Scheduling

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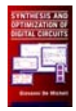


Additional sources:

- Lecture notes by Kia Bazargan, U of M
  - Source: http://www.ece.umn.edu/users/kia/Courses/EE5301
- Notes by Rajesh Gupta, UC San Diego
  - Original source: http://www.cecs.uci.edu/~rgupta/ics280.html

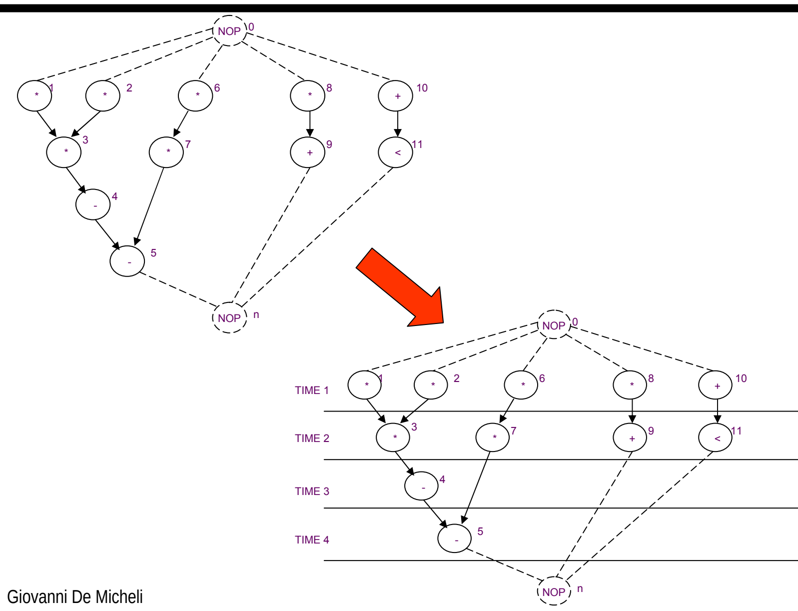
Module 1

- Objectives:
  - The scheduling problem
    - Case analysis
  - Scheduling without constraints
  - Scheduling with timing constraints

Scheduling

- **Circuit model:**
  - Sequencing graph
  - Cycle-time is given
  - Operation delays expressed in cycles
- **Scheduling:**
  - Determine the start times for the operations
  - Satisfying all the sequencing (timing and resource) constraint
- **Goal:**
  - Determine ***area/latency*** trade-off

Example



Taxonomy

- Unconstrained scheduling
- Scheduling with timing constraints:
  - Latency
  - Detailed timing constraints
- Scheduling with resource constraints
- Related problems:
  - Chaining
  - Synchronization
  - Pipeline scheduling

Operation Scheduling

- Input:
  - Sequencing graph G(V, E), with n vertices
  - Cycle time τ.
  - Operation delays $D = \{d_i: i=0..n\}$.
- Output:
  - Schedule ϕ determines start time t_i of operation v_i.
  - Latency $\lambda = t_n - t_0$.
- Goal: determine area / latency tradeoff
- Classes:
  - Non-hierarchical and unconstrained
  - Latency constrained
  - Resource constrained
  - Hierarchical

Simplest method

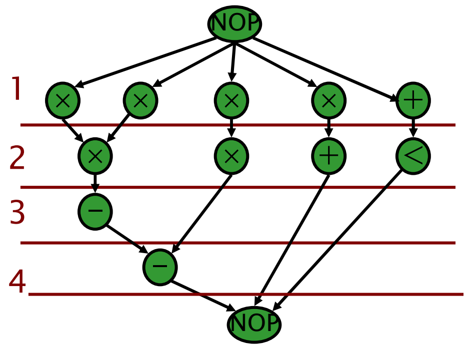
- All operations have bounded delays
- All delays are in cycles:
  - Cycle-time is given
- No constraints – no bounds on area
- Goal:
  - Minimize latency

Min Latency Unconstrained Scheduling

- Simplest case: no constraints, find min latency
- Given set of vertices ***V***, delays ***D*** and a partial order > on operations E, find an integer labeling of operations $\phi: V \rightarrow Z^+$ Such that:
  - $t_i = \phi(v_i)$.
  - $t_i \geq t_j + d_j \quad \forall (v_j, v_i) \in E$.
  - $\lambda = t_n - t_0$ is minimum.
- Solvable in polynomial time
- Bounds on latency for resource constrained problems
- ASAP algorithm used: topological order

ASAP Schedules

- Schedule v_0 at t_0=0.
- While (v_n not scheduled)
  - Select v_i with all scheduled predecessors
  - Schedule v_i at t_i = max $\{t_j+d_j\}$, v_j being a predecessor of v_i.
- Return t_n.



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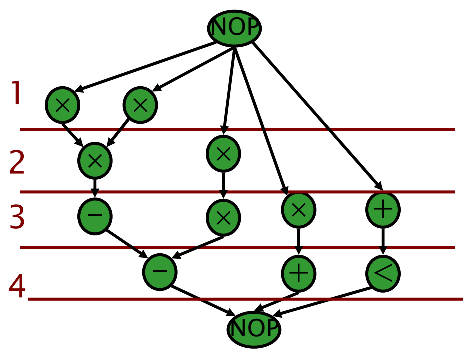
ALAP Schedules

- Schedule v_n at t_n=λ.
- While (v_0 not scheduled)
  - Select v_i with all scheduled successors
  - Schedule v_i at t_i = min $\{t_j$-$d_j\}$, v_j being a succecessor of v_i.



