Goals of the lecture

- Logical Clocks (Lamport's clocks)
- Concurrency vs Simultaneity
- Total Ordering
- Physical Clocks
- Vector Clocks

Logical Clocks

A *global clock* C: $S \to \mathcal{N}$ that satisfies:

$$\forall s,t \in S : s \prec_1 t \vee s \leadsto t \Rightarrow C(s) < C(t)$$

$\mathcal{C}$: the set of all global clocks
Equivalent to :

$$\forall s,t \in S : s \to t \Rightarrow \forall C \in \mathcal{C} : C(s) < C(t) \qquad \textbf{(CC)}$$

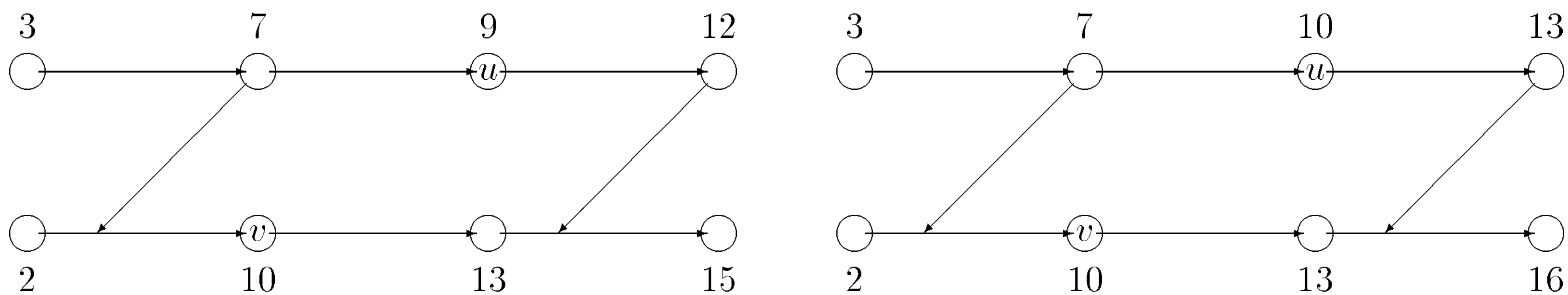
- Lemma: $\mathcal{C}$ is non-empty iff $(S, \to)$ is an irreflexive partial order.
- happened-before relation

Concurrency $\equiv$ simultaneity for some observer

$$\forall u, v \in S : u || v \Rightarrow \exists C \in \mathcal{C} : (C(u) = C(v))$$

If two local states are concurrent, $\Rightarrow$ there exists a global clock such that both states are assigned the same timestamp. This will show the converse of (CC), i.e.,

$$\forall s, t \in S : s \not\rightarrow t \Rightarrow \exists C \in \mathcal{C} : \neg(C(s) < C(t))$$



Transitivity ?

Logical Clock

- Useful for various algorithms
- Actions taken for each event type:

For any initial state s:

$s.c = 0;$

Rule for a send event (s, snd, t): /* s.c is sent as part of msg */

$t.c := s.c + 1;$

Rule for a receive event $(s, rcv(u), t)$:

$t.c := \max(s.c, u.c) + 1;$

Rule for an internal event (s, int, t):

$t.c := s.c + 1;$

The following claim is easy to verify: (Converse ?)

$$\forall s, t \in S : s \rightarrow t \Rightarrow s.c < t.c$$

Ordering the events totally

- Extend the logical clock with process number
  - the timestamp of any event is a tuple $< e.c, e.p >$
- the total order $<$ is obtained as:

$$(e.c, e.p) < (f.c, f.p)$$

$$\Leftrightarrow$$

$$(e.c < f.c) \vee ((e.c = f.c) \wedge (e.p < f.p)).$$

Physical Clocks

- What if some messages do not follow the algorithm ?
- Given approximately correct physical clocks, one can synchronize clocks such that $u \rightarrow v$ implies $C(u) < C(v)$.
  - κ = upper bound on the drift rate of any clock
  - μ = minimum transmission time for any message
  - t = physical time at which the message is sent

