

ECE382M.20: System-on-Chip (SoC) Design

Lecture 12 – Operation Scheduling

Source: G. De Micheli, *Integrated Systems Center, EPFL*
“*Synthesis and Optimization of Digital Circuits*”, McGraw Hill, 2001.

Additional sources:

Notes by Kia Bazargan, <http://www.ece.umn.edu/users/kia/Courses/EE5301>

Notes by Rajesh Gupta, UCSD, <http://www.cecs.uci.edu/~rgupta/ics280.html>

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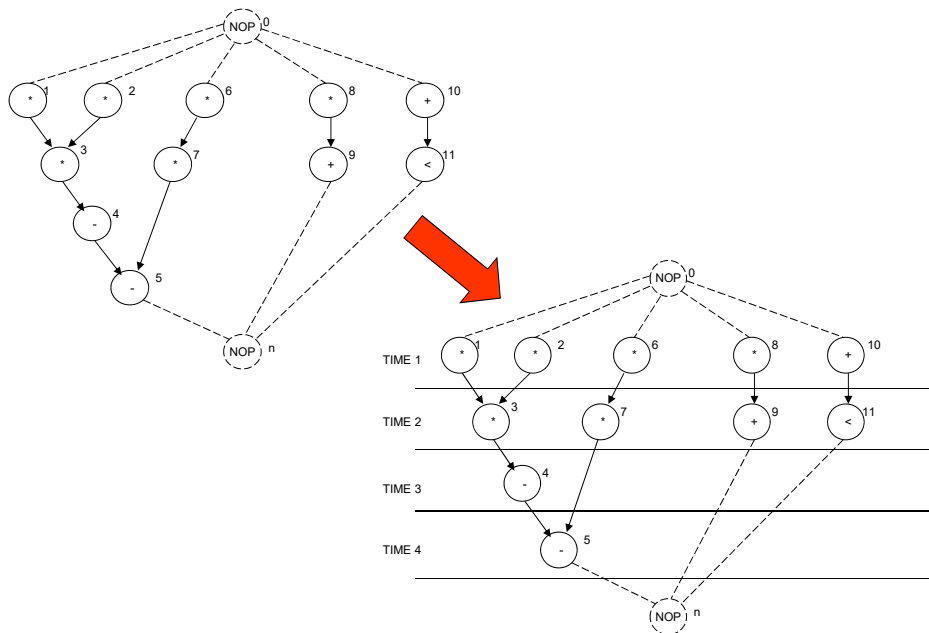
Lecture 12: Outline

- **The scheduling problem**
 - Case analysis
- **Unconstrained scheduling**
 - ASAP and ALAP schedules
- **Resource constrained (RC) scheduling**
 - List scheduling
- **Time constrained (TC) scheduling**
 - Force-directed scheduling
- **Advanced scheduling problems**
 - Chaining
 - Pipelining

Scheduling

- **Circuit model:**
 - Sequencing graph
 - Cycle-time is given
 - Operation delays expressed in cycles
- **Scheduling:**
 - Determine the start times for the operations
 - Satisfying all the sequencing (timing and resource) constraint
- **Goal:**
 - Determine *area/latency* trade-off

Example



Operation Scheduling

- **Input:**
 - Sequencing graph $G(V, E)$, with n vertices
 - Cycle time τ
 - Operation delays $D = \{d_i; i=0..n\}$
- **Output:**
 - Schedule ϕ determines start time t_i of operation v_i .
 - Latency $\lambda = t_n - t_0$.
- **Goal: determine area / latency tradeoff**
- **Classes:**
 - Non-hierarchical and unconstrained
 - Latency constrained
 - Resource constrained
 - Hierarchical

Simplest Method

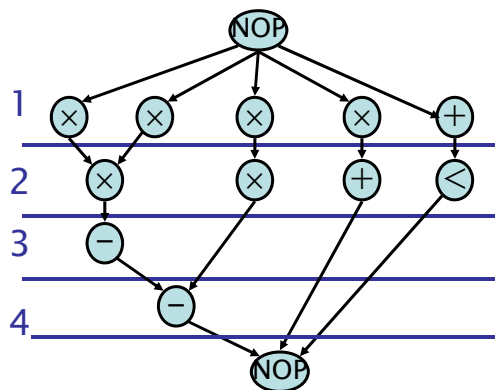
- **All operations have bounded delays**
- **All delays are in cycles:**
 - Cycle-time is given
- **No constraints – no bounds on area**
- **Goal:**
 - Minimize latency

Min Latency Unconstrained Scheduling

- **Simplest case: no constraints, find min latency**
- **Given set of vertices V , delays D and a partial order $>$ on operations E ,**
- **find an integer labeling of operations $\phi: V \rightarrow \mathbb{Z}^+$ such that:**
 - $t_i = \phi(v_i)$
 - $t_i \geq t_j + d_j \quad \forall (v_j, v_i) \in E$
 - and $\lambda = t_n - t_0$ is minimum
- **Solvable in polynomial time**
 - Bounds on latency for resource constrained problems
 - ASAP algorithm used: topological order

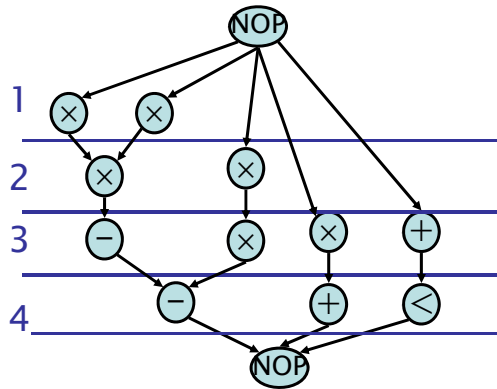
ASAP Schedules

- **Schedule v_0 at $t_0=0$**
- **While (v_n not scheduled)**
 - Select v_i with all scheduled predecessors
 - Schedule v_i at $t_i = \max \{t_j + d_j\}$, v_j being a predecessor of v_i
- **Return t_n**