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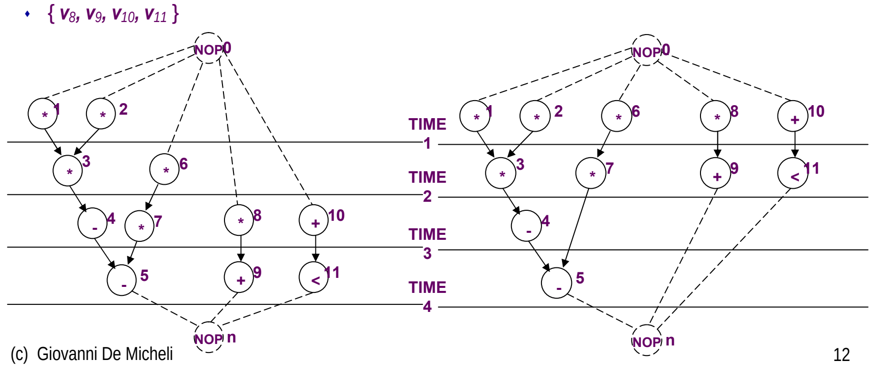
Remarks

- ALAP solves a latency-constrained problem
- Latency bound can be set to latency computed by ASAP algorithm
- Mobility:
  - Defined for each operation
  - Difference between ALAP and ASAP schedule
- Slack on the start time

Example

- Operations with zero mobility:
  - { v_1, v_2, v_3, v_4, v_5 }
  - Critical path
- Operations with mobility one:
  - { v_6, v_7 }
- Operations with mobility two:
  - { v_8, v_9, v_{10}, v_{11} }



Scheduling under detailed timing constraints

- Motivation:
  - Interface design
  - Control over operation start time
- Constraints:
  - Upper/lower bounds on start-time difference of any operation pair
- Feasibility of a solution

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Constraint graph model

- Start from sequencing graph
  - Model delays as weights on edges
- Add forward edges for ***minimum*** constraints:
  - Edge (v_i, v_j) with weight $l_{ij} \rightarrow t_j \geq t_i + l_{ij}$
- Add backward edges for maximum constraints:
  - That is, for constraint from v_i to v_j
    add backward edge (v_j, v_i) with weight: $-u_{ij}$
    - because $t_j \leq t_i + u_{ij} \rightarrow t_i \geq t_j - u_{ij}$

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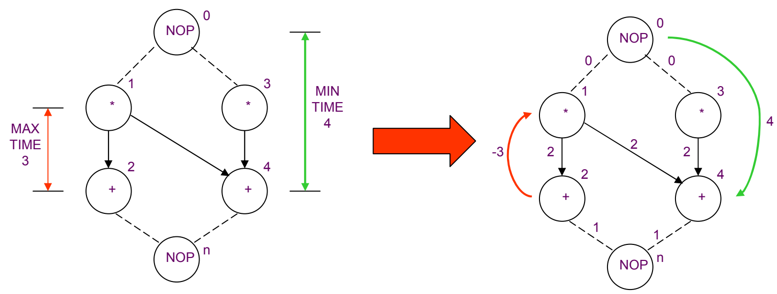
Example



Vertex	Start time
v_0	1
v_1	1
v_2	3
v_3	1
v_4	5
v_n	6

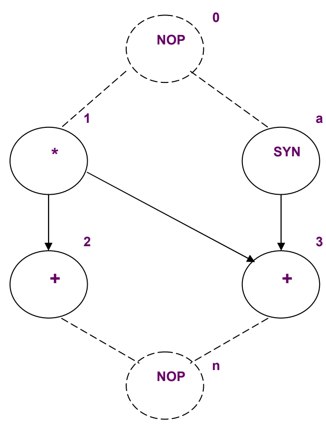
Methods for scheduling under detailed timing constraints

- Assumption:
  - All delays are fixed and known
- Set of linear inequalities
- Longest ***path*** problem
- Algorithms:
  - Bellman-Ford, Liao-Wong
- Extensions:
  - Unbounded delays, relative scheduling

Method for scheduling with unbounded-delay operations

- Unbounded delays:
  - Synchronization
  - Unbounded-delay operations (e.g. loops)
- Anchors:
  - Unbounded-delay operations
- Relative scheduling:
  - Schedule ops w.r.t. to the anchors
  - Combine schedules

Example



- $t_3 = \max \{ t_1 + d_1; t_a + d_a \}$

Relative scheduling method

- For each vertex:
  - Determine ***relevant anchor set R*** (v_i)
  - Anchors affecting start time
  - Determine time offsets from anchors
- Start-time:
  - Expressed by : $t_i = \max \{ t_a + d_a + t_i \}$
  - Computed only at run-time because delays of anchors are unknown

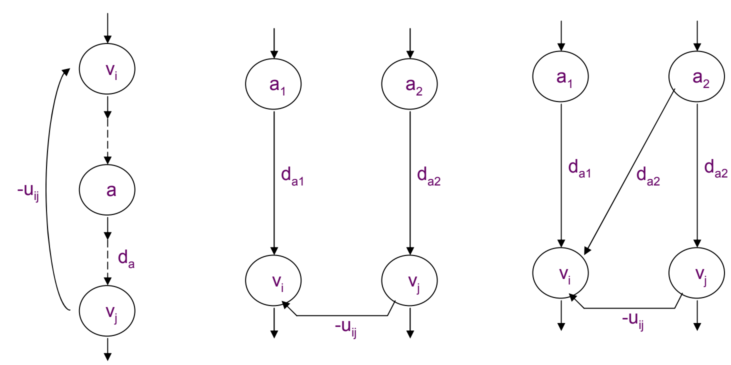
Relative scheduling under timing constraints

- Problem definition:
  - Detailed timing constraints
  - Unbounded delay operations
- Solution:
  - May or may not exist
  - Problem may be ill-specified

Relative scheduling under timing constraints

- **Feasible problem:**
  - A solution exists when unknown delays are zero
- **Well-posed problem:**
  - A solution exists for any value of the unknown delays
- **Theorem:**
  - A constraint graph can be well-posed if there are no cycles with unbounded weights

Example



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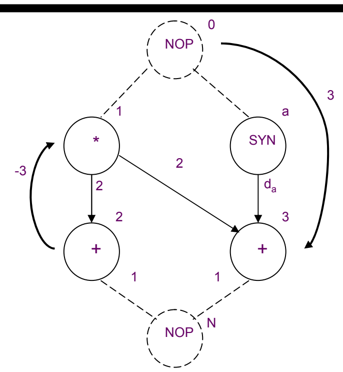
Relative scheduling approach

