# EE382V: System-on-a-Chip (SoC) Design

## Lecture 16 – Operation Scheduling

*Source: G. De Micheli, Integrated Systems Center, EPFL*
*"Synthesis and Optimization of Digital Circuits", McGraw Hill, 2001.*
*Additional sources:*
*Notes by Kia Bazargan, http://www.ece.umn.edu/users/kia/Courses/EE5301*
*Notes by Rajesh Gupta, UCSD, http://www.cecs.uci.edu/~rgupta/ics280.html*

Andreas Gerstlauer
Electrical and Computer Engineering
University of Texas at Austin
gerstl@ece.utexas.edu

UT ECE

## Lecture 16: Outline

- **The scheduling problem**
  - Case analysis
- **Unconstrained scheduling**
  - ASAP and ALAP schedules
- **Resource constrained (RC) scheduling**
  - List scheduling
- **Time constrained (TC) scheduling**
  - Force-directed scheduling
- **Advanced scheduling problems**
  - Chaining
  - Pipelining



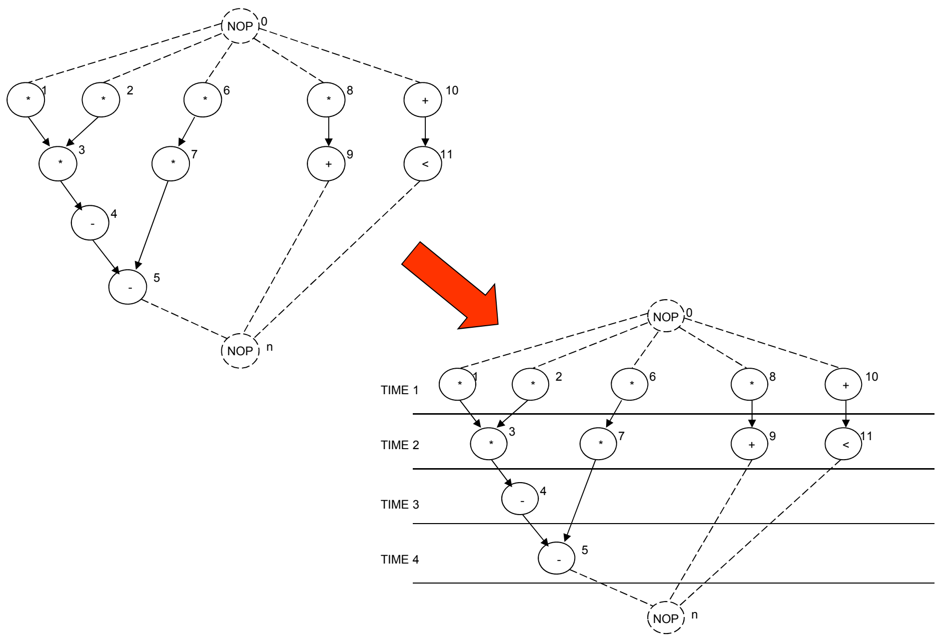
 

## Scheduling

- **Circuit model:**
  - Sequencing graph
  - Cycle-time is given
  - Operation delays expressed in cycles
- **Scheduling:**
  - Determine the start times for the operations
  - Satisfying all the sequencing (timing and resource) constraint
- **Goal:**
  - Determine *area/latency* trade-off



## Example



## Operation Scheduling

- **Input:**
  - Sequencing graph G(V, E), with $n$ vertices
  - Cycle time $\tau$
  - Operation delays $D = \{d_i: i=0..n\}$
- **Output:**
  - Schedule $\phi$ determines start time $t_i$ of operation $v_i$.
  - Latency $\lambda = t_n - t_0$.
- **Goal: determine area / latency tradeoff**
- **Classes:**
  - Non-hierarchical and unconstrained
  - Latency constrained
  - Resource constrained
  - Hierarchical



## Simplest Method

- **All operations have bounded delays**
- **All delays are in cycles:**
  - Cycle-time is given
- **No constraints – no bounds on area**
- **Goal:**
  - Minimize latency