ALAP Schedules

- **Schedule v_n at $t_n=l$**
- **While (v_0 not scheduled)**
 - Select v_i with all scheduled successors
 - Schedule v_i at $t_i = \min \{t_j - d_j\}$, v_j being a successor of v_i



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Remarks

- **ALAP solves a latency-constrained problem**
 - Latency bound can be set to latency computed by ASAP algorithm
- **Mobility**
 - Defined for each operation
 - Difference between ALAP and ASAP schedule
 - Slack on the start time

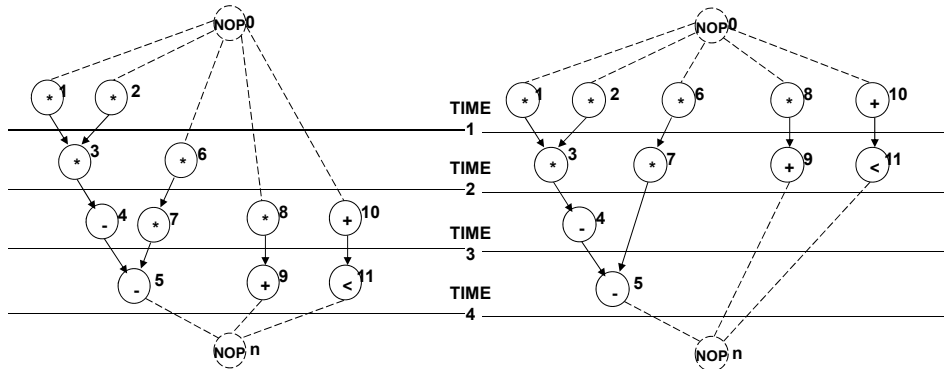
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Example

- **Operations with zero mobility:**
 - $\{v_1, v_2, v_3, v_4, v_5\}$
 - Critical path
- **Operations with mobility one:**
 - $\{v_6, v_7\}$
- **Operations with mobility two:**
 - $\{v_8, v_9, v_{10}, v_{11}\}$



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Lecture 12: Outline

- ✓ The scheduling problem
- ✓ Unconstrained scheduling
- **Resource constrained (RC) scheduling**
 - Exact formulations
 - ILP
 - Hu's algorithm
 - Heuristic methods
 - List scheduling
- Time constrained (TC) scheduling
- Advanced scheduling problems

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Scheduling under Resource Constraints

- **Classical scheduling problem**
 - Fix area bound – minimize latency (ML-RCS)
 - Minimum latency resource constrained scheduling
 - The amount of available resources affects the achievable latency
- **Dual problem:**
 - Fix latency bound – minimize resources (MR-LCS)
 - Minimum resources latency constrained scheduling
- **Assumption:**
 - All delays bounded and known

ML-RCS

- **Given**
 - a set of ops V with integer delays D
 - a partial order on the operations E
 - upper bounds $\{a_k; k = 1, 2, \dots, n_{res}\}$ on resource usage
- **Find an integer labeling $\phi : V \rightarrow \mathbf{Z}^+$ such that:**
 - $t_i = \phi(v_i)$,
 - $t_i \geq t_j + d_j$ for all i, j s.t. $(v_j, v_i) \in E$,
 - $|\{v_i | T(v_i) = k \text{ and } t_i \leq l < t_j + d_j\}| \leq a_k$
 - for all types $k = 1, 2, \dots, n_{res}$ and steps l
 - and t_n is minimum
- **Intractable problem**

ILP Formulation

- **Binary decision variables**
 - $X = \{x_{il} \mid i = 1, 2, \dots, n; l = 1, 2, \dots, \bar{\lambda} + 1\}$
 - x_{il} is **TRUE** only when operation v_i starts in step l of the schedule (i.e. $l = t_i$)
 - $\bar{\lambda}$ is an upper bound on latency
- **Start time of operation v_i : $\sum_l l \cdot x_{il}$**

ILP Constraints

- **Operations start only once**

$$\sum_l x_{il} = 1 \quad i = 1, 2, \dots, n$$
- **Sequencing relations must be satisfied**

$$t_i \geq t_j + d_j \rightarrow t_i - t_j - d_j \geq 0 \quad \text{for all } (v_j, v_i) \in E$$

$$\sum_l l \cdot x_{il} - \sum_l l \cdot x_{jl} - d_j \geq 0 \quad \text{for all } (v_j, v_i) \in E$$
- **Resource bounds must be satisfied**
 Simple case (unit delay)

$$\sum_{i:T(v_i)=k} x_{il} \leq a_k \quad k = 1, 2, \dots, n_{res}; \quad \text{for all } l$$

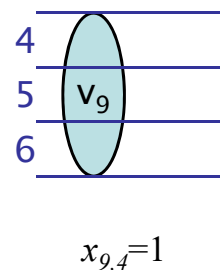
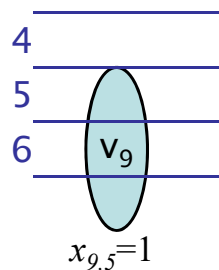
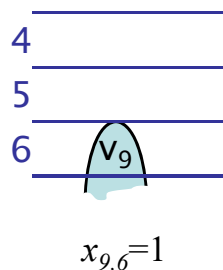
Start Time vs. Execution Time

- For each operation v_i , only one start time
- If $d_i=1$, then the following questions are the same:
 - Does operation v_i start at step l ?
 - Is operation v_i running at step l ?
- But if $d_i > 1$, the two questions should be formulated as:
 - Does operation v_i start at step l ?
 - Does $x_{il} = 1$ hold?
 - Is operation v_i running at step l ?
 - Does the following hold?

$$\sum_{m=l-d_i+1}^l x_{im} \stackrel{?}{=} 1$$

Operation v_i Still Running at Step l ?

