Democratic Elections in Faulty Distributed Systems

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Outline

- Motivation

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- Social Choice and Social Welfare

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- Social Choice with Byzantine Faults

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  - Pruned-Kemeny-Young Scheme for Byzantine Social Welfare

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- Simulation Results
- Conclusion

Motivation – Leader Election

Conventional Problem

Node with the highest id should be the leader. All the nodes in the system should agree on the leader.

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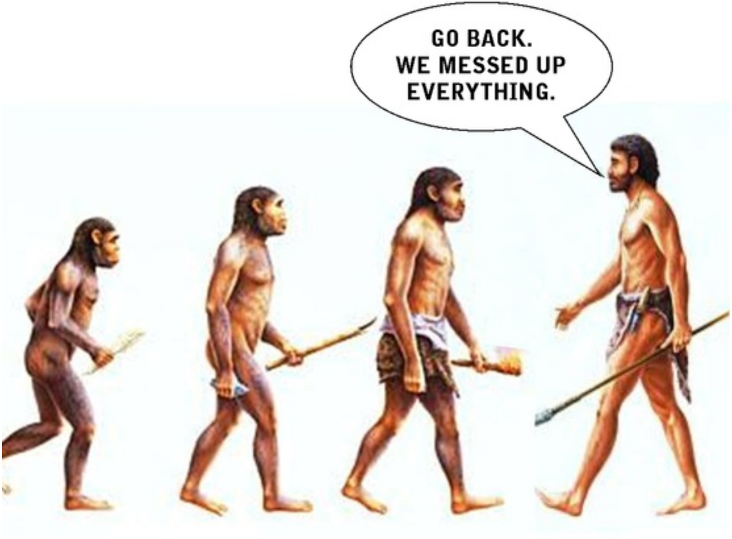
Node with the highest id should be the leader. All the nodes in the system should agree on the leader.

- Philosophers of Ancient Athens would protest!

Motivation – Leader Election



Democratic Leader Election

- *Elect* a leader
  - Each node has individual preferences
  - Conduct an election where every node votes

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- Enter ‘Byzantine’ Voters!

Why Not Use Top-Choice Approach?

'Multivalued Byzantine Agreement', Turpin and Coan 1984,
'$k-$set Consensus', Prisco et al. 1999

- Every voter sends her *top* choice
- Run Byzantine Agreement
  - Agree on the choice with most votes

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	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

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Elect choice with most votes (at top) : c or b

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1^{st} choice	b	b	b				a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice				b	b	b	

Elect choice with most votes (at top) : c or b But ...

$\#(a > b) = 4, \quad \#(b > a) = 3$

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	P_1	P_2	P_3	P_4	P_5	P_6	P_7
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2^{nd} choice	a	a	a	a	a	a	
3^{rd} choice	c	c	c				c

Elect choice with most votes (at top) : c or b But ...

$\#(a > b) = 4, \quad \#(b > a) = 3 \quad$ and $\#(a > c) = 4, \quad \#(c > a) = 3$

Model & Constructs

System

- n processes (voters)
- f Byzantine processes (voters) : *bad*
- Non-faulty processes (voters) : *good*
- $f < n/3$

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Jargon

$\mathcal{A}$: Set of candidates
Ranking: Total order over the set of candidates.
Vote: A voter's preference ranking over candidates.
Ballot : Collection of all votes.
Scheme : Mechanism that takes a ballot as input and outputs a winner.

Conducting Distributed Democratic Elections

- Use Interactive Consistency
  - Agree on everyone's vote[1]
  - Agree on the ballot
- Use a *scheme* to decide the winner

[1] We use Gradecast based Byzantine Agreement by Ben-Or et al.

Byzantine Social Choice

Social Choice

Given a ballot, declare a candidate as the winner of the election.

Arrow 1950-51, Buchanan 1954, Graaff 1957

Byzantine Social Choice

Given a set of n processes of which at most f are faulty, and a set $\mathcal{A}$ of k choices, design a protocol elects one candidate as the social choice, while meeting the 'protocol requirements'.

Byzantine Social Welfare

Social Welfare

Given a ballot, produce a *total order* over the set of candidate.

Arrow 1950-51, Buchanan 1954, Graaff 1957, Farquharson 1969

Byzantine Social Welfare

Given a set of n processes of which at most f are faulty, and a set $\mathcal{A}$ of k choices, design a protocol that produces a *total order* over $\mathcal{A}$, while meeting the 'protocol requirements'.