We require

$$C_i(t + \mu) > C_j(t) \text{ for all } i, j, t.$$

From the bound on the drift we know that

$$C_i(t + \mu) > C_i(t) + (1 - \kappa)\mu.$$

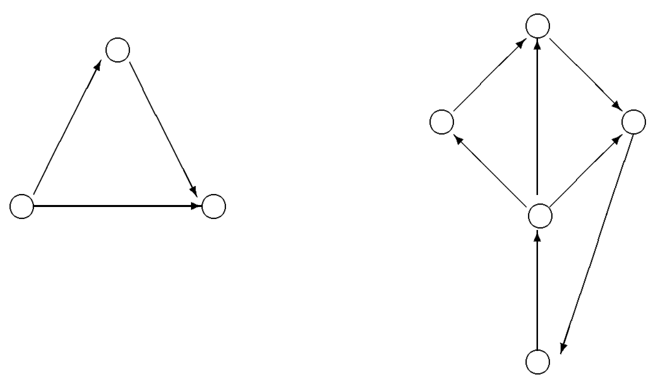
Thus, we need $C_i(t) + (1 - \kappa)\mu > C_j(t)$.

That is, $C_j(t) - C_i(t) < (1 - \kappa)\mu$.

Clock Synchronization Algorithm

The synchronization constant $(\epsilon) < (1 - \kappa)\mu$.

- Algorithm:
  - send out a timestamped message along its outgoing link at least every τ seconds.
  - Every message takes time between μ and $\mu + \xi$.
  - On receipt of a message timestamped with T_m, the clock is updated as maximum of the previous value and $T_m + \mu$.
- Let the network be strongly connected with d as the diameter. Then, it can be shown that $\epsilon = d(2\kappa\tau + \xi)$ for all $t > t_0 + \tau d$ assuming that $\mu + \xi << \tau$.



Vector Clocks

- Logical clocks satisfy

$$s \rightarrow t \Rightarrow s.c < t.c.$$

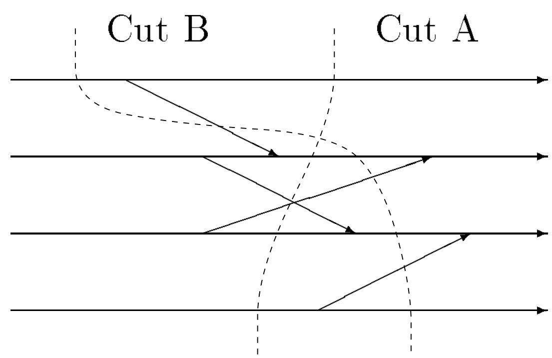
  However, the converse is not true.

- Vector clock satisfy:

$$s \rightarrow t \Leftrightarrow s.v < t.v.$$

Consistent Cuts

- $(E, \prec)$
  - down-set Y in this partial order will be called a prefix.
  - The set of all prefixes is a lattice.
  - $\sup Y$ for any prefix Y is called a *cut*.
- $(E, \rightarrow)$ where $\rightarrow$ is the causal-precedes.
  - A down-set Y in this partial order is called a consistent prefix.
  - Similarly, $\sup Y$ is called a consistent cut.
  - The set of all consistent prefixes is also a lattice.
    $F \subseteq E$ is a consistent cut iff $\forall e, f \in F : \neg(e \rightarrow f)$.



Vector Algorithm

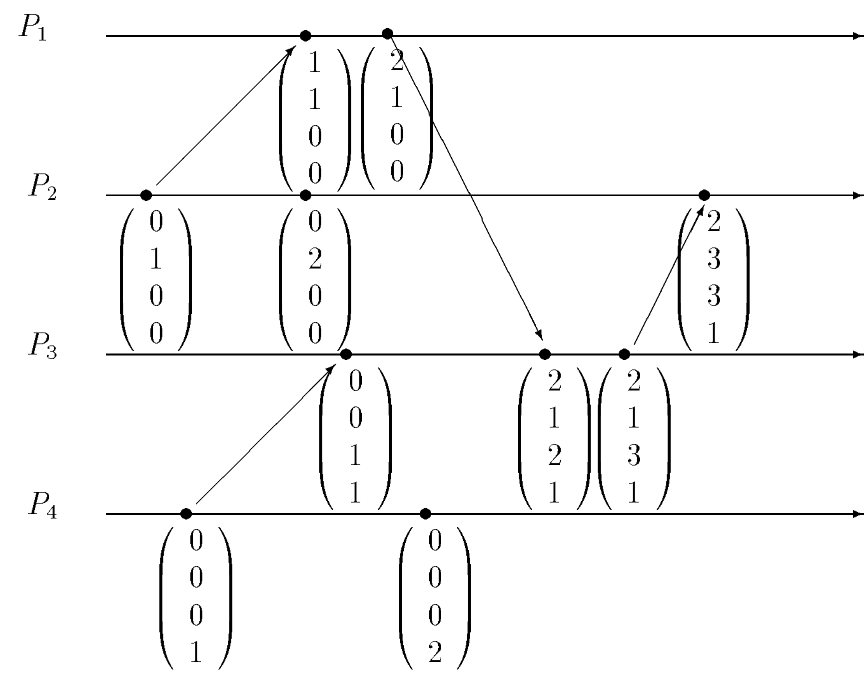
- Let there be N processes
- Algorithm:

