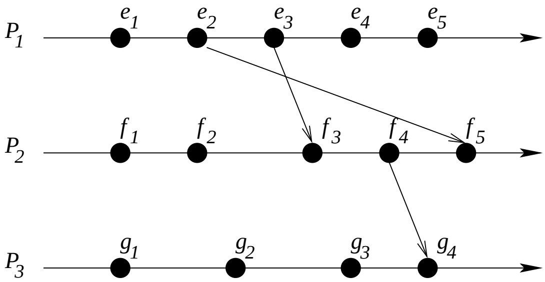
What is a Distributed Computation?

- Distributed Program:
  a computer program that runs on a distributed system
- Distributed Computation:
  A single execution of a distributed program
- Assumptions:
  No shared memory,
  No shared clock,
  Asynchrony in communication

Modeling a Distributed Computation

A computation is $(E, \rightarrow)$ where E is the set of events and $\rightarrow$ (happened-before) is the smallest relation that includes:

- e occurred before f in the same process implies $e \rightarrow f$.
- e is a send event and f the corresponding receive implies $e \rightarrow f$.
- if there exists g such that $e \rightarrow g$ and $g \rightarrow f$, then $e \rightarrow f$.



[Lamport 78]

Modeling a computation as a Poset

$(E, \rightarrow)$ is an irreflexive poset ($\rightarrow$ is an irreflexive and transitive binary relation on E)

Can we exploit the theory of ordered sets?

- join/meet of elements, width of a poset, dimension of a poset, order ideals

Example: Order ideal of a poset corresponds to a consistent global state. The set of all order ideals form a distributive lattice under set containment relation.

Can we exploit the theory of distributive lattices for analyzing consistent global states ?

- representing sublattices, lattice congruences

Talk Outline

Background: Posets

A poset (partially ordered set) is a tuple $(X, \leq)$ where X is any set and $\leq$ is a binary relation on X with the following properties:

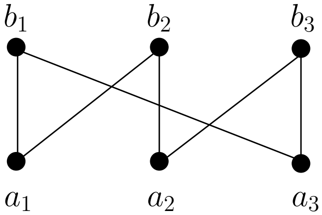
- reflexive,
- antisymmetric and
- transitive

$(X, <)$ is an irreflexive poset when $<$ is irreflexive and transitive.

Examples:

1. $(\mathbb{N}, <)$:
   set of natural numbers under usual less than relation
2. $(\mathbb{N}^k, <)$:
   set of k-dimensional vectors under component-wise comparison
   $(2, 3, 0) < (3, 3, 1)$
   $(2, 3, 0) \not< (1, 4, 2)$
3. $(E, \rightarrow)$:
   set of events of a distributed computation under the happened-before relation

Background: Poset Terminology



- $x||y$ (x incomparable with y): $\neg(x < y) \wedge \neg(y < x)$
- chain: $Y \subseteq X$ is a chain if every distinct pair of elements from Y is comparable
- antichain: $Y \subseteq X$ is an antichain if every distinct pair of elements from Y is incomparable
- height of a poset: size of the longest chain in the poset
- width of a poset: size of the longest antichain in the poset
- width antichain: antichains of size equal to the width

Background: Lattices

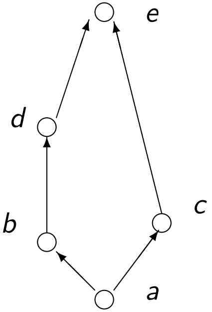
For any $z \in X$, z is the join of x and y, i.e., $z = x \sqcup y$ iff

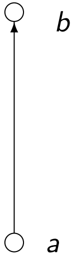
- $x \leq z$ and $y \leq z$
- $\forall z' \in X, (x \leq z' \wedge y \leq z') \Rightarrow z \leq z'$.

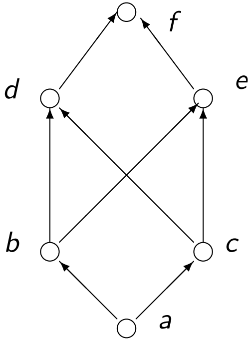
The meet of two elements $z = x \sqcap y$ is defined dually.

A poset $(X, \leq)$ is a lattice iff it is closed under meets and joins.

$\forall x, y \in X$, $x \sqcup y \in X$ and $x \sqcap y \in X$.

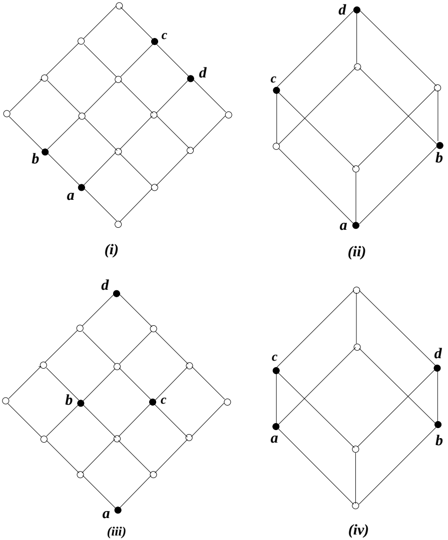






Background: Sublattices

Which subsets form sublattices?

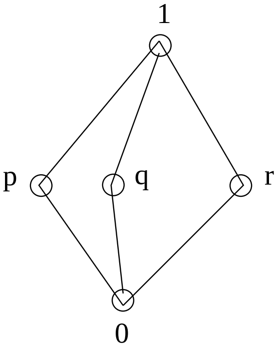


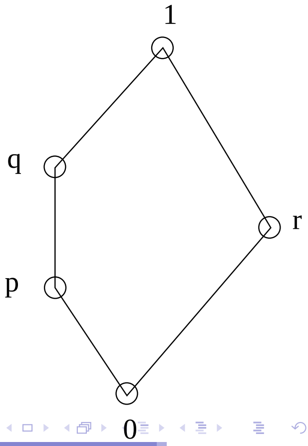
Background: Distributive Lattices

A lattice $(L, \leq)$ is a distributive lattice iff
$\forall x, y, z \in L : x \sqcup (y \sqcap z) = (x \sqcup y) \sqcap (x \sqcup z)$.

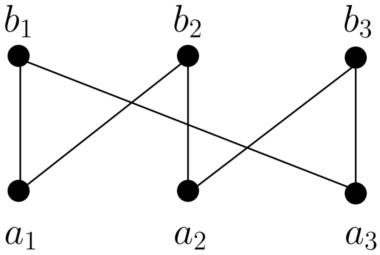
Fact

- A lattice is distributive iff it does not have a pentagon or a diamond as a sublattice.