Protocol Requirements

1. *Agreement*: All good processes decide on the same choice/ranking.

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2. *Termination*: The protocol terminates in a finite number of rounds.

Validity Condition

Validity: Requirement on the choice/ranking decided, based upon the votes of good processes.

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- S: If v is the top choice of all good voters, then v must be the winner.
- S': If v is the last choice of all good voters, then v must **not** be the winner.
- M': If v is last choice of majority of good voters, then v must **not** be the winner.

Validity Conditions

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

Table: Ballot of 7 votes (P_6, P_7 Byzantine)

Validity Conditions

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
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M (Elect majority of *good* voters) : elect b

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M (Elect majority of *good* voters) : elect b

P (Do not elect a candidate that is not the *top* choice of any *good* voters) : *do not* elect a

$BSC(k, V)$

Byzantine Social Choice problem with k candidates, and validity condition/requirement V.

$BSC(2, M)$:

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- M: elect top choice of majority of *good* votes
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Reason:
$f \geq n/4 \Rightarrow$ can not differentiate b/w *good* and *bad* votes

$BSC(2, M')$:

- M': **do not** elect the last choice of majority of *good* votes
- Impossible to solve for $f \geq n/4$

Byzantine Social Choice – Possibilities

$BSC(k, S \wedge M')$:

- S: if v is first choice of all *good* voters, elect v
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Approach:

- Round 1 : Agree on *last* choices of all voters

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- Remove any candidates that appears $\lfloor (n - f)/2 + 1 \rfloor$ times or more

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- Round 2 : Use *top* choices from remaining candidates, agree and decide

$BSC(k, V)$ Results – Summarized

Requirement	Unsolvable	Solvable
S	-	$k \geq 2$
S'	-	$k \geq 2$
M	$f \geq n/4 \wedge k \geq 2$	-
M'	$f \geq n/4 \wedge k = 2$	$k \geq 3$
P	$f \geq 1 \wedge k \geq n$	$f < min(n/k, n/3) \wedge 2 \leq k < n$

Table: Impossibilities & Possibilities for $BSC(k, V)$

Given a ballot, produce a total order over the set of candidates

Byzantine Social Welfare – Schemes

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3^{rd} choice	c	c	c	b	b	b	c

Result : $b \succ a \succ c$

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Distance (d) between rankings: # of pair-orderings on which rankings differ

Pairwise Comparison, Condorcet, circa 1785

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b	a	– differ on
c	c	(a, b)

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Distance (d) between rankings: # of pair-orderings on which rankings differ

Pairwise Comparison, Condorcet, circa 1785

r	r'	d
a	c	2
b	b	– differ on
c	a	(a, b) and (b, c)

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Median (m) of ballot: Ranking that has least distance from overall pair-wise comparisons in the ballot

Kemeny-Young Scheme

(1) J. Kemeny, 1959, (2) H. Young, 1995

Goal: Get as close to the median as possible.

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For a given ballot B:

$$score(r, B) = \sum_{p \in P_r} (\text{frequency of } p \text{ in } B)$$

S_k: set of all permutations of k candidates ($k!$ permutations)

foreach ranking $r \in S_k$ **do**
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done
select ranking with maximum $score_r$ value as the outcome

Kemeny-Young Scheme – Example

Candidates: $\{a, b, c\}$

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

$$\#(a \succ b) = 4, \quad \#(b \succ a) = 3, \quad \#(a \succ c) = 4,$$
$$\#(c \succ a) = 3, \quad \#(b \succ c) = 4, \quad \#(c \succ b) = 3$$

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$$\#(c \succ a) = 3, \quad \#(b \succ c) = 4, \quad \#(c \succ b) = 3$$

Permutations:

a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

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Permutations:

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12					

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Permutations:

a	a	b	b	c	c
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c	b	c	a	b	a
12	11	11	10	10	9

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Permutations:

a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a
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Kemeny-Young Scheme Result: $a \succ b \succ c$

Pruned-Kemeny-Young Scheme (this paper)

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	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

Pruned-Kemeny-Young – Example

$n = 7, \qquad f = 2$

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a

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b	c	a	c	a	b
c	b	c	a	b	a
		11			