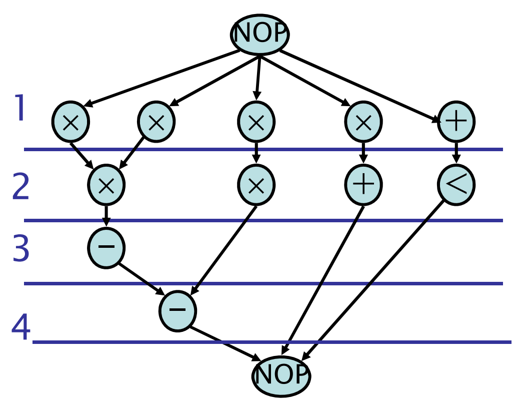
## Min Latency Unconstrained Scheduling

- Simplest case: no constraints, find min latency
- Given set of vertices $V$, delays $D$ and a partial order > on operations $E$,
- find an integer labeling of operations $\phi$: $V \rightarrow Z^+$ such that:
  - $t_i = \phi(v_i)$
  - $t_i \geq t_j + d_j \quad \forall (v_j, v_i) \in E$
  - and $\lambda = t_n - t_0$ is minimum
- ➢ Solvable in polynomial time
  - Bounds on latency for resource constrained problems
  - ASAP algorithm used: topological order



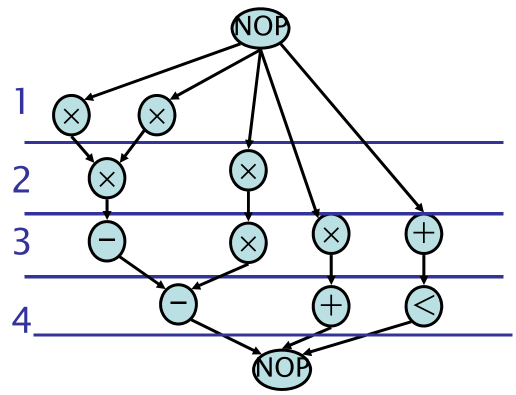
## ASAP Schedules

- Schedule $v_0$ at $t_0$=0
- While ($v_n$ not scheduled)
  - Select $v_i$ with all scheduled predecessors
  - Schedule $v_i$ at $t_i = \max \{t_j + d_j\}$, $v_j$ being a predecessor of $v_i$
- Return $t_n$



## ALAP Schedules

- **Schedule $v_n$ at $t_n = l$**
- **While ($v_0$ not scheduled)**
  - Select $v_i$ with all scheduled successors
  - Schedule $v_i$ at $t_i = \min \{t_j - d_j\}$, $v_j$ being a succecessor of $v_i$



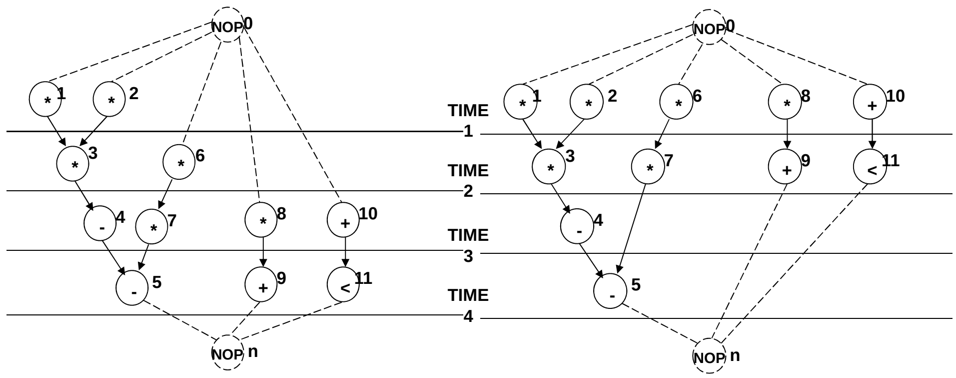



## Remarks

- **ALAP solves a latency-constrained problem**
  - Latency bound can be set to latency computed by ASAP algorithm
- **Mobility**
  - Defined for each operation
  - Difference between ALAP and ASAP schedule
  - ➢ Slack on the start time