Background: Order Ideals of a Poset



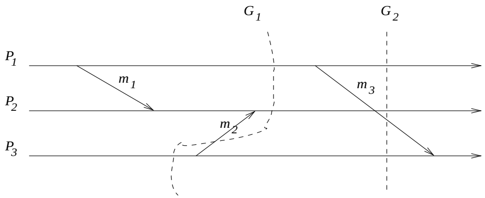
Let $(X, <)$ be any poset. A subset $Y \subseteq X$ an order ideal (or a downset) if

$$z \in Y \wedge y < z \Rightarrow y \in Y.$$

Are these order ideals?
$Y_1 = \{a_1, b_1\}$
$Y_2 = \{a_1, a_3, b_1\}$
$Y_3 = \{\}$
$Y_4 = X$
$Y \subseteq X$ an order filter if $z \in Y \wedge z < y \Rightarrow y \in Y$

Example: Order Ideals

Consistent Global State (CGS) of a Distributed System



Consistent global state = subset of events executed so far

A subset G of E is a consistent global state (also called a consistent cut) if

$\forall e, f \in E : (f \in G) \wedge (e \rightarrow f) \Rightarrow (e \in G)$

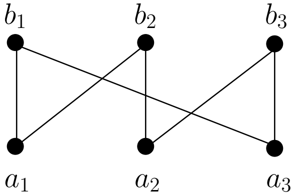
Background: Lattice of order ideals

Theorem

The set of all order ideals of any poset forms a distributive lattice *under the set containment relation.*

The set of ideals forms a lattice

- if X and Y are ideals then so are $X \cap Y$ and $X \cup Y$
  meet $\rightarrow$ intersection join $\rightarrow$ union



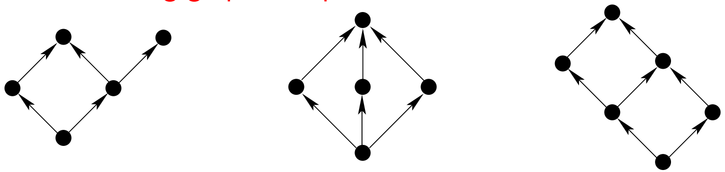
$Y_1 = \{a_1, a_3, b_1\}$
$Y_2 = \{a_1, a_2, b_2\}$
$Y_1 \cup Y_2 = \{a_1, a_2, a_3, b_1, b_2\}$
$Y_1 \cap Y_2 = \{a_1\}$

Ideal Lattice

The lattice of ideals is distributive

- union distributes over intersection

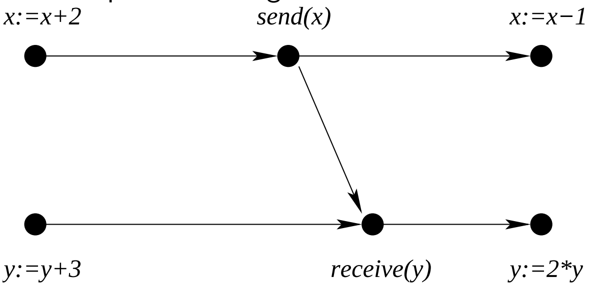
which of the following graphs are possible CGS lattices?



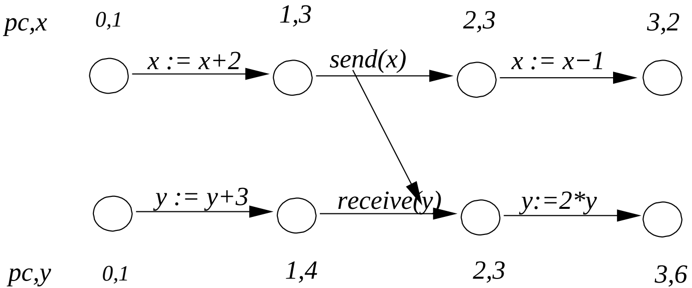
Corollary: The set of all CGS of a computation forms a distributive lattice.

Modeling using States vs Events

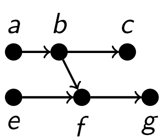
One can model a computation using states rather than events



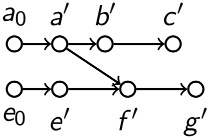
Equivalent state based model



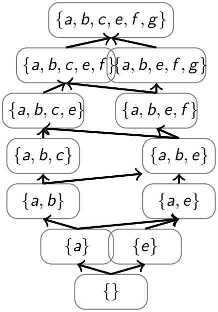
Consistent Global States in the State based Model



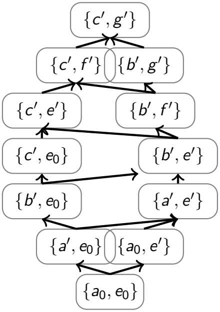
(a) Event Based Model



(b) State Based Model



(c) CGS



(d) CGS

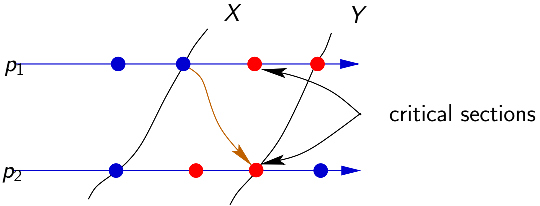
Talk Outline

Global Predicate Detection

Predicate: A global condition expressed using variables on processes

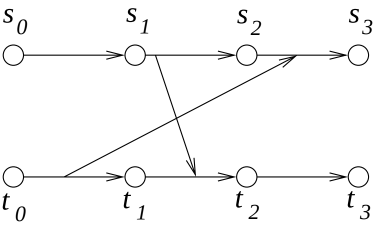
e.g., more than one process is in critical section,
there is no token in the system

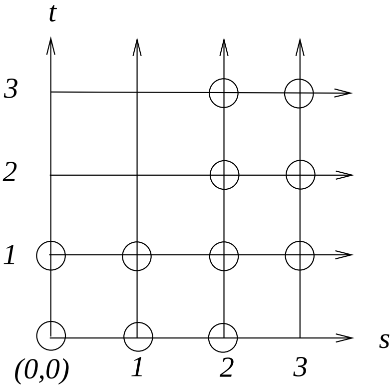
Problem: find a consistent cut that satisfies the given predicate



The global predicate may express: a software fault or a global breakpoint

Two interpretations of predicates





Possibly:Φ: exists a path from the initial state to the final state along which Φ is true on some state

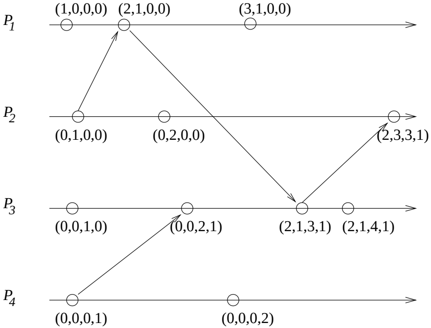
Definitely:Φ : for all paths from the initial state to the final state Φ is true on some state