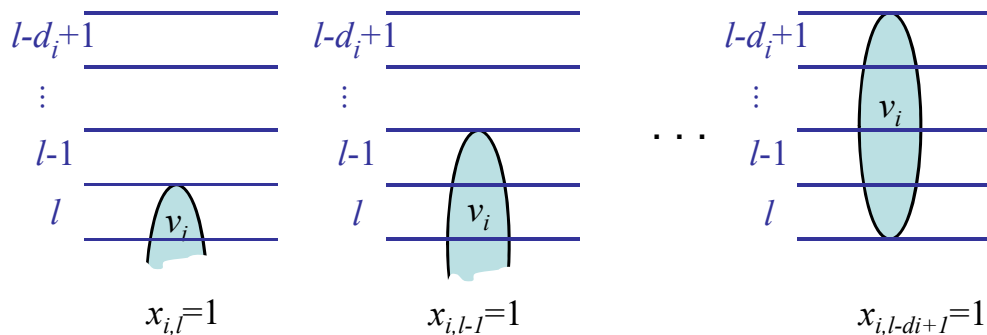
- Is v_9 running at step 6?
 - Is $x_{9,6} + x_{9,5} + x_{9,4} = 1$?



- Note:
 - Only one (if any) of the above three cases can happen
 - To meet resource constraints, we have to ask the same question for ALL steps, and ALL operations of that type

Operation v_i Still Running at Step l ?

- Is v_i running at step l ?
 - Is $x_{i,l} + x_{i,l-1} + \dots + x_{i,l-d_i+1} = 1$?



ILP Formulation of ML-RCS

- **Constraints:**

- Unique start times: $\sum_l x_{il} = 1, \quad i = 0, 1, \dots, n$

- Sequencing (dependency) relations must be satisfied

$$t_i \geq t_j + d_j \quad \forall (v_j, v_i) \in E \Rightarrow \sum_l l \cdot x_{il} \geq \sum_l l \cdot x_{jl} + d_j$$

- Resource constraints

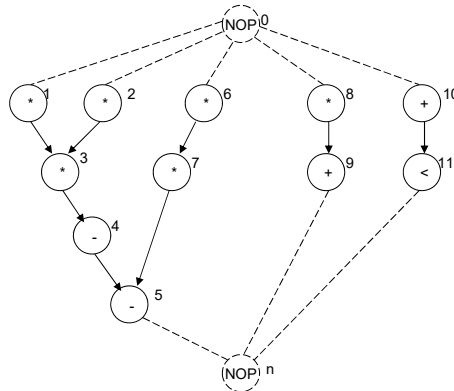
$$\sum_{i:T(v_i)=k} \sum_{m=l-d_i+1}^l x_{im} \leq a_k, \quad k = 1, \dots, n_{res}, \quad l = 1, \dots, \bar{\lambda} + 1$$

- **Objective:** $\min c^T t$

- t = start times vector, c = cost weight (e.g., $[0 \ 0 \ \dots \ 1]$)

- When $c = [0 \ 0 \ \dots \ 1]$, $c^T t = \sum_l l \cdot x_{nl}$

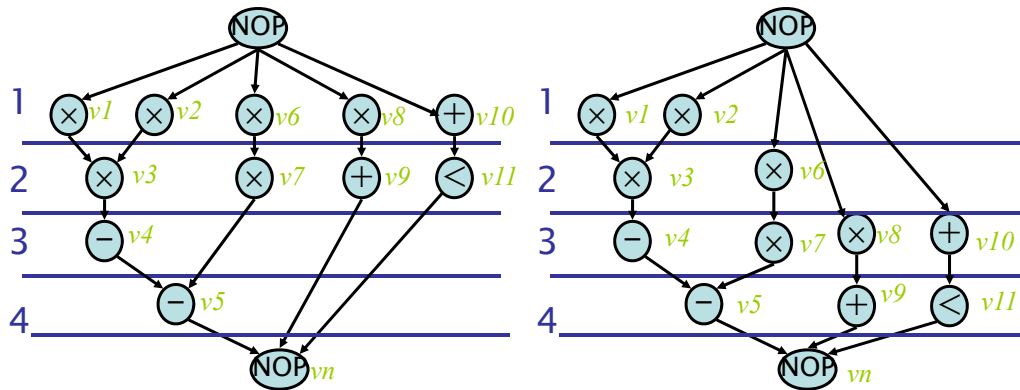
ILP Example



- **Resource constraints**
 - 2 ALUs; 2 Multipliers
 - $a_1 = 2$; $a_2 = 2$
- **Single-cycle operation**
 - $d_i = 1$ for all i

ILP Example

- Assume $\bar{\lambda} = 4$
- **First, perform ASAP and ALAP**
 - (we can write the ILP without ASAP and ALAP, but using ASAP and ALAP will simplify the inequalities)



ILP Example: Unique Start Times

- Without using ASAP and ALAP values:

$$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} = 1$$

$$x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} = 1$$

...

...