## Example

- **Operations with zero mobility:**
  - { $v_1$, $v_2$, $v_3$, $v_4$, $v_5$ }
  - Critical path
- **Operations with mobility one:**
  - { $v_6$, $v_7$ }
- **Operations with mobility two:**
  - { $v_8$, $v_9$, $v_{10}$, $v_{11}$ }






## Lecture 16: Outline

- ✓ The scheduling problem
- ✓ Unconstrained scheduling
- **Resource constrained (RC) scheduling**
  - Exact formulations
    - ILP
    - Hu's algorithm
  - Heuristic methods
    - List scheduling
- Time constrained (TC) scheduling
- Advanced scheduling problems



## Scheduling under Resource Constraints

- **Classical scheduling problem**
  - Fix area bound – minimize latency (ML-RCS)
    - Minimum latency resource constrained scheduling
  - The amount of available resources affects the achievable latency
- **Dual problem:**
  - Fix latency bound – minimize resources (MR-LCS)
    - Minimum resources latency constrained scheduling
- **Assumption:**
  - All delays bounded and known



## ML-RCS

- **Given**
  - a set of ops $V$ with integer delays $D$
  - a partial order on the operations $E$
  - upper bounds $\{ a_k;\ k = 1, 2, \ldots, n_{res} \}$ on resource usage
- **Find an integer labeling $\phi : V \rightarrow Z^+$ such that:**
  - $t_i = \phi(v_i)$,
  - $t_i \geq t_j + d_j$ for all $i,j$ s.t. $(v_j, v_i) \in E$,
  - $| \{ v_i \,|\, T(v_i) = k$ and $t_i \leq l < t_j + d_j \} | \leq a_k$
    - for all types $k = 1,2,\ldots,n_{res}$ and steps $l$
  - ➢ and $t_n$ is minimum
- ➢ **Intractable problem**

## ILP Formulation

- **Binary decision variables**
  - $X = \{ x_{il}, \quad i = 1,2, .... n; \; l = 1,2, ..., \overline{\lambda} + 1\}$
  - $x_{il}$ is **TRUE** only when operation $v_i$ starts in step $l$ of the schedule ( i.e. $l = t_i$ )
  - $\overline{\lambda}$ is an upper bound on latency
- **Start time of operation $v_i$ : $\Sigma_l \, l \cdot x_{il}$**



## ILP Constraints

- **Operations start only once**

  $\Sigma \, x_{il} = 1 \quad i = 1, 2, \ldots, n$
- **Sequencing relations must be satisfied**

  $t_i \geq t_j + d_j \rightarrow \quad t_i - t_j - d_j \geq 0$ for all $(v_j, v_i) \in E$

  $\Sigma \, l \cdot x_{il} - \Sigma \, l \cdot x_{jl} - d_j \geq 0$ for all $(v_j, v_i) \in E$
- **Resource bounds must be satisfied**

  Simple case (unit delay)

  $\Sigma_{\substack{l \\ i:T(v_i)=k}} \, x_{il} \leq a_k \quad k = 1,2, \ldots n_{res}$ ; for all $l$



## Start Time vs. Execution Time

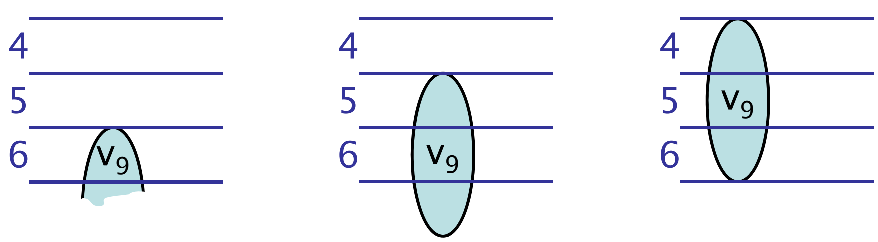
- **For each operation $v_i$, only one start time**
- **If $d_i$=1, then the following questions are the same:**
  - Does operation $v_i$ start at step $l$?
  - Is operation $v_i$ running at step $l$?
- **But if $d_i > 1$, the two questions should be formulated as:**
  - Does operation $v_i$ start at step $l$?
    – Does $x_{il} = 1$ hold?
  - Is operation $v_i$ running at step $l$?
    – Does the following hold?