Detecting Possibly:Φ

- Centralized Checker Process
- Send relevant events to the checker process
- Include dependency information for events
- Checker process enumerates consistent global states

Tracking Dependency

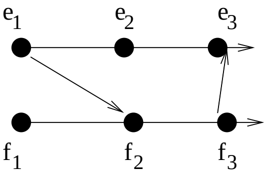
Problem: Given $(E, \rightarrow)$, assign timestamps v to events in E such that $\forall e, f \in E : e \rightarrow f \equiv v(e) < v(f)$



Online Timestamps: Vector Clocks [Fidge 89, Mattern 89]:

- all events: increment $v[i]$ after each event
- send events: piggyback v with the outgoing message
- receive events: compute the max with the received timestamp

Detecting *Possibly* : *B* — Enumeration of Consistent Global States

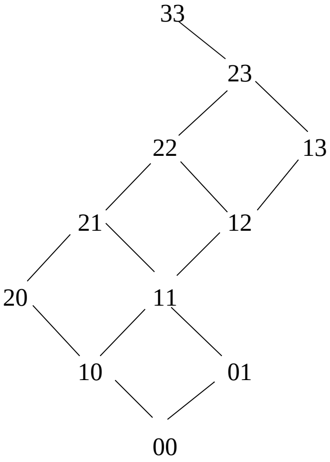


(a)

BFS: 00, 01, 10, 11, 20, 12, 21, 13, 22, 23, 33

DFS: 00, 10, 20, 21, 22, 23, 33, 11, 12, 13, 01

(c)



(b)

Challenges for Lattice Enumeration

- The number of CGS is exponential in the number of processes
- The lattice cannot be stored in the main memory
- What if the poset is infinite?

Cooper and Marzullo's Algorithm

[Cooper and Marzullo 91]

Implicit BFS Traversal

current: list of the global states at the current level.

Initially, *current* has only one global state, the initial global state

repeat

    enumerate *current*;

    last := *current*;

    current = global states reached from *last* in one step;

until (*current* is empty)

Cooper and Marzullo's Algorithm

[Cooper and Marzullo 91]

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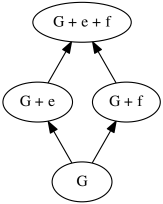
until (*current* is empty)

Problems:

Repeated Enumeration: a CGS can be reached from multiple global states.

Space Complexity: need to store a level of the lattice – exponential in the number of processes

Avoiding Repeated Enumeration



Idea: explore events only in a sorted order

an event e is explored from a global state G iff e is bigger than all the events in G

Revised BFS Algorithm

Q: set of CGS at the current level initially $\{(0, 0, \dots, 0)\}$;
σ: a topological sort of all events in $(E, \rightarrow)$

while $(Q \neq \emptyset)$ do
    $G := remove\_first(Q)$;
    for all events e enabled in G do // generate CGS at the next level
        if $(\forall f \in maximal(G) : \sigma(f) < \sigma(e))$ then
            $H := G \cup \{e\}$;
            $append(Q, H)$;
    endfor;
endwhile;

Time : $O(n^2 M)$ Space complexity: $O(n w_L)$
n: number of processes M: number of CGS
w_L: width of the lattice L.
Problem: w_L is exponential in n in the worst case.

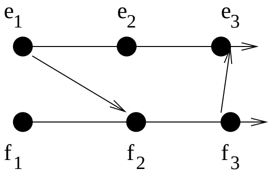
Implicit Depth First Search

Idea: Instead of storing width, store the height of the lattice
Use implicit Depth-First-Search [Alagar and Venkaesan 94]

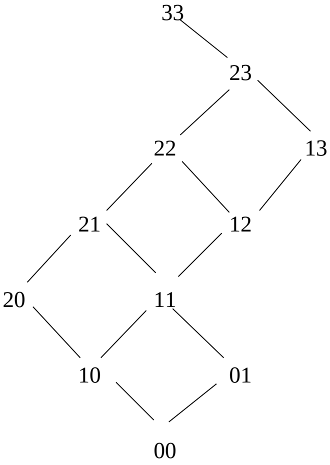
```
 G: array[1..n] of integer; // current global state
 pred: array[1..n] of integer; // predecessor info for the next event
function dfsTraversal(int k) //event e at P_k enabled in the current state
      G[k]++;
      enumerate(G);
      compute pred[k] using the vector clock for e
      forall (j ≠ k): if (e_j depends on e) then pred[j]--;
      forall (j) with next event e_j
           if (pred[j] = 0) and σ(e) < σ(e_j))
                dfsTraversal(j);
      restore values of G and pred;
end;
```

Problem: Stack can grow to $O(E)$ the number of events in the computation

Lexical Enumeration of Consistent Global States



(a)



(b)

BFS: 00, 01, 10, 11, 20, 12, 21, 13, 22, 23, 33

DFS: 00, 10, 20, 21, 22, 23, 33, 11, 12, 13, 01

Lexical: 00, 01, 10, 11, 12, 13, 20, 21, 22, 23, 33

(c)

Lexical Order

$G <_l H$ iff

$$\exists k : (\forall i : 1 \leq i \leq k-1 : G[i] = H[i]) \wedge (G[k] < H[k]).$$

Lemma

$\forall G, H : G \subseteq H \;\Rightarrow\; G \leq_l H.$

Algorithm for Lex Order

nextLex(G): next consistent global state in lexical order

```
var
    G : consistent global state initially (0, 0, ..., 0);
enumerate(G);
while (G < ⊤)
    G := nextLex(G);
    enumerate(G);
endwhile ;
```

No intermediate consistent global nodes stored

Computing next consistent global state in lexical order

Lemma

Given any global state K (possibly inconsistent), the set of all consistent global states that are greater than or equal to K in the CGS lattice is a sublattice.

Corollary

- There exists a minimum consistent global state H that is greater than or equal to a given global state K.

Notation

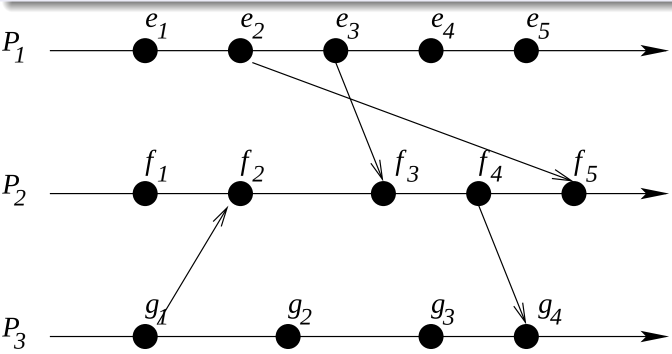
- $succ(G, k)$: advance along P_k and reset components for P_i $(i > k)$ to 0.
  e.g. $succ(\langle 7, 5, 8, 4\rangle, 2) = \langle 7, 6, 0, 0\rangle$
  $succ\langle 7, 5, 8, 4\rangle, 3)$ is $\langle 7, 5, 9, 0\rangle$.
- $leastConsistent(K)$: the least consistent global state greater than or equal to a given global state K in the $\subseteq$ order.