...

$$x_{11,1} + x_{11,2} + x_{11,3} + x_{11,4} = 1$$

- Using ASAP and ALAP:

$$x_{1,1} = 1$$

$$x_{2,1} = 1$$

$$x_{3,2} = 1$$

$$x_{4,3} = 1$$

$$x_{5,4} = 1$$

$$x_{6,1} + x_{6,2} = 1$$

$$x_{7,2} + x_{7,3} = 1$$

$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$

$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$

....

ILP Example: Dependency Constraints

- Using ASAP and ALAP, the non-trivial inequalities are: (assuming unit delay for + and *)

$$2.x_{7,2} + 3.x_{7,3} - x_{6,1} - 2.x_{6,2} - 1 \geq 0$$

$$2.x_{9,2} + 3.x_{9,3} + 4.x_{9,4} - x_{8,1} - 2.x_{8,2} - 3.x_{8,3} - 1 \geq 0$$

$$2.x_{11,2} + 3.x_{11,3} + 4.x_{11,4} - x_{10,1} - 2.x_{10,2} - 3.x_{10,3} - 1 \geq 0$$

$$4.x_{5,4} - 2.x_{7,2} - 3.x_{7,3} - 1 \geq 0$$

$$5.x_{n,5} - 2.x_{9,2} - 3.x_{9,3} - 4.x_{9,4} - 1 \geq 0$$

$$5.x_{n,5} - 2.x_{11,2} - 3.x_{11,3} - 4.x_{11,4} - 1 \geq 0$$

ILP Example: Resource Constraints

- Resource constraints (assuming 2 adders and 2 multipliers)

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq 2$$

$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \leq 2$$

$$x_{7,3} + x_{8,3} \leq 2$$

$$x_{10,1} \leq 2$$

$$x_{9,2} + x_{10,2} + x_{11,2} \leq 2$$

$$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \leq 2$$

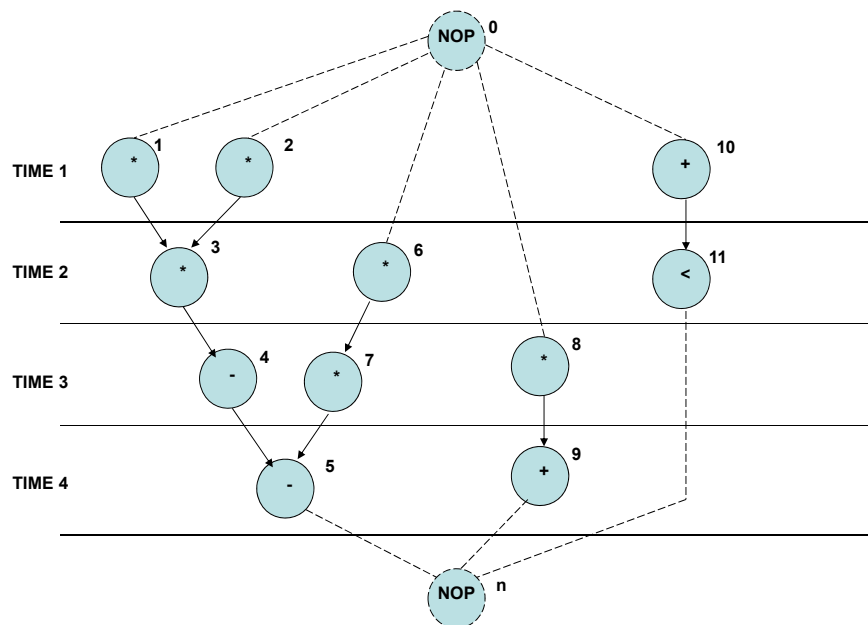
$$x_{5,4} + x_{9,4} + x_{11,4} \leq 2$$

- Objective:

- Since $\lambda=4$ and sink has no mobility, any feasible solution is optimum, but we can use the following anyway:

$$\text{Min } x_{n,1} + 2 \cdot x_{n,2} + 3 \cdot x_{n,3} + 4 \cdot x_{n,4}$$

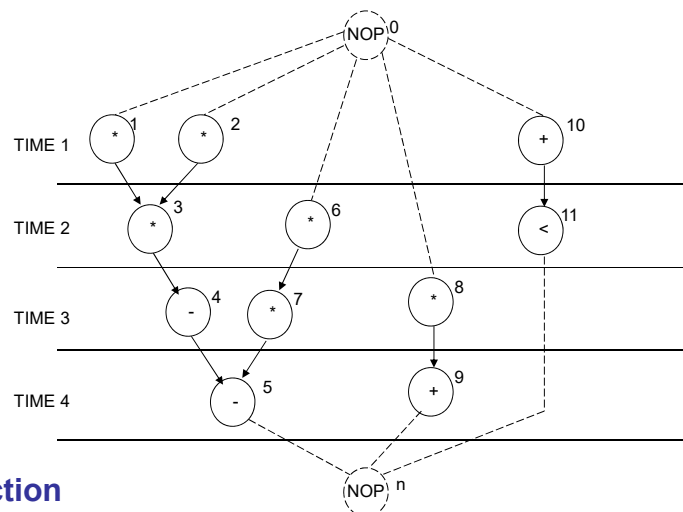
ILP Example: Solution



MR-LCS Dual ILP formulation

- **Minimize resource usage under latency constraint**
- **Additional constraint**
 - Latency bound must be satisfied
 - $\sum_l l x_{nl} \leq \lambda + 1$
- **Resource usage is unknown in the constraints**
 - Resource usage is the objective to minimize

MR-LCS ILP Example



- **Cost function**
 - Multiplier area = 5
 - ALU area = 1
 - Objective function: $5a_1 + a_2$