- Analyze graph:
  - Detect anchors
  - Well-posedness test
  - Determine dependencies from anchors
- Schedule ops with respect to relevant anchors:
  - Bellman-Ford, Liao-Wong, Ku algorithms
- Combine schedules to determine start times:

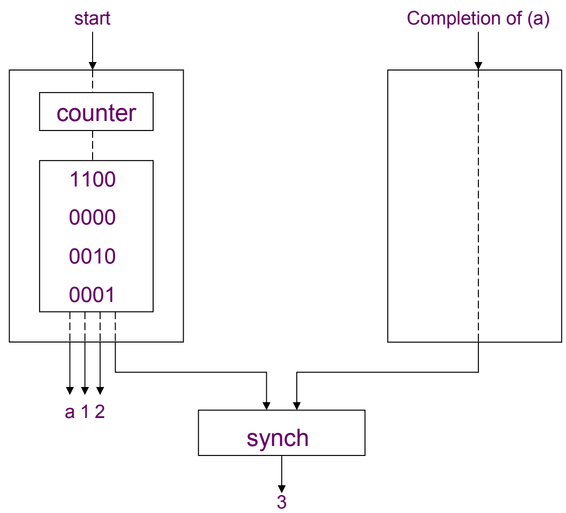
$$t_i = \max_{a \in R(v_i)} \{ t_a + d_a + t_i \}$$

Example



Vertex v_i	Relevant Anchor Set $R(v_i)$	Offsets t_0	t_a
a	$\{v_0\}$	0	-
v_1	$\{v_0\}$	0	-
v_2	$\{v_0\}$	2	-
v_3	$\{v_0, a\}$	3	0

Example of control-unit



Module 2

- Objectives:
  - Scheduling with resource constraints
  - Exact formulation:
    - ILP
    - Hu's algorithm
  - Heuristic methods
    - List scheduling
    - Force-directed scheduling

Scheduling under resource constraints

- Classical scheduling problem:
  - Fix area bound – minimize latency (ML-RCS)
- The amount of available resources affects the achievable latency
- Dual problem:
  - Fix latency bound – minimize resources (MR-LCS)
- Assumption:
  - All delays bounded and known

Minimum latency resource-constrained scheduling (ML-RCS)

- Given a set of ops V with integer delays D, a partial order on the operations E, and upper bounds $\{ a_k; k = 1, 2, \ldots, n_{res} \}$ on resource usage:
- Find an integer labeling of the operation $\varphi : V \rightarrow z^+$ such that :

$t_i = \varphi(v_i)$,

$t_i \geq t_j + d_j$ for all i,j *s.t.* $(v_j, v_i) \in E$,

$| \{ v_i \,|\, T(v_i) = k$ and $t_i \leq l < t_j + d_j \} | \leq a_k$ for all types $k = 1,2,\ldots,n_{res}$ and steps l

and t_n ***is minimum***

Scheduling under resource constraints

- Intractable problem
- Algorithms:
  - Exact:
    - Integer linear program
    - Hu (restrictive assumptions)
  - Approximate :
    - List scheduling
    - Force-directed scheduling

ILP formulation

- Binary decision variables:

  $X = \{ x_{il}, \quad i = 1,2,\ldots. n; \; l = 1,2,\ldots, \bar{\lambda} + 1\}$

  x_{il} is TRUE only when operation v_i starts in step l of the schedule (i.e. $l = t_i$)

  $\bar{\lambda}$ is an upper bound on latency

- Start time of operation v_i : $\Sigma_l \; l \cdot x_{il}$

ILP formulation constraints

- Operations start only once

  $\Sigma\, x_{il} = 1 \quad i = 1, 2, \ldots, n$

- Sequencing relations must be satisfied

  $t_i \geq t_j + d_j \quad \rightarrow \quad t_i - t_j - d_j \geq 0 \quad$ *for all* $(v_j, v_i) \in E$

  $\Sigma\, l \bullet x_{il} - \Sigma\, l \bullet x_{jl} - d_j \geq 0 \quad$ *for all* $(v_j, v_i) \in E$

- Resource bounds must be satisfied

  Simple case (unit delay)

  $\sum_{i:T(v_i)=k} x_{il} \leq a_k \quad k = 1,2,\ldots n_{res}\,; \quad$ *for all* l

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Start Time vs. Execution Time

- For each operation v_i , only one start time
- If $d_i=1$, then the following questions are the same:
  - Does operation v_i start at step l?
  - Is operation v_i running at step l?
- But if $d_i>1$, then the two questions should be formulated as:
  - Does operation v_i start at step l?
    - Does $x_{il} = 1$ hold?
  - Is operation v_i running at step l?
    - Does the following hold?

$$\sum_{m=l-d_i+1}^{l} x_{im} \stackrel{?}{=} 1$$

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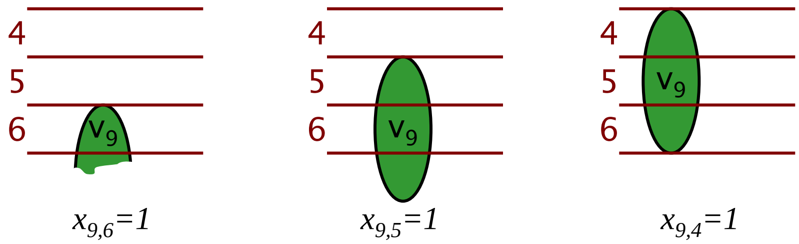
Operation v_i Still Running at Step l ?