Computation of $nextLex(G)$

Theorem

$$nextLex(G) = leastConsistent(succ(G, k))$$

where k is the index of the process with the smallest priority which has an event enabled in G.



Example: Let $G = (4, 3, 3)$. Then $k = 2$, $succ(G, k) = (4, 4, 0)$
Therefore, $nextLex(G) = (4, 4, 1)$.

Algorithm for Lex Order

nextLex(G): next consistent global state in lexical order

var

G : consistent global state initially $(0, 0, ..., 0)$;

enumerate(G);

while $(G < \top)$

$k :=$ smallest priority process with an event enabled in G

$G := leastConsistent(succ(G, k))$

enumerate(G);

endwhile ;

k, $succ(G, k)$ and $leastConsistent()$ can be computed in $O(n^2)$ time using vector clocks.

[Garg03]

Parallel and Online Algorithms

Partition the lattice into multiple interval sublattices

Assume that events arrive in a total order σ consistent with $\rightarrow$.

for every event e

- $G_{min}(e) =$ smallest consistent global state that contains e
- $G_{bnd}(e) = \{f | \sigma(f) \leq \sigma(e)\}$

Theorem[Chang and Garg 14]: Consider the set of all interval lattices, $I(e)$, $\{G | G_{min}(e) \subseteq G \subseteq G_{bnd}(e)\}$.

These interval lattices are mutually disjoint and cover the entire lattice of all consistent global states.

ParaMount: A parallel implementation for detecting predicates in concurrent systems [Chang and Garg 14]

Talk Outline

Predicate Detection for Special Cases

Exploit the structure/properties of the predicate

- stable predicate: [Chandy and Lamport 85]

  once the predicate becomes true, it stays true

  e.g., deadlock

- observer independent predicate [Charron-Bost *et al* 95]

  occurs in one interleaving $\Longrightarrow$ occurs in all interleavings

  e.g., stable predicates, disjunction of local predicates

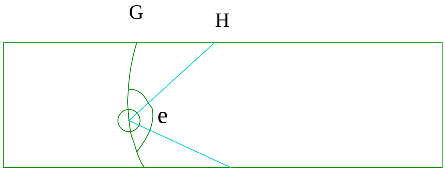
- linear predicate [Chase and Garg 95]

  closed under meet, e.g., there is no leader in the system

- relational predicate: $x_1 + x_2 + \cdots + x_n \geqslant k$ [Chase and Garg 95] [Tomlinson and Garg 96]

  e.g., violation of k-mutual exclusion

Linearity



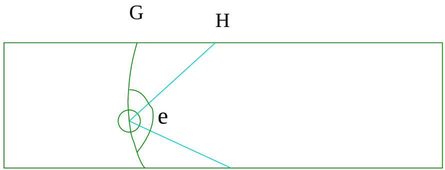
Crucial Element $crucial(G, e, B)$
For a consistent cut $G \subsetneq E$ and a predicate B, $e \in E - G$ is crucial for G if:

$$\forall H \supseteq G : (e \in H) \vee \neg B(H).$$

Linear Predicates A predicate B is linear if for all consistent cuts $G \subsetneq E$,

$$\neg B(G) \;\Rightarrow\; \exists e \in E - G : crucial(G, e, B).$$

Examples of Linear Predicates: Conjunctive Predicates



- mutual exclusion problem: (P1 in CS) and (P2 in CS)
- missing primary: (P1 is secondary) and (P2 is secondary) and (P3 is secondary)

Channel Predicates: Observing hallways

Many properties require channels

Example: termination detection – all processes are idle and all channels are empty

Channel predicate: boolean function on the state of the unidirectional channel

channel state : sequence of messages sent - set of messages received

Linearity: Given any channel state in which the predicate is false, either

- the next event at the receiver is crucial, or
- the next event at the sender is crucial

Linear Channel Predicates

- Empty channels

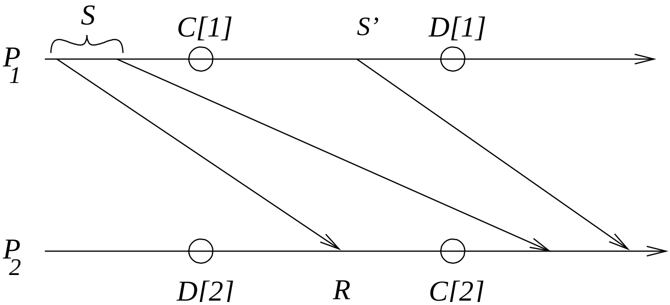
  If false, then it cannot be made true by sending more messages.
  The next event at the receiver is crucial.
- Channel has more than three red messages

  The next event at the sender is crucial.
- Channel has exactly three red messages

  If less than three, the next event at the sender is crucial,
  If more than three, the next event at the receiver is crucial

Non-linear Channel Predicates

$B \equiv$Channel has an odd number of messages



The set of cuts satisfying the predicate is not linear.

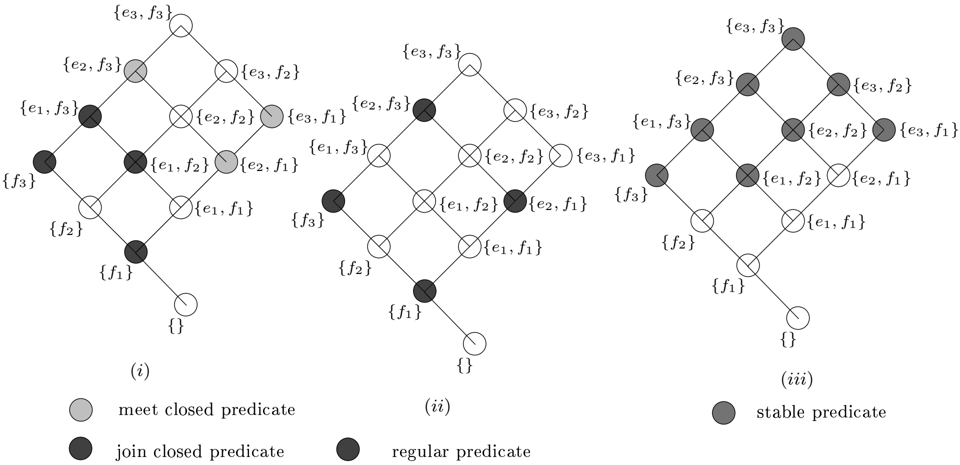
Linearity of Predicates and Meet-Closure

Theorem: [Chase and Garg 95] A predicate B is linear if and only if it is meet-closed (in the lattice of all consistent cuts).