ILP Solving

- **Use standard ILP packages**
- **Transform into LP problem**
- **Advantages**
 - Exact method
 - Others constraints can be incorporated
- **Disadvantages**
 - Works well up to few thousand variables

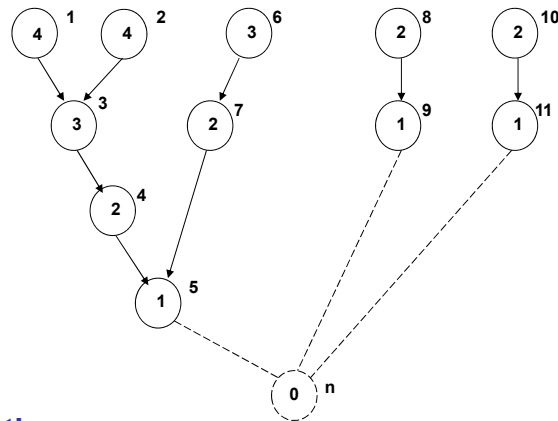
Hu's Algorithm

- **Simple case of the scheduling problem**
 - Operations of unit delay
 - Operations (and resources) of the same type
- **Hu's algorithm**
 - Greedy, polynomial *and* optimal (exact)
 - Computes lower bound on number of resources for given latency
OR
Computes lower bound on latency subject to resource constraints
- **Basic idea**
 - Label operations based on their distances from the sink
 - Try to schedule nodes with higher labels first (i.e., most “critical” operations have priority)

Hu's Algorithm with \bar{a} Resources

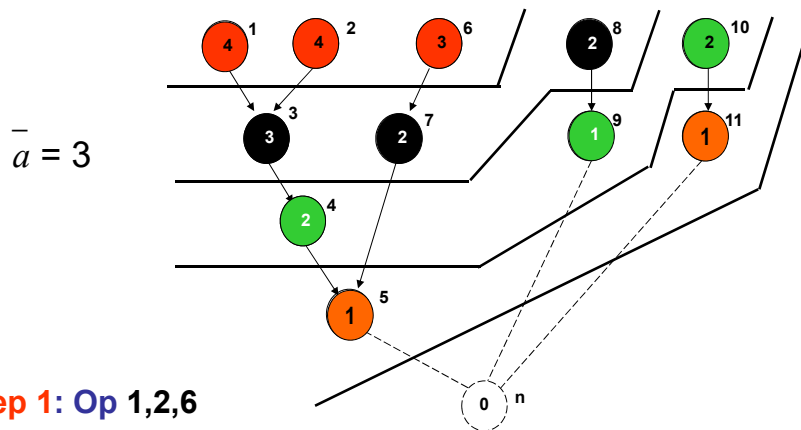
- Label operations with distance to sink
- Set step $l = 1$
- Repeat until all ops are scheduled
 - $U =$ unscheduled vertices in V
 - Predecessors have been scheduled (or no predecessors)
 - Select $S \subseteq U$ resources with
 - $|S| \leq \bar{a}$
 - Maximal labels
 - Schedule the S operations at step l
 - Increment step $l = l + 1$

Hu's Algorithm Example



- **Assumptions**
 - One resource type only
 - All operations have unit delay
- **Labels**
 - Distance to sink

Hu's Algorithm Example



Step 1: Op 1,2,6

Step 2: Op 3,7,8

Step 3: Op 4,9,10

Step 4: Op 5,11

List Scheduling

- **Heuristic method for:**
 - Min *latency* subject to *resource bound* (ML-RCS)
 - Min *resource* subject to *latency bound* (MR-LCS)
- **Greedy strategy (like Hu's)**
 - Does not guarantee optimality (unlike Hu's)
- **General graphs (unlike Hu's)**
 - Resource constraints on different resource types
 - Operations of arbitrary delay
- **Priority list heuristics**
 - Priority decided by criticality (similar to Hu's)
 - Longest path to sink, longest path to timing constraint
 - $O(n)$ time complexity

List Scheduling for Minimum Latency

```

LIST_L( G(V, E), a ) {
  l = 1;
  repeat {
    for each resource type k = 1, 2, ..., n_res {
      Determine ready operations Ul,k;
      Determine unfinished operations Tl,k;
      Select Sk ⊆ Ul,k vertices, s.t. |Sk| + |Tl,k| ≤ ak;
      Schedule the Sk operations at step l;
    }
    l = l + 1;
  }
  until (vn is scheduled);
  return (t);
}

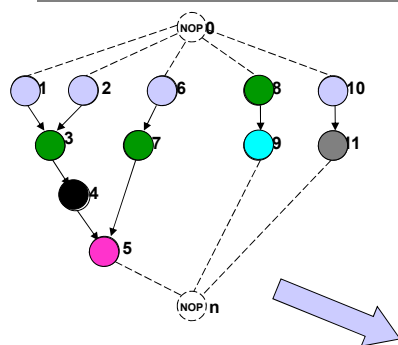
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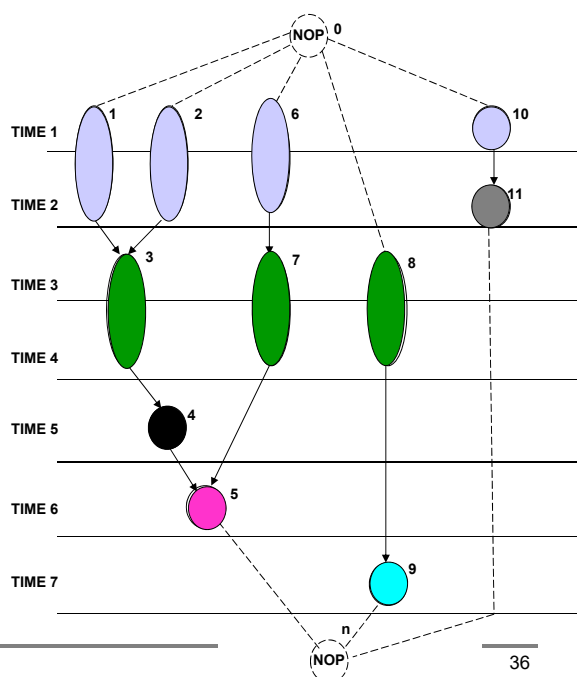
List Scheduling Example



Resource bounds

3 multipliers with delay 2

1 ALU with delay 1



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Lecture 12: Outline

- ✓ The scheduling problem
- ✓ Unconstrained scheduling
- ✓ Resource constrained (RC) scheduling
- Time constrained (TC) scheduling
 - ✓ Exact methods
 - ✓ ILP formulations
 - ✓ Hu's algorithm
 - Heuristics
 - List scheduling
 - Force-directed scheduling
- Advanced scheduling problems

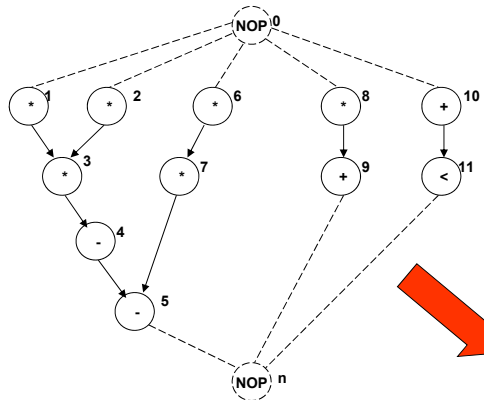
List Scheduling for Minimum Resources