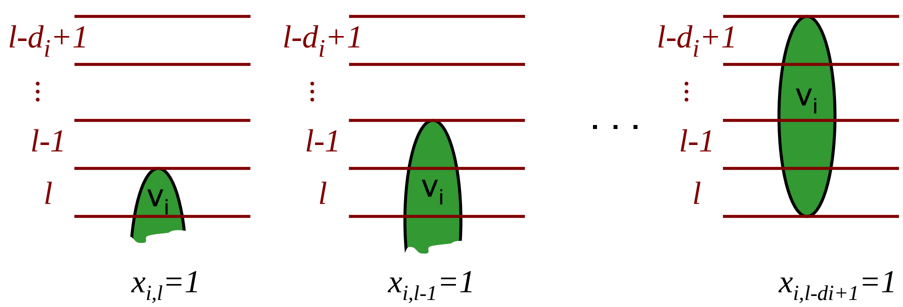
- Is v_9 running at step 6?
  - Is $x_{9,6} + x_{9,5} + x_{9,4} = 1$?



- Note:
  - Only one (if any) of the above three cases can happen
  - To meet resource constraints, we have to ask the same question for ALL steps, and ALL operations of that type

Operation v_i Still Running at Step l ?

- Is v_i running at step l ?
  - Is $x_{i,l} + x_{i,l-1} + ... + x_{i,l-di+1} = 1$?



ILP Formulation of ML-RCS

- Constraints:
  - Unique start times: $\sum_l x_{il} = 1, \quad i = 0,1,\ldots,n$
  - Sequencing (dependency) relations must be satisfied

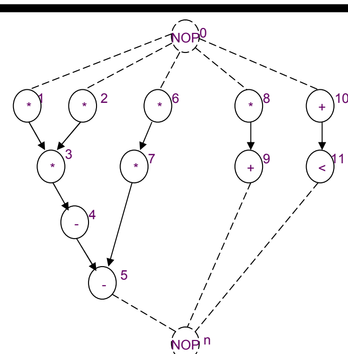
$$t_i \geq t_j + d_j \;\; \forall (v_j, v_i) \in E \Rightarrow \sum_l l \cdot x_{il} \geq \sum_l l \cdot x_{jl} + d_j$$

  - Resource constraints

$$\sum_{i:T(v_i)=k} \; \sum_{m=l-d_i+1}^{l} x_{im} \leq a_k, \quad k = 1,\ldots,n_{res}, \quad l = 1,\ldots,\overline{\lambda}+1$$

- Objective: min $c^T t$.
  - t =start times vector, c =cost weight (e.g., [0 0 ... 1])
  - When c =[0 0 ... 1], $c^T t = \sum_l l \cdot x_{nl}$

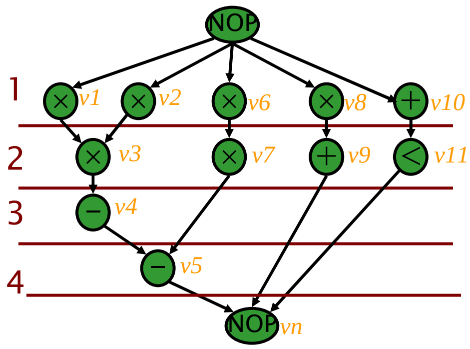
Example

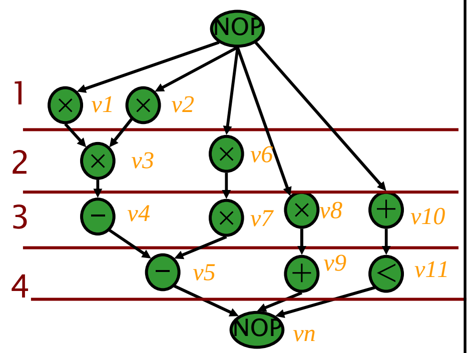


- Resource constraints:
  - 2 ALUs; 2 Multipliers
  - $a_1 = 2$; $a_2 = 2$
- Single-cycle operation
  - $d_i = 1$ for all i

ILP Example

- Assume $\overline{\lambda} = 4$
- First, perform ASAP and ALAP
  - (we can write the ILP without ASAP and ALAP, but using ASAP and ALAP will simplify the inequalities)





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ILP Example: Unique Start Times Constraint

- Without using ASAP and ALAP values:

$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} = 1$

$x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} = 1$

...

...

...

$x_{11,1} + x_{11,2} + x_{11,3} + x_{11,4} = 1$

- Using ASAP and ALAP:

$x_{1,1} = 1$

$x_{2,1} = 1$

$x_{3,2} = 1$

$x_{4,3} = 1$

$x_{5,4} = 1$

$x_{6,1} + x_{6,2} = 1$

$x_{7,2} + x_{7,3} = 1$

$x_{8,1} + x_{8,2} + x_{8,3} = 1$

$x_{9,2} + x_{9,3} + x_{9,4} = 1$

....

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ILP Example: Dependency Constraints

- Using ASAP and ALAP, the non-trivial inequalities are: (assuming unit delay for + and *)

$$2.x_{7,2} + 3.x_{7,3} - x_{6,1} - 2.x_{6,2} - 1 \geq 0$$
$$2.x_{9,2} + 3.x_{9,3} + 4.x_{9,4} - x_{8,1} - 2.x_{8,2} - 3.x_{8,3} - 1 \geq 0$$
$$2.x_{11,2} + 3.x_{11,3} + 4.x_{11,4} - x_{10,1} - 2.x_{10,2} - 3.x_{10,3} - 1 \geq 0$$
$$4.x_{5,4} - 2.x_{7,2} - 3.x_{7,3} - 1 \geq 0$$
$$5.x_{n,5} - 2.x_{9,2} - 3.x_{9,3} - 4.x_{9,4} - 1 \geq 0$$
$$5.x_{n,5} - 2.x_{11,2} - 3.x_{11,3} - 4.x_{11,4} - 1 \geq 0$$