Special Classes of Predicates

- a predicate P is meet-closed if all the cuts that satisfy the predicate are closed under intersection. $(C_1 \models P \wedge C_2 \models P) \Rightarrow (C_1 \cap C_2) \models P$.
- A predicate P is join-closed if all cuts that satisfy the predicate are closed under union.
  i.e., $(C_1 \models P \wedge C_2 \models P) \Rightarrow (C_1 \cup C_2) \models P$.
- A predicate is regular if it is join-closed and meet-closed.
- A predicate P is stable, if
  $\forall C_1, C_2 \in L : C_1 \models P \wedge C_1 \subseteq C_2 \Rightarrow C_2 \models P$.

Example: Special Classes of Predicates



Detecting Linear Predicates

(Advancement Property) There exists an efficient (polynomial time) function to determine the crucial event.

Theorem: Any linear predicate that satisfies advancement property can be detected efficiently.

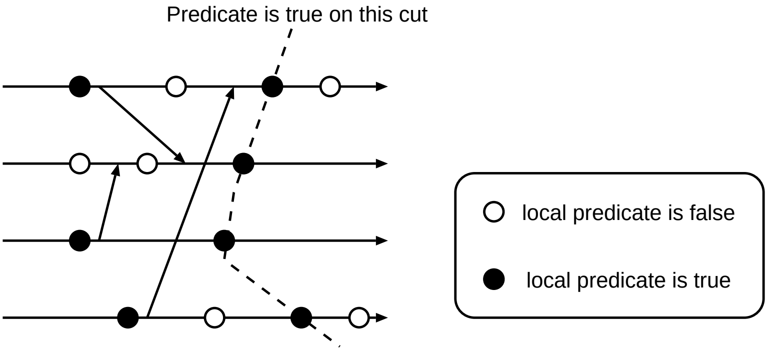
Example: A conjunctive predicate, $l_1 \wedge l_2 \wedge \ldots \wedge l_n$, where l_i is local to P_i.

Importance of Conjunctive Predicates

Sufficient for detection of the following global predicates

- boolean expression of local predicates which can be expressed as a disjunction of a small number of conjunctions.
  Example: x, y and z are in three different processes. Then,
  $even(x) \wedge ((y < 0) \vee (z > 6))$
  $\equiv$
  $(even(x) \wedge (y < 0)) \vee \, (even(x) \wedge (z > 6))$
- predicate satisfied by only a small number of values
  Example: x and y are in different processes.
  $(x = y)$ is not a *local* predicate but x and y are binary.

Conditions for Conjunctive Predicates



Possibly

$(l_1 \wedge l_2 \wedge \ldots l_n)$ is true iff there exist s_i in P_i such that l_i is true in state s_i, and s_i and s_j are incomparable for distinct i, j.

Weak Conjunctive Predicates: Centralized Algorithm

Each non-checker process maintains its local *vector*
send the vector clock to the checker process whenever

- local predicate is true
- at most once in each message interval.

Optimization: Sufficient to send the vector once after any message is sent
Space complexity: $O(n)$
message complexity: $O(m_s)$, m_s = number of program messages sent.
Time complexity: detection of local predicates, maintain vector clock $O(n)$

[Garg and Waldecker 94]

Checker Process

n queues of vectors

Steps

1. Begin with the initial global state
2. Eliminate any vector that happened before any other vector along the current global state.

Predicate is true for the first time

- all vectors are pairwise concurrent

Predicate is false

- if we eliminate the final vector from any process

Overhead: Checker processes

Space complexity

- n queues, each containing at most m vectors

Time complexity

- The algorithm for checker requires at most $O(n^2 m)$ comparisons.
- Any algorithm which determines whether there exists a set of incomparable vectors of size n in n chains of size at most m, makes at least $mn(n-1)/2$ comparisons.

Disadvantages of above algorithm

Centralized

- Checker process may become a bottleneck

Space requirements

- Queues at the checker process may grow large

Message complexity

- many additional messages to the checker process

Token-based Algorithm

A monitor process is active only if it has the token. Token consists of two vectors G and *color*.

G: global state vector

- $G[i] = k$ indicates that state (i, k) is part of the current cut.

color: indicates which states have been eliminated.

- If $color[i] = red$ then state $(i, G[i])$ has been eliminated and can never satisfy the global predicate.
- If $color[i] = green$, then there is no state in G such that $(i, G[i])$ happened before that state.

Monitor Process Algorithm: M_i

```
var     candidate:array[1..n] of integer;
on receiving the token (G,color)
    while (color[i] = red) do
        receive candidate from application process P_i
        if (candidate.vclock[i] > G[i]) then
            G[i] := candidate.vclock[i]; color[i]:=green;
    endwhile
    for j ≠ i:
        if (candidate.vclock[j] > G[j]) then
            G[j] := candidate.vclock[j];
            color[j]:=red;
        endif
    endfor
    if (∃ j: color[j] = red) then send token to P_j
    else detect := true;
```

Analysis of Single-Token WCP Algorithm

Theorem

For any computation $(E, \rightarrow)$ and any conjunctive predicate B, if B holds in $(E, \rightarrow)$, then the Single-Token WCP algorithm returns the least CGS that satisfies B. If B is false, then the algorithm returns false.

Work complexity: $O(n^2 m)$
Every time a state is eliminated, $O(n)$ work is performed. There are at most mn states.
Message complexity: $O(mn)$.
Communication bit complexity: $O(n^2 m)$.
  size of both the token and the candidate messages is $O(n)$.
Space complexity: $O(mn)$ space per process.
m: maximum number of vectors per process, n: number of processes

Other WCP algorithms

A completely distributed algorithm [Chase and Garg 94]

- Uses Dijkstra and Scholten's termination detection algorithm

Keeping queues shorter [Chiou and Korfhage 95]

- eliminate vectors that are useless

Avoiding control messages [Hurfin, Mizuno et al 96]

- piggyback info/token with application messages

Talk Outline

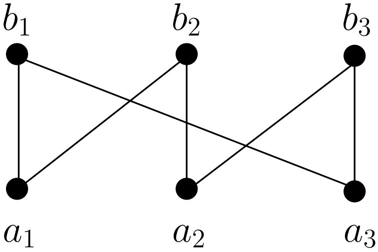
Relational Predicates: Binary Variables

Problem: Given $(S, \to)$
$B \equiv x_1 + x_2 + x_3 \ldots x_n \geq k$
where x_i resides on process P_i.
Example:
x_i: P_i is using the shared resource.
Are there k or more processes using the resource concurrently?
Equivalent Problem: Is there an antichain $H \subseteq S$ such that the size of H it at least k and x is true on local states in H.