Pruned-Kemeny-Young – Example

$n = 7, \qquad f = 2$

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
1^{st} choice	b	b	b	c	c	c	a
2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

a	*a*	*b*	*b*	*c*	*c*
b	*c*	*a*	*c*	*a*	*b*
c	*b*	*c*	*a*	*b*	*a*
9	8	11	6	10	6

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$n = 7, \qquad f = 2$

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2^{nd} choice	a	a	a	a	a	a	b
3^{rd} choice	c	c	c	b	b	b	c

a	a	b	b	c	c
b	c	a	c	a	b
c	b	c	a	b	a
9	8	11	6	10	6

Pruned-Kemeny Scheme Result: $b \succ a \succ c$

Evaluating Scheme Efficacy

Suppose ω is an *ideal* ranking over k candidates

- ω as the election outcome $\Rightarrow$ maximum social welfare

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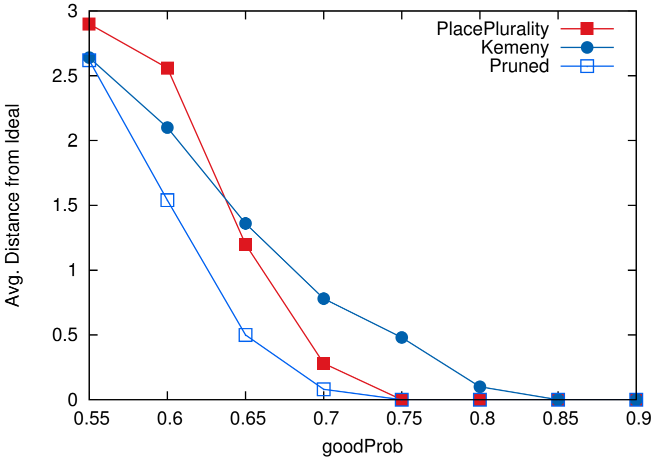
- Analyze outcomes generated by schemes

$$\#\text{ of voters} = 100,\ \#\text{ of } \textit{bad} \text{ voters} = 33,\ \textit{badProb} = 0.9$$

Simulation Results

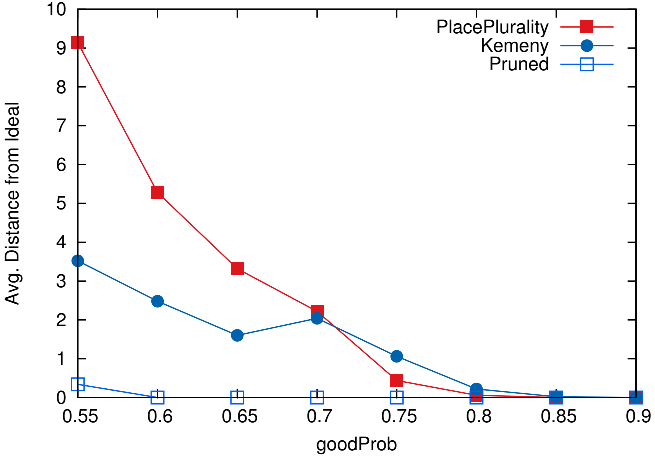
Average (of 50 ballots) distances of produced outcomes from the ideal ranking



(a) # of Candidates = 3

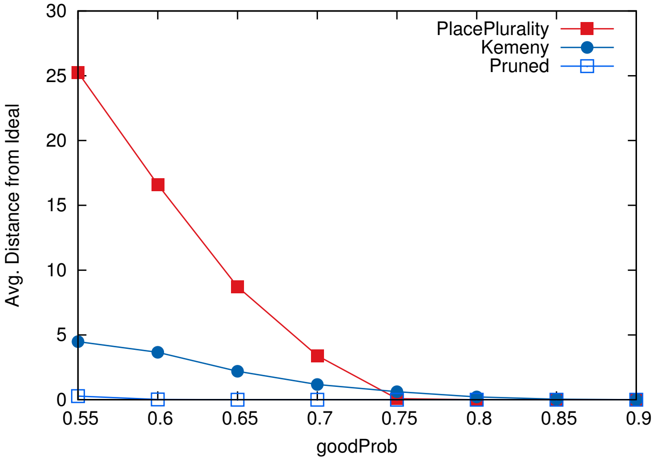
Simulation Results, contd.