ILP Example: Resource Constraints

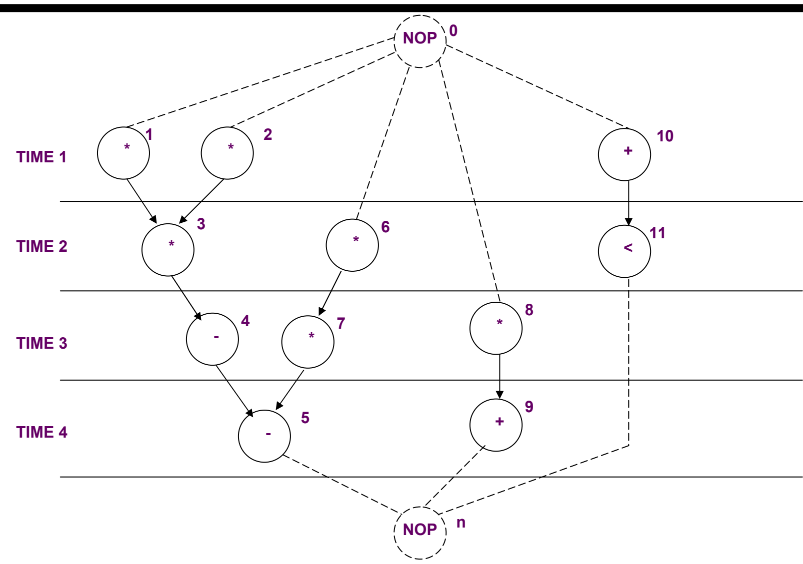
- Resource constraints (assuming 2 adders and 2 multipliers)

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq 2$$
$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \leq 2$$
$$x_{7,3} + x_{8,3} \leq 2$$
$$x_{10,1} \leq 2$$
$$x_{9,2} + x_{10,2} + x_{11,2} \leq 2$$
$$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \leq 2$$
$$x_{5,4} + x_{9,4} + x_{11,4} \leq 2$$

- Objective:
  - Since λ=4 and sink has no mobility, any feasible solution is optimum, but we can use the following anyway:

$$Min \quad x_{n,1} + 2.x_{n,2} + 3.x_{n,3} + 4.x_{n,4}$$

Example



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MR-LCS dual ILP formulation

- Minimize resource usage under latency constraint
- Additional constraint:
  - Latency bound must be satisfied
  - $\Sigma_l \, l \, x_{nl} \leq \lambda + 1$
- Resource usage is unknown in the constraints
- Resource usage is the objective to minimize

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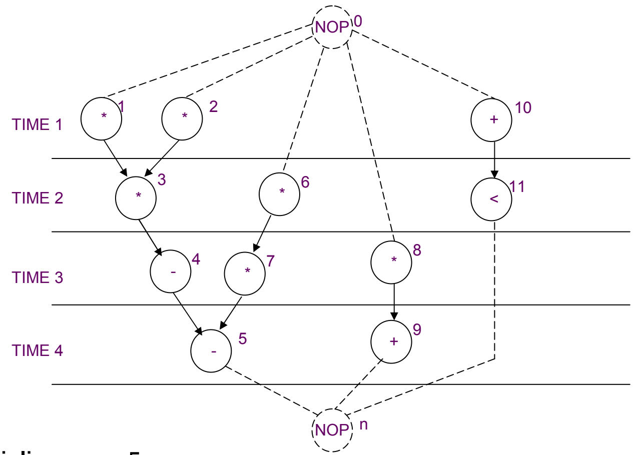
Example



- Multiplier area = 5
- ALU area = 1.
- Objective function: $5a_1 + a_2$

ILP Solution

- Use standard ILP packages
- Transform into LP problem
- Advantages:
  - Exact method
  - Others constraints can be incorporated
- Disadvantages:
  - Works well up to few thousand variables

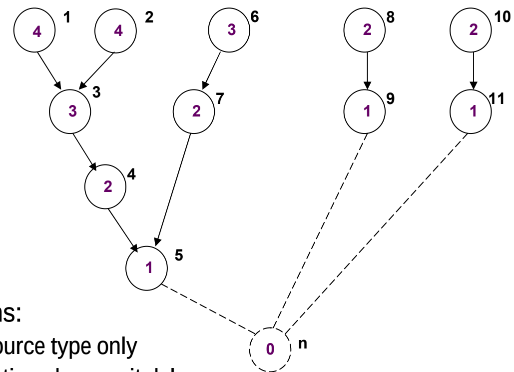
Hu's Algorithm

- Simple case of the scheduling problem
  - Operations of unit delay
  - Operations (and resources) of the same type
- Hu's algorithm
  - Greedy, polynomial AND optimal (exact)
  - Computes lower bound on number of resources for given latency
    OR
    Computes lower bound on latency subject to resource constraints
- Basic idea:
  - Label operations based on their distances from the sink
  - Try to schedule nodes with higher labels first
    (i.e., most "critical" operations have priority)

Hu's algorithm with $\bar{a}$ resources

- Label operations with distance to sink
- Set step $l = 1$
- Repeat until all ops are scheduled:
  - U = unscheduled vertices in V
    - predecessors have been scheduled (or no predecessors)
  - Select $S \subseteq U$ resources with
    - $|S| \leq \bar{a}$
    - Maximal labels
  - Schedule the S operations at step l
  - Increment step $l = l + 1$

Example



- Assumptions:
  - One resource type only
  - All operations have unit delay
- Labels:
  - Distance to sink

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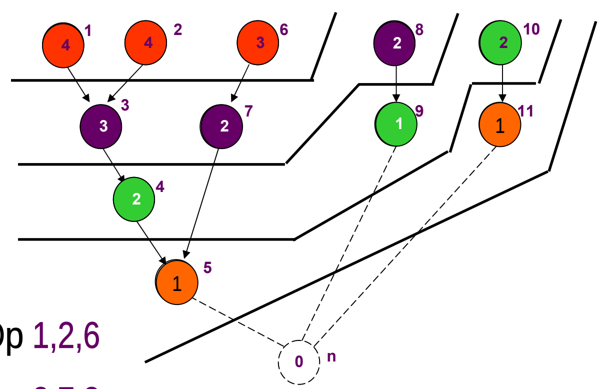
Example