[Tomlinson and Garg 96]

Using Dilworth's Theorem

Dilworth's Chain Partition Theorem: For any poset $(X, \leq)$,
size of a maximum sized antichain (width)
$=$
the minimum number of chains that covers the poset



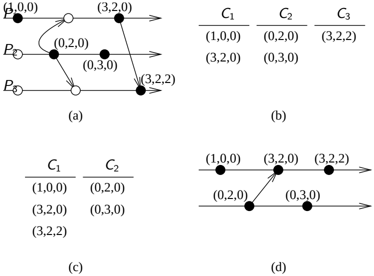
k queues of vector clocks can be merged into $k-1$ queues iff there is no antichain of size k.

Relational Predicate Algorithm

Input: n queues of vector clocks;
Output: true iff $\sum_i x_i \geq k)$
for $i := 1$ to $n - k + 1$ do
    pick smallest k chains and merge them into $k - 1$ chains;
    if not possible then
        found an antichain of size k;
        return true;//the antichain $=$ CGS where the predicate holds
endfor;
return false;// only $k - 1$ chains left

Generalized Merging

Theorem: Let the poset be presented as k queues of vector clocks. There exists an efficient algorithm that can merge N queues into $N-1$ queues in an online fashion whenever possible.
[Tomlinson and Garg 96]



How to merge queues of vectors?

P_1	P_2	P_3
a:(1,0,0)	d:(0,1,0)	f:(2,0,0)
b:(1,1,0)	e:(2,2,0)	g:(2,3,0)
c:(1,2,0)		

Naive Strategy

Move a minimal element into any output queue in which it can be inserted.
After insertion of a, d, b, c:

Q_1	Q_2
a:(1,0,0)	d:(0,1,0)
b:(1,1,0)	
c:(1,2,0)	

P_1	P_2	P_3
		f:(2,0,0)
	e:(2,2,0)	g:(2,3,0)

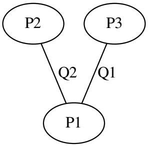
Merge is Possible

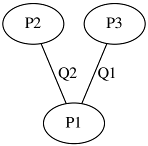
Q_1	Q_2
a:(1,0,0)	d:(0,1,0)
f:(2,0,0)	b(1,1,0)
e:(2,2,0)	c:(1,2,0)
g:(2,3,0)	

$b:(1,1,0)$ is inserted in Q_2 and not Q_1.

Queue Insert Graph

$G = (V, E)$: undirected graph called queue insert graph
V: set of k input queues
E: undirected edges on V
Invariant 1: G is a spanning tree
$\Rightarrow$ there are exactly $k - 1$ edges in G. Each edge is labeled with a unique output queue
Invariant 2: Let (P_i, P_j) be labeled with Q_k
All elements of P_i and P_j are bigger than all elements of Q_k $\Rightarrow$ Any element from P_i or P_j can be inserted at the tail of Q_k.





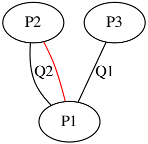
Q_1	Q_2
a:(1,0,0)	d:(0,1,0)

P_1	P_2	P_3
b:(1,1,0)	e:(2,2,0)	f:(2,0,0)
c:(1,2,0)		g:(2,3,0)

Using Queue Insert Graph

$b : (1, 1, 0) \in P_1 < e : (2, 2, 0) \in P_2$
delete $b : (1, 1, 0)$ from P_1 and insert in an output queue.
Which one?

1. Add an edge between P_i and P_j in the spanning tree.
2. A unique cycle is formed. Let (P_i, P_k) be the other edge incident on P_i in that cycle.
3. Remove (P_i, P_k). Transfer its label to (P_i, P_j) and insert the vector in the corresponding output queue.



Verify: Queue Insert Graph invariant is preserved.

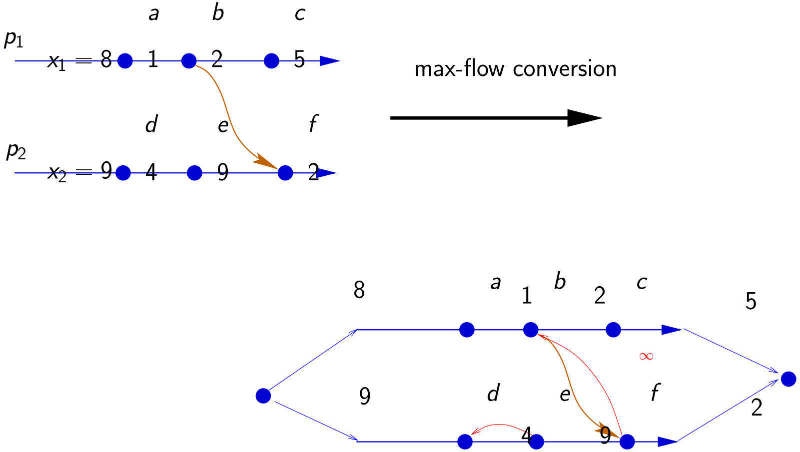
Relational Predicates: Nonbinary Variables

Let x_i: number of tokens at P_i

$\Sigma x_i < k$: loss of tokens

Algorithm: max-flow technique [Groselj 93, Chase and Garg 95],

Consistent cut with minimum value = min cut in the flow graph



Talk Outline

1. Motivation
2. Background: Posets and Lattices
3. Global Predicate Detection Problem
   - Cooper and Marzullo's Algorithm
   - Alagar and Venkatesan's Algorithm
   - Lexical Enumeration of Consistent Global States
4. Predicate Detection for Special Classes
   - Linear Predicates
   - Relational Predicates
5. Slicing
6. Basis Temporal Logic
   - Syntax and Semantics
   - Semiregular Predicates
   - Algorithm to detect BTL

Summary

- Space efficient algorithms for general predicates
- Time efficient algorithms for special classes of predicates

Problem: What if the predicate does not belong to one of the special classes?

