Goals of the lecture: Self-stabilization

- Fault-tolerance
- Definition of self-stabilizing
- Algorithm with K-state Machines
  - Proof
- Algorithm with 3-state Machines
  - Proof

References: Dijkstra 74, Dijkstra 86

Fault-tolerance

- systems which recover from faults
- self-stabilization: highly fault-tolerant
  - a fault can change any data
- system viewed as consisting of legal and illegal states
- self-stabilzation: should reach a legal state in finite moves

Terminology

- Underlying topology: connection graph
- neighbors
- privilege: boolean function of
  - own state,
  - states of its neighbors
- legal state: application dependent

Requirements on legal state

- In each legal state, one or more privileges
- any move from a legal state leads to a legal state
- each privilege present in at least one legal state
- for any pair of legal states, there exist a sequence of transferring moves

Definition of self-stabilzation: Regardless of initial state, and privilege selected each time, the system is guaranteed to reach a legal state after a finite number of moves.

Example: Mutual Exclusion

legal state: exactly one privilege

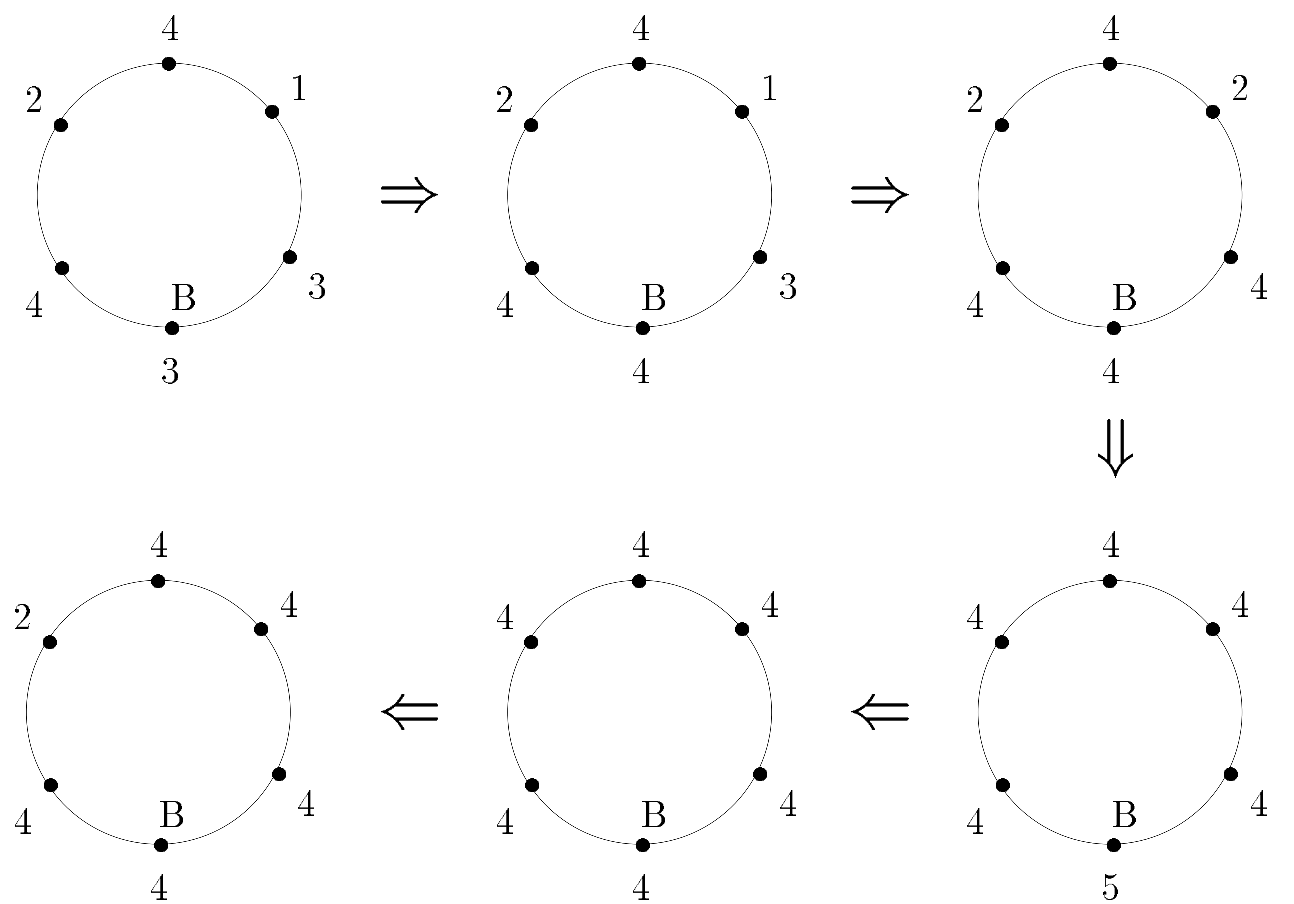
- N+1 machines numbered 0..N
- L,S,R: states of left, self, right
- bottom machine: machine 0
- format:
  if privilege then corresponding move fi

Algorithm I: K-state machine $(K > N)$

Bottom:
if (L=S) then S := S+1 mod K fi

For other machines:
if $(L \neq S)$ then S := L fi

Example



Proof

Lemma 0: If the system is in a legal state, then it will stay legal.

Lemma 1: A sequence of moves in which Bottom does not move is finite.