$\bar{a} = 3$



Step 1: Op 1,2,6

Step 2: Op 3,7,8

Step 3: Op 4,9,10

Step 4: Op 5,11

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List scheduling algorithms

- Heuristic method for:
  - Min ***latency*** subject to ***resource bound*** (ML-RCS)
  - Min ***resource*** subject to ***latency bound*** (MR-LCS)
- Greedy strategy (like Hu's)
  - Does not guarantee optimality (unlike Hu's)
- General graphs (unlike Hu's)
  - Resource constraints on different resource types
  - Operations of arbitrary delay
- Priority list heuristics
  - Priority decided by criticality (similar to Hu's)
  - Longest path to sink, longest path to timing constraint
  - $O(n)$ time complexity

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List scheduling algorithm for minimum latency

LIST_L(G(V, E), a) {
    l = 1;
    repeat {
        for each resource type k = 1, 2, …, n_{res} {
            Determine ready operations $U_{l,k}$;
            Determine unfinished operations $T_{l,k}$;
            Select $S_k \subseteq U_{l,k}$ vertices, s.t. $|S_k| + |T_{l,k}| \leq a_k$;
            Schedule the S_k operations at step l;
        }
        $l = l + 1$;
    }
    until (v_n is scheduled) ;
    return (t);
}

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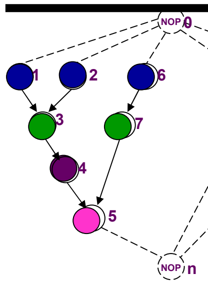
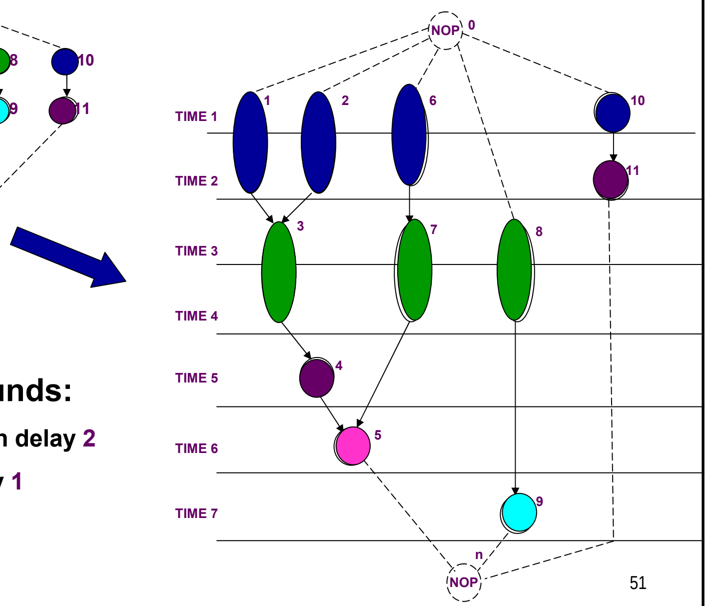
Example





Resource bounds:

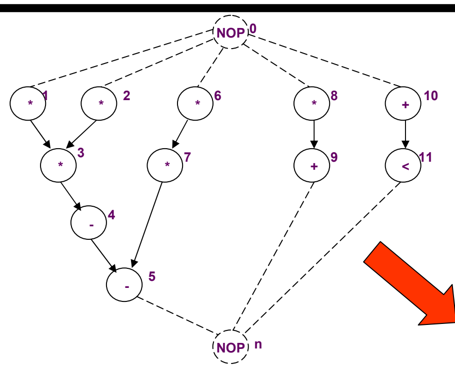
3 multipliers with delay 2

1 ALU with delay 1

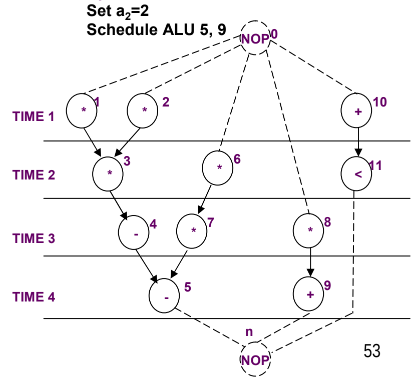
List scheduling algorithm for minimum resource usage

LIST_R(G(V, E), $\overline{\lambda}$) {
  a = 1;
  Compute the latest possible start times t^L by ALAP (G(V, E), $\overline{\lambda}$);
  if (t_0 < 0)
    return ($\emptyset$);
  l = 1;
  repeat {
    for each resource type k = 1, 2, …, n_{res} {
      Determine ready operations $U_{l,k}$;
      Compute the slacks { $s_i = t_i - l$ for all $v_i \in U_{lk}$};
      Schedule the candidate operations with zero slack and update a;
      Schedule the candidate operations not needing additional resources;
      }
    $l = l + 1$;
  }
  until (v_n is scheduled) ;
  return (t, a);
}

Example



Step 1
Two multiplications on CP
Set $a_1 = 2$
Schedule Mult 1,2
Schedule ALU 10
Step 2
Schedule Mult 3, 6
Schedule ALU 11
Step 3
Schedule Mult 7,8
Schedule ALU 4
Step 4
Set a_2=2
Schedule ALU 5, 9



Assumptions
Unit-delay resources
Maximum latency = 4
Start with :
a_1 = 1 multiplier
a_2 = 1 ALUs

Force-Directed Scheduling

- Heuristic, similar to list scheduling
  - Can handle ML-RCS and MR-LCS
  - For ML-RCS, schedules step-by-step
  - BUT, selection of the operations tries to find the ***globally*** best set of operations
- Idea [Paulin]
  - Find the mobility $\mu_i = t_i^L - t_i^S$ of operations (ALAP-ASAP)
  - Look at the operation type probability distributions
  - Try to flatten the operation type distributions
- Definition: operation probability density
  - $p_i(l)$ = Pr { v_i executes in step l }
  - Assume uniform distribution: $p_i(l) = \dfrac{1}{\mu_i + 1} \quad for\ l \in [t_i^S, t_i^L]$

Force-Directed Scheduling: Definitions

- Operation-type distribution (sum of operation probabilities for each type)
  - $$q_k(l) = \sum_{i:T(v_i)=k} p_i(l)$$
- Operation probabilities over control steps:
  - $$p_i = \{p_i(0), p_i(1), \ldots, p_i(n)\}$$
- Distribution graph of type ***k*** over all steps:
  - $$\{q_k(0), q_k(1), \ldots, q_k(n)\}$$
  - $q_k(l)$ can be thought of as ***expected*** operator cost for implementing operations of type ***k*** at step ***l***.