Talk Outline

Basis Temporal Logic: Motivation

RCTL can handle only regular predicates. Even a simple formula such as $p \vee q$ is not regular.
Need for a logic:

- Sufficiently expressive
- Easy to write formulas in that logic
- Can detect them with polynomial time complexity
  polynomial in the number of processes, not the size of the formula

Basis Temporal Logic: Syntax

AP: Set of Atomic Propositions
Atomic Propositions are evaluated on a single global state.
A predicate in BTL is defined recursively as follows:

1. $\forall l \in AP$, l is a BTL predicate
2. If P and Q are BTL predicates then $P \vee Q$, $P \wedge Q$, $\Diamond P$ and $\neg P$ are also BTL predicates

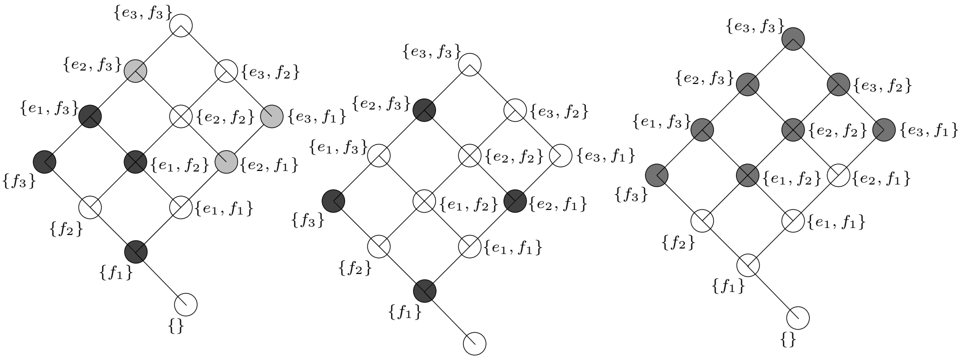
Example: $B = \neg\Diamond(\bigwedge red_i) \wedge token_0$
[Ogale and Garg 07]

Basis Temporal Logic: Semantics

$E, \rightarrow$: Poset (distributed computation)
L: Lattice of consistent global states of $(E, \rightarrow)$
C: A consistent global state of $(E, \rightarrow)$
$\lambda : L \rightarrow 2^{AP}$ set of atomic propositions true in any consistent global state

- $(C, L, \lambda) \models l \Leftrightarrow l \in \lambda(C)$ for an atomic proposition l
- $(C, L, \lambda) \models P \wedge Q \Leftrightarrow C \models P$ and $C \models Q$
- $(C, L, \lambda) \models P \vee Q \Leftrightarrow C \models P$ or $C \models Q$
- $(C, L, \lambda) \models \neg P \Leftrightarrow \neg(C \models P)$
- $(C, L, \lambda) \models \Diamond P \Leftrightarrow \exists C' \in L : (C \subseteq C'$ and $C' \models P)$
  There exists a future consistent global state in which P is true.

Special Classes of Predicates



(i) (ii) (iii)

meet closed predicate

join closed predicate

regular predicate

stable predicate

Basis of a Predicate

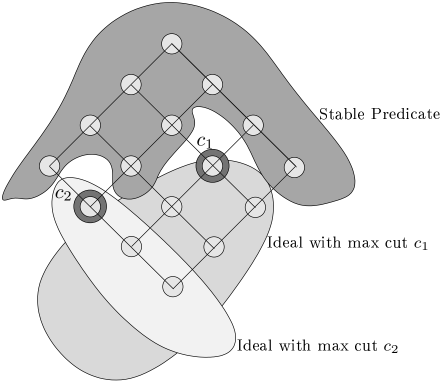
Given a computational lattice L, corresponding to a computation E, and a predicate P, a subset $S[P]$ of L is a basis of P if

1. Compactness: The size of $S[P]$ is polynomial in the size of computation E.
2. Efficient Membership: Given any consistent global state $C \in L$, there exists a polynomial time algorithm that takes $S[P]$, E and C as input and determines whether $(C, L) \models P$.

Examples

- Predicate for an Order Ideal:
  Sufficient to keep the largest CGS that satisfies P
- Regular Predicate:
  Sufficient to keep the slice (or join-irreducibles) of $(E, \rightarrow)$ with respect to P

Stable Predicates



Representing stable predicates Not necessarily a basis!

Given a stable predicate P and the computational lattice L, a stable structure is the set of ideals $\mathcal{I}$ such that a cut satisfies P iff it does not belong to any of the ideals in $\mathcal{I}$. Therefore, $C \models P \Leftrightarrow \neg(C \in \bigcup_{I \in \mathcal{I}} I)$.

Talk Outline

Semiregular Predicates

P is a semiregular predicate if it can be expressed as a conjunction of a regular predicate with a stable predicate.

Examples:

- All processes are never *red* concurrently at any future state and process P_0 has the token. That is, $P = \neg\Diamond(\bigwedge red_i) \wedge token_0$.
- At least one process is beyond phase k (stable) and all the processes are red.

claim: All regular predicates and stable predicates are semiregular.

Properties of Semiregular Predicates

- A semiregular predicate is join-closed   regular and stable predicates are join-closed
- if P and Q are semiregular then so is $P \wedge Q$.   both regular and stable predicates are closed under conjunction

Properties of Semiregular Predicates

If P is a semiregular predicate then $\Diamond P$ and $\Box P$ are semiregular.

- If P is semiregular, P has a unique maximal cut, say C_{max} and $\Diamond P$ is an ideal of the lattice that contains all cuts less than or equal to C_{max}.
- $\Box P$ is a stable predicate for any P, and therefore it is also semiregular.

Semiregular Structure

A semiregular structure, g, is a tuple $(\langle slice, \mathcal{I} \rangle)$ consisting of a slice and a stable structure, such that
the predicate is true in cuts that belong to their intersection.
$C \in g \Leftrightarrow (C \in slice) \wedge \neg(C \in \bigcup_{I \in \mathcal{I}} I)$.

Algorithm to Detect BTL: Base Case

/*The input predicate P_{in} has all negations pushed
- inside to the $\Diamond$ operator or to the atomic propositions */
/* each semiregular structure is represented as a tuple $\langle slice, maxCuts \rangle$
- where $maxCuts$ is the set of maximal cuts
- of the ideals $\mathcal{I}$ representing the stable structure */

function `getBasis`(Predicate P_{in})
output: $S[P_{in}]$, a set of semiregular structures
  Case 1. (Base case: local predicates) : $P_{in} = l$ or $P_{in} = \neg l$
    $S[P_{in}] := \{\langle slice(P), \{\}\rangle\}$
  Case 2. $P_{in} = P \vee Q$
  Case 3. $P_{in} = P \wedge Q$
  Case 4. $P_{in} = \Diamond P$
  Case 5. $P_{in} = \neg\Diamond P$
return $S[P_{in}]$

Algorithm to Detect BTL: Conjunctions and Disjunctions

function $\texttt{getBasis}$(Predicate P_{in})
output: $S[P_{in}]$, a set of semiregular structures