Average (of 50 ballots) distances of produced outcomes from the ideal ranking



(b) # of Candidates = 5

Average (of 50 ballots) distances of produced outcomes from the ideal ranking



(c) # of Candidates = 8

Conclusion

- Introduction of democratic election problem in distributed systems

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- Pruned-Kemeny-Young Scheme for Byzantine Social Welfare problem

Future Work

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  - NP-Hard

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- Pruned-Kemeny-Young (and Kemeny-Young)
  - NP-Hard
  - Yet produce 'better' results
  - Explore techniques for finding 'better' outcomes in polynomial steps

Thanks!

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: Invalid method declaration
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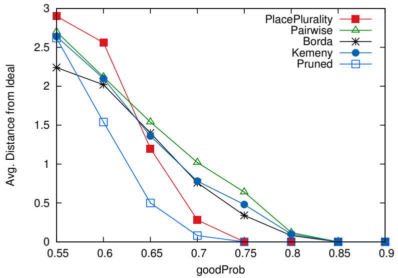
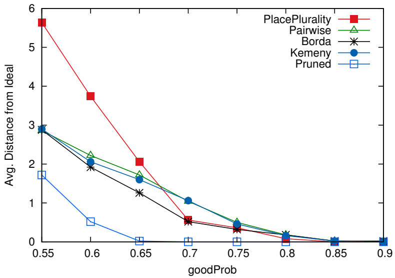
Backup

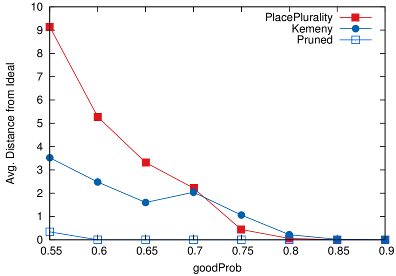


(d) # of Candidates = 3

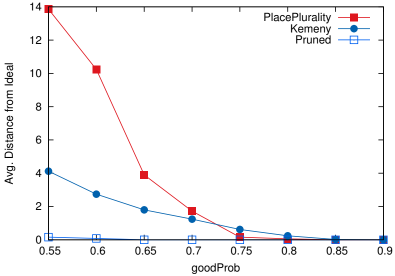


(e) # of Candidates = 4

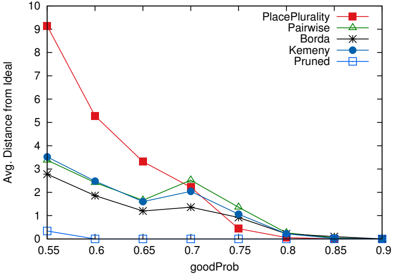
Average (of 50 ballots) distances of produced outcomes from the ideal ranking



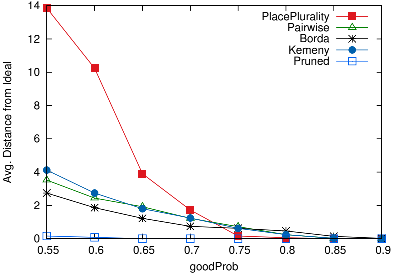
(f) # of Candidates = 5



(g) # of Candidates = 6

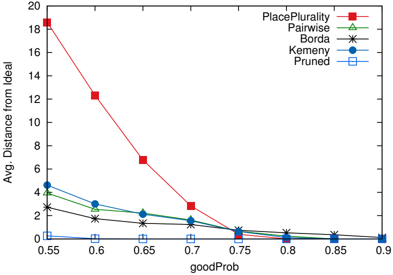


(h) # of Candidates = 5

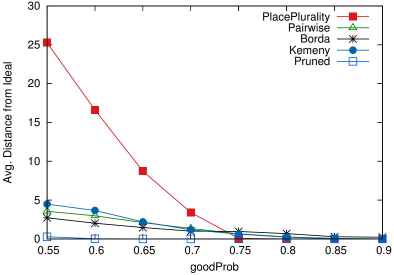


(i) # of Candidates = 6

Backup



(j) # of Candidates = 7



(k) # of Candidates = 8

Related Work

- Arrow's Impossibility Theorem, and his work on Social Choice and Welfare Theory
  - 1950, 1951
- Pairwise Comparison Schemes, Social Welfare Schemes, Theory of Voting, Welfare Economics
  - Condorcet circa 1785, Buchanan 1954, Graaff 1957, Kemeny 1959, Farquharson 1969, Ishikawa et al. 1979, Young 1988
- Multivalued Byzantine Agreement Schemes, Byzantine Leader Election, k-set Consensus
  - Turpin and Coan 1984, Ostrovsky et al. 1994, Russell et al. 1998, Kapron et al. 2008, Prisco et al. 1999