Example

$$q_{add}(1) = \frac{1}{3} = 0.33$$

$$q_{add}(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$q_{add}(3) = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 2$$

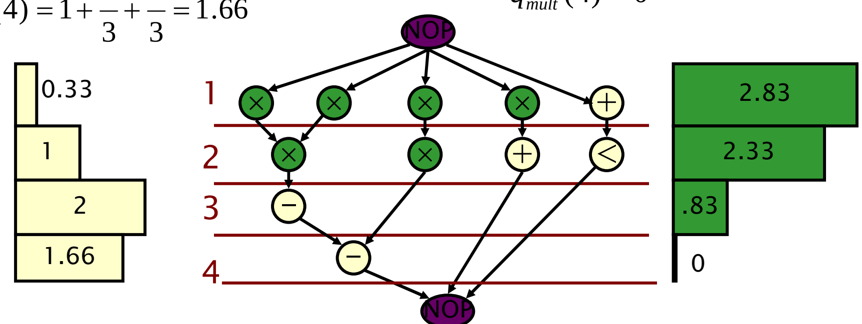
$$q_{add}(4) = 1 + \frac{1}{3} + \frac{1}{3} = 1.66$$

$$q_{mult}(1) = 1 + 1 + \frac{1}{2} + \frac{1}{3} = 2.83$$

$$q_{mult}(2) = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} = 2.33$$

$$q_{mult}(3) = \frac{1}{2} + \frac{1}{3} = 0.83$$

$$q_{mult}(4) = 0$$



Force-Directed Scheduling Algorithm

- Very similar to LIST_L(G(V,E), a)
  - Compute mobility of operations using ASAP and ALAP
  - Computer operation probabilities and type distributions
  - Select and schedule operations
  - Update operation probabilities and type distributions
  - Go to next control step
- Difference with list scheduling in selecting operations
  - Select operations with least force
  - ***O***(***n***^{2}) time complexity due to pair-wise force computations

Force

- Used as ***priority*** function
- Force is related to concurrency:
  - Sort operations for least force
- Mechanical analogy:
  - Force = constant x displacement
    - Constant = operation-type distribution
    - Displacement = change in probability

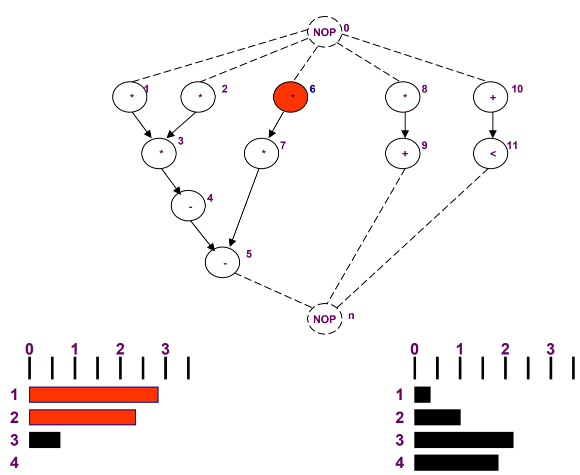
Two Types of Forces

- **Self-force:**
  - Sum of forces to feasible schedule steps
  - Self-force for operation v_i in step l
    - Sum over type distribution x delta probability

      $\Sigma_{m \text{ in interval}}\; q_k(m)\;(\delta_{lm} - p_i(m))$
    - Higher self-force indicates higher mobility

- **Predecessor/successor-force:**
  - Related to the predecessors/successors
    - Fixing an operation timeframe restricts timeframe of predecessors/successors
    - Ex: Delaying an operation implies delaying its successors
    - Computed by changes in self-forces of neighbors

Example: Schedule operation v_6



Operation v_6 can be scheduled in step 1 or step 2

Example: operation v_6

- Op v_6 can be scheduled in the first two steps
  - $p(1) = 0.5$; $p(2) = 0.5$; $p(3) = 0$; $p(4) = 0$
- Distribution: $q(1) = 2.8$; $q(2) = 2.3$
- Assign v_6 to step 1:
  - variation in probability 1 – 0.5 = 0.5 for step 1
  - variation in probability 0 – 0.5 = -0.5 for step 2
- Self-force: 2.8 * 0.5 – 2.3 * 0.5 = + 0.25
- No successor force

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Example: operation v_6

- Assign v_6 to step 2:
  - variation in probability 0 – 0.5 = -0.5 for step 1
  - variation in probability 1 – 0.5 = 0.5 for step 2
- Self-force: - 2.8 * 0.5 + 2.3 * 0.5 = - 0.25
- Successor-force:
  - Operation v_7 assigned to step 3
  - Succ. force is 2.3 (0- 0.5) + 0.8 (1 – 0.5) = - .75
- Total force = -1

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Example: operation v_6

- Total force in step 1 = + 0.25
- Total force in step 2 = -1
- Conclusion:
  - Least force is for step 2
  - Assigning v_6 to step 2 reduces concurrency

Force-directed scheduling algorithm for minimum resources

```
FDS ( G ( V, E ), λ ) {
    repeat {
        Compute/update the time-frames;
        Compute the operation and type probabilities;
        Compute the self-forces, p/s-forces and total forces;
        Schedule the op. with least force;
    } until (all operations are scheduled)
    return (t);
}
```