$$\sum_{m=l-d_i+1}^{l} x_{im} \stackrel{?}{=} 1$$



## Operation $v_i$ Still Running at Step $l$ ?

- **Is $v_9$ running at step 6?**
  - Is $x_{9,6} + x_{9,5} + x_{9,4} = 1$ ?




$x_{9,6}=1$ $x_{9,5}=1$ $x_{9,4}=1$

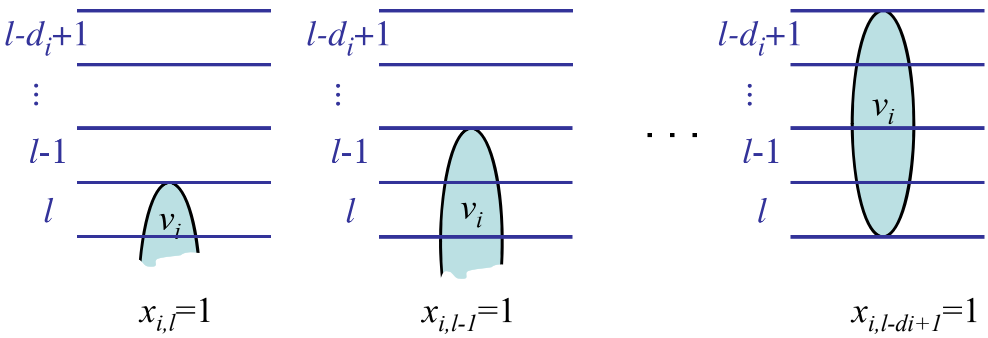
- **Note:**
  - Only one (if any) of the above three cases can happen
  - To meet resource constraints, we have to ask the same question for ALL steps, and ALL operations of that type



## Operation $v_i$ Still Running at Step $l$ ?

- **Is $v_i$ running at step $l$ ?**
  - Is $x_{i,l} + x_{i,l-1} + \ldots + x_{i,l-di+1} = 1$ ?




EE382V: SoC Design, Lecture 16 © K. Bazargan 19

## ILP Formulation of ML-RCS

- **Constraints:**
  - Unique start times: $\sum_l x_{il} = 1, \quad i = 0,1,\ldots,n$
  - Sequencing (dependency) relations must be satisfied

$$t_i \geq t_j + d_j \;\; \forall (v_j, v_i) \in E \Rightarrow \sum_l l.x_{il} \geq \sum_l l.x_{jl} + d_j$$

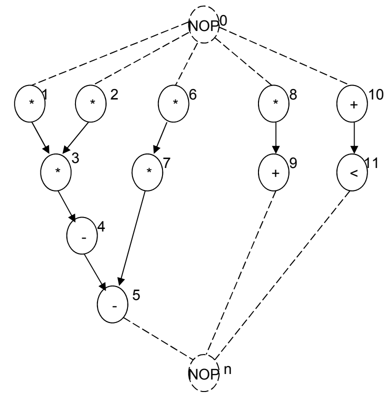
  - Resource constraints

$$\sum_{i:T(v_i)=k} \;\; \sum_{m=l-d_i+1}^{l} x_{im} \leq a_k, \quad k = 1,\ldots,n_{res}, \quad l = 1,\ldots,\bar{\lambda}+1$$

- **Objective:** $\min c^T t$
  - $t$ = start times vector, $c$ = cost weight (e.g., [0 0 ... 1])
  - When $c = [0\ 0\ \ldots\ 1]$, $c^T t = \sum_l l.x_{nl}$