Case 1. (Base case: local predicates) : $P_{in} = l$ or $P_{in} = \neg l$

$S[P_{in}] := \{\langle slice(P), \{\}\rangle\}$

Case 2. $P_{in} = P \vee Q$

$S[P] := \texttt{getBasis}(P); S[Q] = \texttt{getBasis}(Q);$

$S[P_{in}] := S[P] \cup S[Q];$

Case 3. $P_{in} = P \wedge Q$

$S[P] := \texttt{getBasis}(P); S[Q] = \texttt{getBasis}(Q);$

$S[P_{in}] := \bigcup_{g_p \in S[P], g_q \in S[Q]} \{(\langle g_p.slice \wedge g_q.slice,$
$g_p.maxCuts \cup g_q.maxCuts\rangle)\};$

Case 4. $P_{in} = \Diamond P$

Case 5. $P_{in} = \neg \Diamond P$

return $S[P_{in}]$

Algorithm to Detect BTL: Modalities

function $\texttt{getBasis}$(Predicate P_{in})
output: $S[P_{in}]$, a set of semiregular structures
  Case 1. (Base case: local predicates) : $P_{in} = l$ or $P_{in} = \neg l$
  Case 2. $P_{in} = P \vee Q$
  Case 3. $P_{in} = P \wedge Q$
  Case 4. $P_{in} = \Diamond P$
    $S[P] := \texttt{getBasis}(P);$
    $S[P_{in}] := \bigcup_{g \in S[P]} \{\langle \Diamond(g.slice), \{\}\rangle\};$
  Case 5. $P_{in} = \neg\Diamond P$
    $S[P] := \texttt{getBasis}(P);$
    /* $slice_{orig}$ is the original computation */
    $S[P_{in}] := \{\langle slice_{orig}, \cup_{g \in S[P]}\{\texttt{maxCutIn}(g.slice)\}\rangle\};$

  Remove all empty semiregular structures from $S[P_{in}]$;

return $S[P_{in}]$

Complexity Analysis

Theorem

The total number of ideals $|I|$ in the basis computed by the algorithm to detect a BTL predicate P with k operators is at most 2^k

Theorem

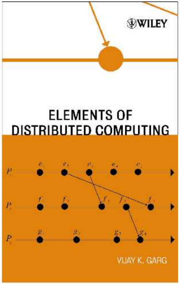
The time complexity of the algorithm to detect a BTL formula is polynomial in the number of events ($|E|$) and the number of processes (n) in the computation.

Conclusions

- Lattice properties are crucial in monitoring distributed computations

Additional Tutorial

- Elements of Distributed Computing Wiley & Sons 2002



- Introduction to Lattice Theory with Computer Science Applications (Expected December 2014)

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- Yen-Jung Chang: Parallel and Online CGS Enumeration
- Himanshu Chauhan, Aravind Natarajan: Distributed Slicing Algorithms

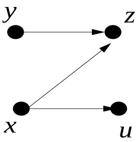
Blue:Graduate Students at PDSL, UT Austin Green: Graduate Students elsewhere Red: Faculty Members

Additional Topics

- Monitoring for liveness properties:
  Infinite (periodic posets)
- Quotient Construction for Distributive Lattices
  Collapsing sublattices such that temporal logic formula is true in the original lattice iff it holds in the reduced lattice
- Online Chain Partition
  Events arrive in an online fashion. Insert them into as few chains as possible

Online Chain Decomposition

- Elements of a poset presented in a total order consistent with the poset
- Assign elements to chains as they arrive
- Can be viewed as a game between
  - Bob: present elements
  - Alice: assign them to chains
- For a poset of width k, Bob can force alice to use $k(k+1)/2$ chains. Any online algorithm can be forced to use k^2 chains [Felsner 97].



Online Chain Decomposition

An efficient online algorithm that uses at most k^2 chains with at most $O(k^2)$ comparisons per event. [Aggarwal and Garg 05]

- Use k sets of queues $B_1, B_2, ..., B_k$. The set B_i has i queues with the invariant that no head of any queue is comparable to the head of any other queue.
- For a new element z, insert it into the first queue q in B_i with its head less than z.
- Swap remaining queues in B_i with queues in B_{i-1}.

Motivation for Control

Who controls the past controls the future, who controls the present controls the past...

George Orwell,

Nineteen Eighty-Four.

- maintain global invariants or proper order of events
  Examples: Distributed Debugging
  ensure that $busy_1 \vee busy_2$ is always true ensure that m_1 is delivered before m_2
  maintain $\neg CS_1 \vee \neg CS_2$
- Fault tolerance
  On fault, rollback and execute under control
- Adaptive policies
  procedure A (B) better under light (heavy) load

Models for Control

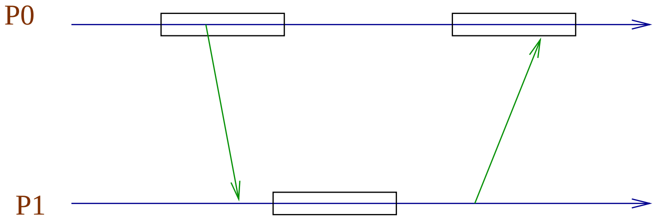
Is the future known ?

- Yes: offline control
  - applications in distributed debugging, recovery, fault tolerance..
- No: online control
- applications: global synchronization, resource allocation

Delaying events vs Changing order of events

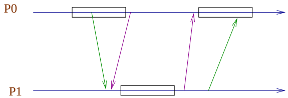
- supervisor simply adds delay between events
- supervisor changes order of events

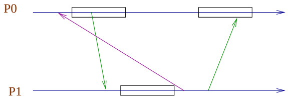
Delaying events: Offline control



Maintain at least one of the process is not red
Can add additional arrows in the diagram such that the control relation should not interfere with existing causality relation
(otherwise, the system deadlocks)

Delaying events: Offline control



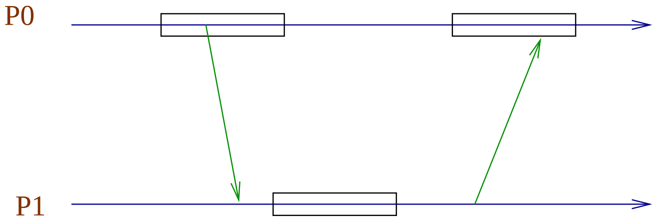


Problem: Instance: Given a computation and a boolean expression q of local predicates

Question: Is there a non-interfering control relation that maintains q

This problem is NP-complete [Tarafdar and Garg 97]

Delaying events: disjunctive predicates



Efficient algorithm for disjunctive predicates

Example: at least one of the philosopher does not have a fork

Result:

a control strategy exists iff there is no set of overlapping false intervals

$overlap(I_1, I_2) = (I_1.lo \rightarrow I_2.hi) \wedge (I_2.lo \rightarrow I_1.hi)$

Result:

There exists an $O(n^2 m)$ algorithm to determine the strategy $n =$ number of processes $m =$ number of states per process