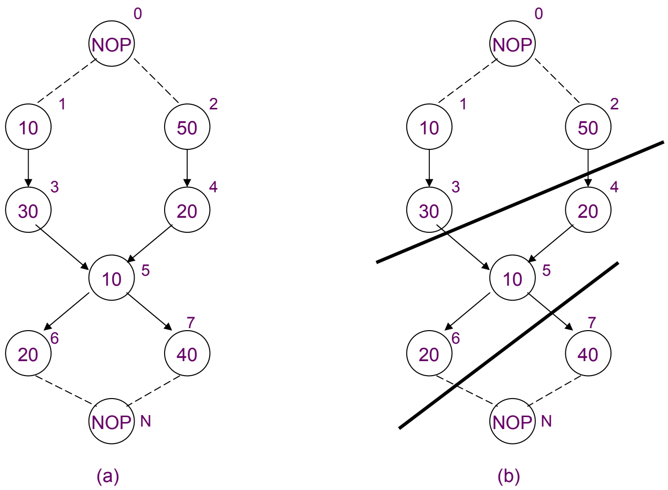
Scheduling Generalizations

- Conditional operations
- Hierarchy
- Resource generalizations
  - Multi-cycling and chaining
  - Pipelined resources
- Model generalizations
  - Pipelining
  - Loops

Multi-Cycling and Chaining

- Consider propagation delays of resources not in terms of cycles
- Use scheduling to ***chain*** multiple operations in the same control step
- Useful technique to explore effect of ***cycle-time*** on area/latency trade-off
- Algorithms:
  - ILP, ALAP/ASAP, list scheduling

Example



◆ Cycle-time: 60

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Pipelining

- Two levels of pipelining:
  - Structural pipelining
    - Pipelined resources
    - Non-pipelined model
  - Functional pipelining
    - Non-pipelined resources
    - Pipelined model

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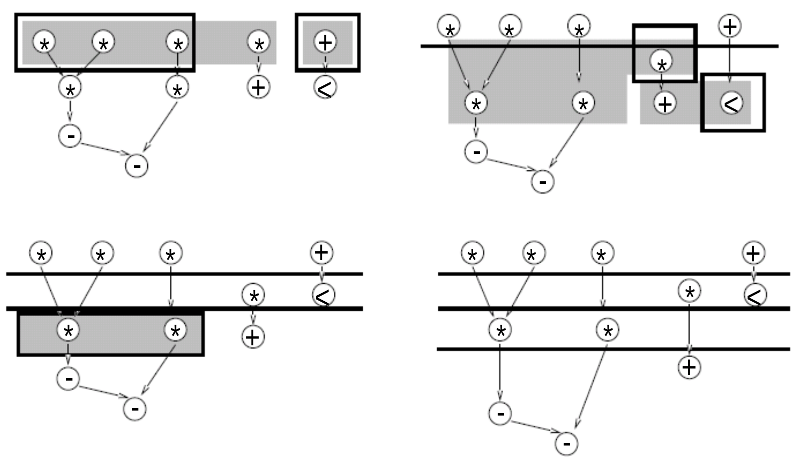
Structural Pipelining

- Non-pipelined model using pipelined resources
- Resources characterized by
  - Execution delay
  - Data introduction interval: ***DII***
- Implications
  - Operations sharing a pipelined resource are serialized (always)
  - Operations do not have data dependency
- Solution using list scheduling
  - Relax criteria for selection of vertices

Structural Pipelining Example



- 3 multipliers w/ 2 cycle delay and ***DII*** = 1

Functional Pipelining

- Pipelined model, non-pipelined resources
- Assume non-hierarchical graphs
- Model characterized by
  - Latency
  - Initiation interval, ***II***
- Restart source before completing sink
  - Implicit loop
- Solutions using ILP or heuristics
  - ILP resource constraints modified to include increased concurrency
  - List or force-directed methods

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Pipelining and concurrency

- ***II*** determines resource usage
  - Smaller ***II*** leads to larger overlaps, higher resource requirements
    $\min\{a_k\} = n_k$, for II=1 (all n_k operations are concurrent)
  - In general, $\bar{a}_k = \left\lceil \frac{n_k}{II} \right\rceil$
- Concurrent operations
  - Operations v_i and v_j are executing concurrently at control step l, if
    $\text{rem}\{ t_i / II \} = \text{rem}\{ t_j / II \} = l$
  - Affects the design of the controller circuitry

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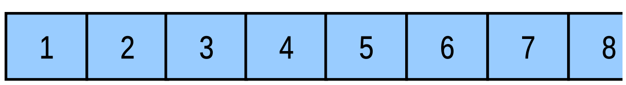
Loop Scheduling

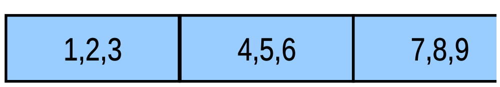
- Potential parallelism ***across*** loop invocations
- Single loop executions
  - Sequential execution
  - Loop unrolling (known iteration count)
    - Merge multiple iterations into one to provide scheduling opportunities
  - Loop pipelining (iteration count might be unknown)
    - Start next iteration while current one is still running
    - Depends on dependencies across iterations
- Merging of multiple loops
  - Run different loops in parallel (no dependencies)

Loop Scheduling Example

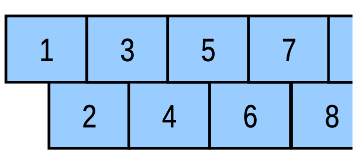
- Sequential



- Unrolled



- Pipelined



Loop Pipelining

- Iteration count = ***N***
- Loop latency = ***N*** · ***λ***
- Pipeline loop iterations with ***II*** < ***λ***
- Latency of the pipelined loop = ***N*** · ***II*** + overhead
- Overhead = $\left\lceil \lambda / II \right\rceil - 1$

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Summary

- Scheduling determines ***area/latency*** trade-off
- Intractable problem in general:
  - Heuristic algorithms
  - ILP formulation (small-case problems)
- Several heuristic formulations
  - List scheduling is the fastest and most used
  - Force-directed scheduling tends to yield good results
- Several extensions
  - Chaining and multi-cycling
  - Pipelining

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