For any initial state s:

$$(\forall i : i \neq s.p : s.v[i] = 0) \wedge (s.v[s.p] = 1)$$

Rule for an internal event (s, int, t):

$$t.v := s.v;$$
$$t.v[t.p]++;$$



Vector Algorithm [Contd.]

Rule for a send event (s, snd, t):

$t.v := s.v;$

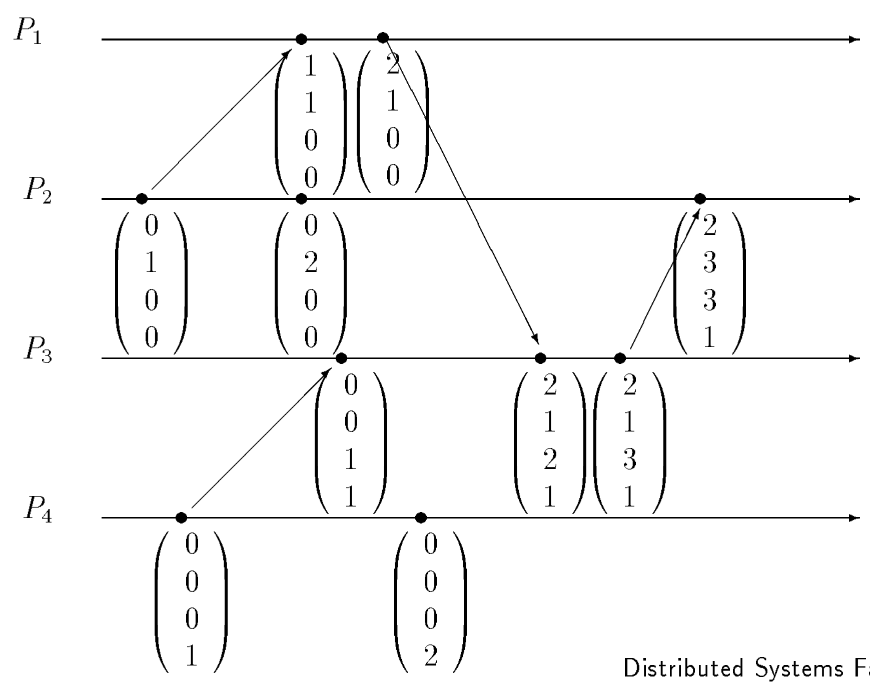
$t.v[t.p] + +;$

Rule for a receive event $(s, rcv(u), t)$:

for $i := 1$ to N

$t.v[i] := \max(s.v[i], u.v[i]);$

$t.v[t.p] + +;$



Properties of the Vector Clock Algorithm

Lemma 1 *Let $s \neq t$. Then,*

$$s \not\rightarrow t \Rightarrow t.v[s.p] < s.v[s.p]$$

Proof:

- $t.p = s.p$: then it follows that $t \prec s$.
- $s.p \neq t.p$. Since $s.v[s.p]$ is the local clock of $P_{s.p}$ and $P_{t.p}$ could not have seen this value as $s \not\rightarrow t$ ∎

Theorem 1 $s \rightarrow t$ **iff** $s.v < t.v$.
Proof: $(s \rightarrow t) \Rightarrow (s.v < t.v)$

- $s \rightarrow t$: there is a message path from s to t. Therefore, $\forall k : s.v[k] \leq t.v[k]$. Furthermore, since $t \not\rightarrow s$, from lemma 1 $t.v[j] > s.v[j]$.
- The converse follows from Lemma 1. ∎

Optimization

Recall $x < y$ if and only if $(\forall i : x[i] \leq y[i]) \land (\exists j : x[j] < y[j])$. If we know the processes the vectors came from, the comparison between two states can be made in constant time.

Lemma 2 $s \rightarrow t$ *iff*

$$(s.v[s.p] \leq t.v[s.p]) \land (s.v[t.p] < t.v[t.p])$$