Lemma 2: Given any configuration, either
(1) no other machine has the same state as the bottom, or
(2) there exists a value which is different from all machines.

Lemma 3: With in a finite number of moves, part one of Lemma 2 will be true.

Theorem 1: Within finite number of moves, the system will reach a legal state.

Algorithm II: 3-state machine

Ring of at least 3 machines
Bottom: B, Normal: N, Top: T
configuration viewed as a string of 0,1,2

Bottom:
if $(B+1=R)$ then $B := B+2$;

Normal:
if $(L=S+1)$ or $(R=S+1)$ then $S := S+1$;

Top:
if $(L=B)$ and $(T \neq B+1)$ then $T := B+1$

Viewing the string with arrows

y = # of left-pointing + 2# of right-pointing

Bottom :

(0)	B $\leftarrow$ R	to	B $\rightarrow$ R	$\Delta y = +1$

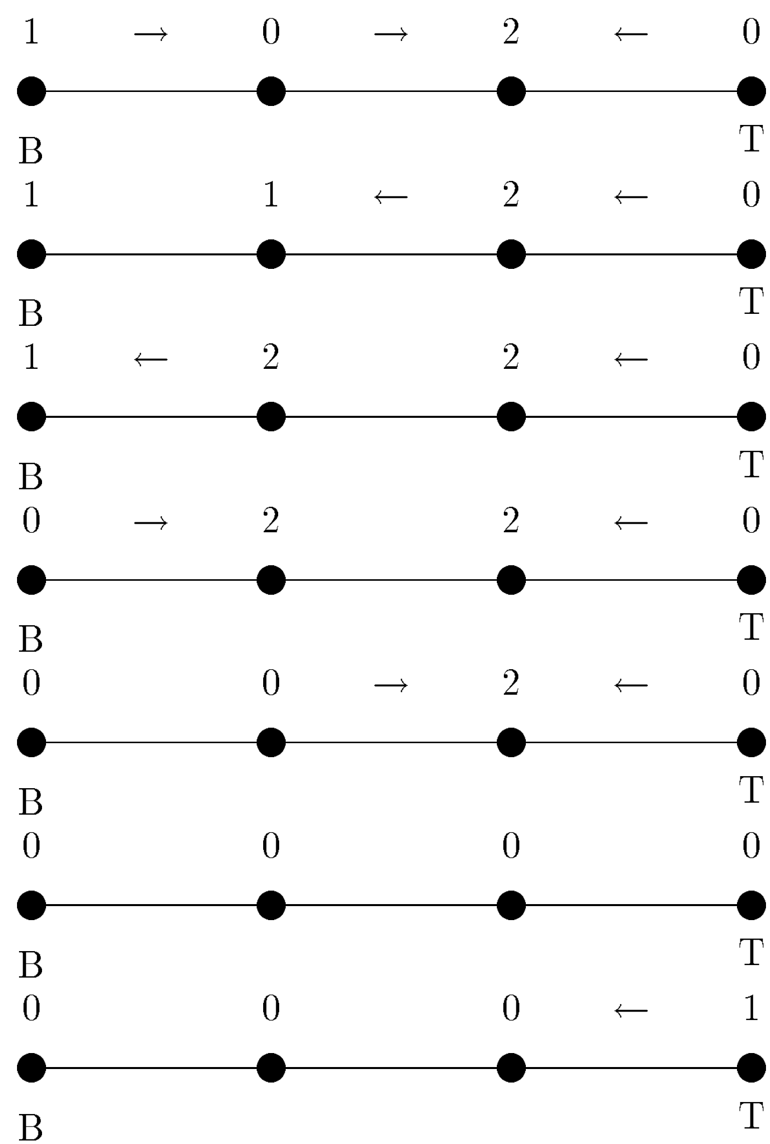
Normal Machine:

(1)	L $\rightarrow$ S R	to	L S $\rightarrow$ R	$\Delta y = 0$
(2)	L S $\leftarrow$ R	to	L $\leftarrow$ S R	$\Delta y = 0$
(3)	L $\rightarrow$ S $\leftarrow$ R	to	L S R	$\Delta y = -3$
(4)	L $\rightarrow$ S $\rightarrow$ R	to	L S $\leftarrow$ R	$\Delta y = -3$
(5)	L $\leftarrow$ S $\leftarrow$ R	to	L $\rightarrow$ S R	$\Delta y = 0$

Top Machine (privilege also depends on B):

(6)	L $\rightarrow$ T	to	L $\leftarrow$ T	$\Delta y = +1$
(7)	L T	to	L $\leftarrow$ T	$\Delta y = +1$

Example



Proof

Claim: Single arrow implies it stays that way.
Claim: string free from arrow creates one in a single move.

Now show that if multiple arrow then y will be decreased in finite moves.

Proof contd

Lemma 0: Between two successive moves of Top at least one move of Bottom takes place.

Lemma 1: A sequence of moves in which Bottom does not move is finite. Proof: sufficient to consider normal machines. (3),(4),(5) decrease number of arrows. (1) and (2) moves finite due to topology.

Theorem: Within finite moves, there is one arrow in the string. Proof:between successive moves of bottom, falsification of "left-most arrow exists and points to the right" happen in (3), (4), or (6). if (6) then done. If (3) or (4), y decreases by 3. y can increase by at most 2 per move of Bottom, thus y is decreased by 1.