```

LIST_R(  $G(V, E), \bar{\lambda}$  ) {
   $a = 1$ ;
  Compute the latest possible start times  $t^L$  by ALAP (  $G(V, E), \bar{\lambda}$  );
  if ( $t_0 < 0$ )
    return ( $\emptyset$ );
   $l = 1$ ;
  repeat {
    for each resource type  $k = 1, 2, \dots, n_{res}$  {
      Determine ready operations  $U_{l,k}$ ;
      Compute the slacks  $\{s_i = t_i - l \text{ for all } v_i \in U_{l,k}\}$ ;
      Schedule candidate operations with zero slack and update  $a$ ;
      Schedule candidate operations not needing add'l resources;
    }
     $l = l + 1$ ;
  }
  until ( $y_n$  is scheduled) ;
  return ( $t, a$ );
}

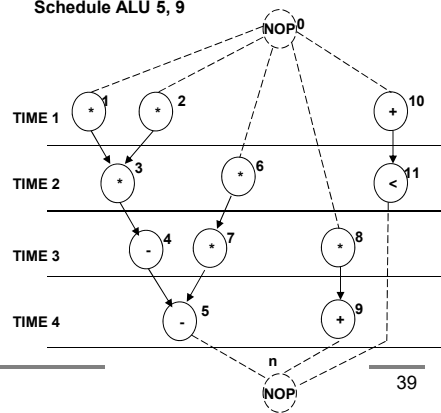
```

List Scheduling Example



- **Assumptions**
 - Unit-delay resources
 - Maximum latency = 4
- **Start with**
 - $a_1 = 1$ multiplier
 - $a_2 = 1$ ALUs

- Step 1**
Two multiplications on CP
Set $a_1 = 2$
Schedule Mult 1,2
Schedule ALU 10
- Step 2**
Schedule Mult 3,6
Schedule ALU 11
- Step 3**
Schedule Mult 7,8
Schedule ALU 4
- Step 4**
Set $a_2 = 2$
Schedule ALU 5,9



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Force-Directed Scheduling (FDS)

- **Heuristic, similar to list scheduling**
 - Can handle ML-RCS and MR-LCS
 - For ML-RCS, schedules step-by-step
 - BUT, selection of the operations tries to find the *globally* best set of operations
- **Idea [Paulin]**
 - Find the mobility $\mu_i = t_i^L - t_i^S$ of operations (ALAP-ASAP)
 - Look at the operation type probability distributions
 - Try to flatten the operation type distributions
- **Definition: operation probability density**
 - $p_i(l) = \Pr \{ v_i \text{ executes in step } l \}$
 - Assume uniform distribution: $p_i(l) = \frac{1}{\mu_i + 1}$ for $l \in [t_i^S, t_i^L]$

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Force-Directed Scheduling: Definitions

- **Operation-type distribution**
(sum of operation probabilities for each type)
 - $q_k(l) = \sum_{i:T(v_i)=k} p_i(l)$
- **Operation probabilities over control steps**
 - $p_i = \{p_i(0), p_i(1), \dots, p_i(n)\}$
- **Distribution graph of type k over all steps**
 - $\{q_k(0), q_k(1), \dots, q_k(n)\}$
 - $q_k(l)$ can be thought of as *expected* operator cost for implementing operations of type k at step l

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Force-Directed Scheduling Example

$$q_{add}(1) = \frac{1}{3} = 0.33$$

$$q_{add}(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$q_{add}(3) = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 2$$

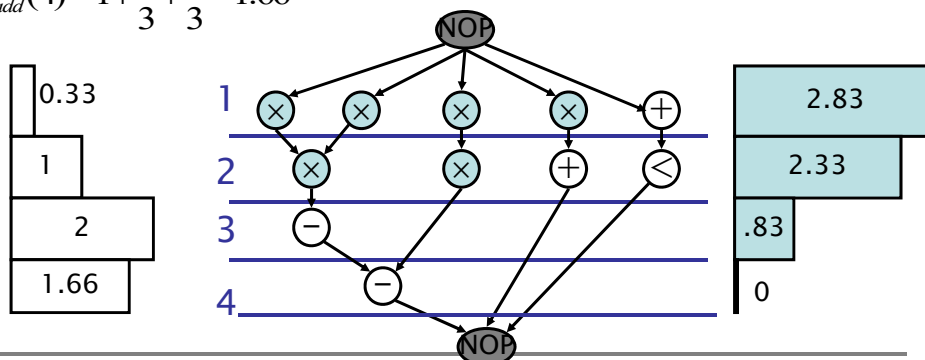
$$q_{add}(4) = 1 + \frac{1}{3} + \frac{1}{3} = 1.66$$

$$q_{mult}(1) = 1 + 1 + \frac{1}{2} + \frac{1}{3} = 2.83$$

$$q_{mult}(2) = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} = 2.33$$

$$q_{mult}(3) = \frac{1}{2} + \frac{1}{3} = 0.83$$

$$q_{mult}(4) = 0$$



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Force-Directed Scheduling Algorithm

- **Very similar to LIST_L($G(V,E), a$)**
 - Compute mobility of operations using ASAP and ALAP
 - Computer operation probabilities and type distributions
 - Select and schedule operation
 - Update operation probabilities and type distributions
 - Go to next step/operation
- **Difference with list scheduling in selecting operations**
 - Select operations with least force
 - $O(n^2)$ time complexity due to pair-wise force computations

Force

- **Used as *priority* function**
- **Force is related to concurrency**
 - Sort operations for least force
- **Mechanical analogy (spring)**
 - Force = constant \times displacement
 - Constant = operation-type distribution
 - Displacement = change in probability

Two Types of Forces

- **Self-force**

- Sum of forces to feasible schedule steps
- Self-force for operation v_i in step l
 - Sum over type distribution \times delta probability

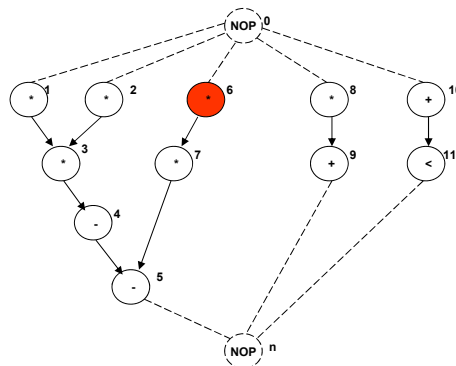
$$\sum_{m \text{ in interval}} q_k(m) (\delta_{lm} - p_i(m))$$

- Higher self-force indicates higher mobility

- **Predecessor/successor-force**

- Related to the predecessors/successors
 - Fixing an operation timeframe restricts timeframe of predecessors/successors
 - Ex: Delaying an operation implies delaying its successors
 - Computed by changes in self-forces of neighbors

Example: Schedule Operation v_6



- **Distribution graphs for multiplier and ALU**



- **Operation v_6 can be scheduled in step 1 or step 2**

Example: Operation v_6

- **Op v_6 can be scheduled in the first two steps**
 - $p(1) = 0.5; p(2) = 0.5; p(3) = 0; p(4) = 0$
- **Distribution**
 - $q(1) = 2.8; q(2) = 2.3$
- **Assign v_6 to step 1**
 - Variation in probability $1 - 0.5 = 0.5$ for step 1
 - Variation in probability $0 - 0.5 = -0.5$ for step 2
- **Self-force**
 - $2.8 * 0.5 - 2.3 * 0.5 = + 0.25$
- **No successor force**
- **Total force = 0.25**

Example: Operation v_6

- **Assign v_6 to step 2**
 - variation in probability $0 - 0.5 = -0.5$ for step 1
 - variation in probability $1 - 0.5 = 0.5$ for step 2
- **Self-force**
 - $-2.8 * 0.5 + 2.3 * 0.5 = - 0.25$
- **Successor-force**
 - Operation v_7 assigned to step 3
 - Succ. force is $2.3 (0 - 0.5) + 0.8 (1 - 0.5) = - .75$
- **Total force = -1**

Example: Operation v_6

- Total force in step 1 = + 0.25
 - Total force in step 2 = -1
- **Conclusion:**
- Least force is for step 2
 - Assigning v_6 to step 2 reduces concurrency

FDS for Minimum Resources

```

FDS (  $G(V, E), \bar{\lambda}$  )
{
  repeat {
    Compute/update the time-frames;
    Compute the operation and type probabilities;
    Compute the self-forces, p/s-forces and total forces;
    Schedule the op. with least force;
  }
  until (all operations are scheduled)
  return (t);
}

```

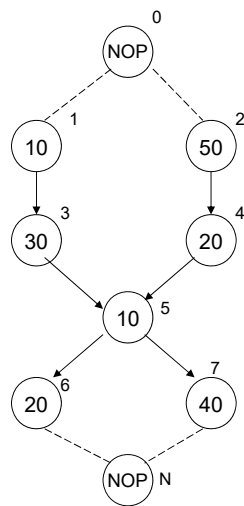
Scheduling Generalizations

- **Detailed timing constraints**
 - Protocol and interface synthesis
 - Bounds on start time differences
 - Forward & backward edges for min/max constraints
- **Operation generalizations**
 - Unbounded delay operations (e.g. synchronization)
 - Relative scheduling w.r. to anchors and combine
 - Conditional operations
- **Resource generalizations**
 - Multi-cycling and chaining
 - Pipelined resources
- **Model generalizations**
 - Hierarchy
 - Pipelining
 - Loops

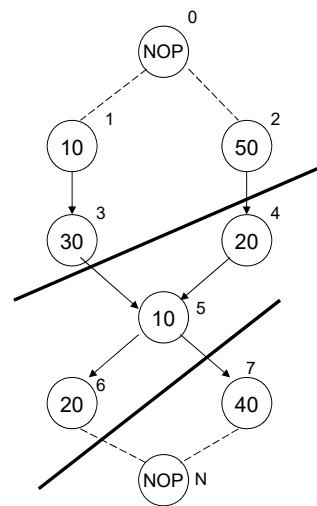
Multi-Cycling and Chaining

- **Consider delays of resources not in terms of cycles**
 - Use scheduling to *chain* multiple operations in the same control step
 - Use scheduling to *multi-cycle* an operation across more than one control step
- **Useful techniques to explore effect of *cycle-time* on area/latency trade-off**
- **Algorithms**
 - ILP
 - ALAP/ASAP
 - List scheduling

Chaining Example



(a)



(b)

- **Cycle-time: 50**

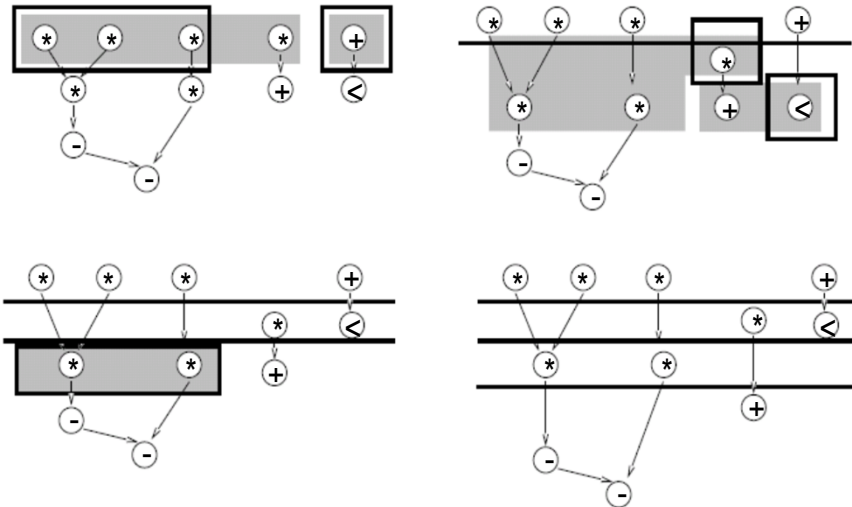
Pipelining

- **Two levels of data pipelining**
 - Structural pipelining
 - Pipelined resources
 - Non-pipelined model
 - Functional pipelining
 - Non-pipelined resources
 - Pipelined model
- **Control pipelining**
 - Pipelined control logic

Structural Pipelining

- **Non-pipelined model using pipelined resources**
- **Resources characterized by**
 - Execution delay
 - Data introduction interval: DII
- **Implications**
 - Operations sharing a pipelined resource are serialized (always)
 - Operations do not have data dependency
- **Solution using list scheduling**
 - Relax criteria for selection of vertices

Structural Pipelining Example



- **3 multipliers w/ 2 cycle delay and $DII = 1$**

Functional (Loop) Pipelining

- **Pipelined model, non-pipelined resources**
- **Assume non-hierarchical graphs**
- **Model characterized by**
 - Latency
 - Initiation interval, II
- **Restart source before completing sink**
 - Implicit loop
 - Limited by loop-carried dependencies
- **Solutions using ILP or heuristics**
 - ILP resource constraints modified to include increased concurrency
 - List or force-directed methods

Pipelining and Concurrency

- **II determines resource usage**
 - Smaller II leads to larger overlaps, higher resource requirements

$$\min\{a_k\} = n_k, \text{ for } II=1 \text{ (all } n_k \text{ operations are concurrent)}$$
 - In general, $\bar{a}_k = \left\lceil \frac{n_k}{II} \right\rceil$
- **Concurrent operations**
 - Operations v_i and v_j are executing concurrently at control step l , if

$$\text{rem}\{t_i/II\} = \text{rem}\{t_j/II\} = l$$
 - Affects the design of the controller circuitry

Loop Scheduling

- **Potential parallelism across loop invocations**
- **Single loop executions**
 - Sequential execution
 - Loop unrolling (known iteration count)
 - Merge multiple iterations into one to provide scheduling opportunities
 - Loop pipelining (iteration count might be unknown)
 - Start next iteration while current one is still running
 - Depends on dependencies across iterations
 - Functional pipelining
- **Merging of multiple loops**
 - Run different loops in parallel (no dependencies)

Loop Scheduling Example

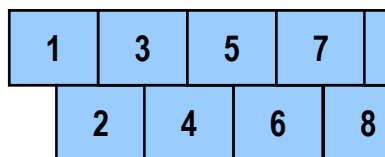
- **Sequential**



- **Unrolled**



- **Pipelined**



- Iteration count = N
- Loop latency = $N \cdot \lambda$
- Pipeline loop iterations with $II < \lambda$
- Latency of the pipelined loop
 - $N \cdot II + \text{overhead}$
 - Overhead = $\lceil \frac{\lambda}{II} \rceil - 1$

Lecture 12: Summary

- **Scheduling determines *area/latency* trade-off**
- **Intractable problem in general**
 - Heuristic algorithms
 - ILP formulation (small-case problems)
- **Several heuristic formulations**
 - List scheduling is the fastest and most used
 - Force-directed scheduling tends to yield good results
- **Several extensions**
 - Chaining and multi-cycling
 - Pipelining