EE382V: SoC Design, Lecture 16 © K. Bazargan 20

## ILP Example

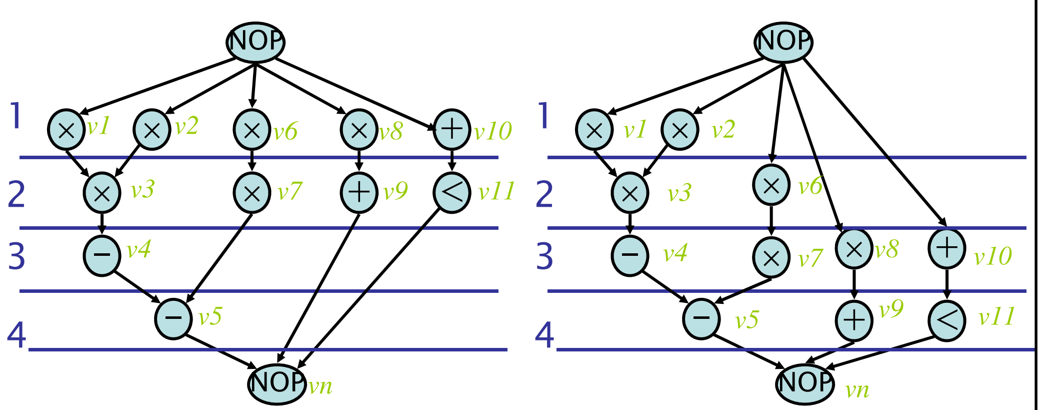



- **Resource constraints**
  - 2 ALUs; 2 Multipliers
  - $a_1 = 2$; $a_2 = 2$
- **Single-cycle operation**
  - $d_i = 1$ for all $i$



## ILP Example

- **Assume** $\overline{\lambda} = 4$
- **First, perform ASAP and ALAP**
  - (we can write the ILP without ASAP and ALAP, but using ASAP and ALAP will simplify the inequalities)



## ILP Example: Unique Start Times

- **Without using ASAP and ALAP values:**

$$x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} = 1$$
$$x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} = 1$$
$$\ldots$$
$$\ldots$$
$$\ldots$$
$$x_{11,1} + x_{11,2} + x_{11,3} + x_{11,4} = 1$$

- **Using ASAP and ALAP:**

$$x_{1,1} = 1$$
$$x_{2,1} = 1$$
$$x_{3,2} = 1$$
$$x_{4,3} = 1$$
$$x_{5,4} = 1$$
$$x_{6,1} + x_{6,2} = 1$$
$$x_{7,2} + x_{7,3} = 1$$
$$x_{8,1} + x_{8,2} + x_{8,3} = 1$$
$$x_{9,2} + x_{9,3} + x_{9,4} = 1$$
$$\ldots.$$

EE382V: SoC Design, Lecture 16 © K. Bazargan 23

## ILP Example: Dependency Constraints

- **Using ASAP and ALAP, the non-trivial inequalities are: (assuming unit delay for + and *)**

$$2.x_{7,2} + 3.x_{7,3} - x_{6,1} - 2.x_{6,2} - 1 \geq 0$$
$$2.x_{9,2} + 3.x_{9,3} + 4.x_{9,4} - x_{8,1} - 2.x_{8,2} - 3.x_{8,3} - 1 \geq 0$$
$$2.x_{11,2} + 3.x_{11,3} + 4.x_{11,4} - x_{10,1} - 2.x_{10,2} - 3.x_{10,3} - 1 \geq 0$$
$$4.x_{5,4} - 2.x_{7,2} - 3.x_{7,3} - 1 \geq 0$$
$$5.x_{n,5} - 2.x_{9,2} - 3.x_{9,3} - 4.x_{9,4} - 1 \geq 0$$
$$5.x_{n,5} - 2.x_{11,2} - 3.x_{11,3} - 4.x_{11,4} - 1 \geq 0$$

EE382V: SoC Design, Lecture 16 © K. Bazargan 24

## ILP Example: Resource Constraints

- **Resource constraints (assuming 2 adders and 2 multipliers)**

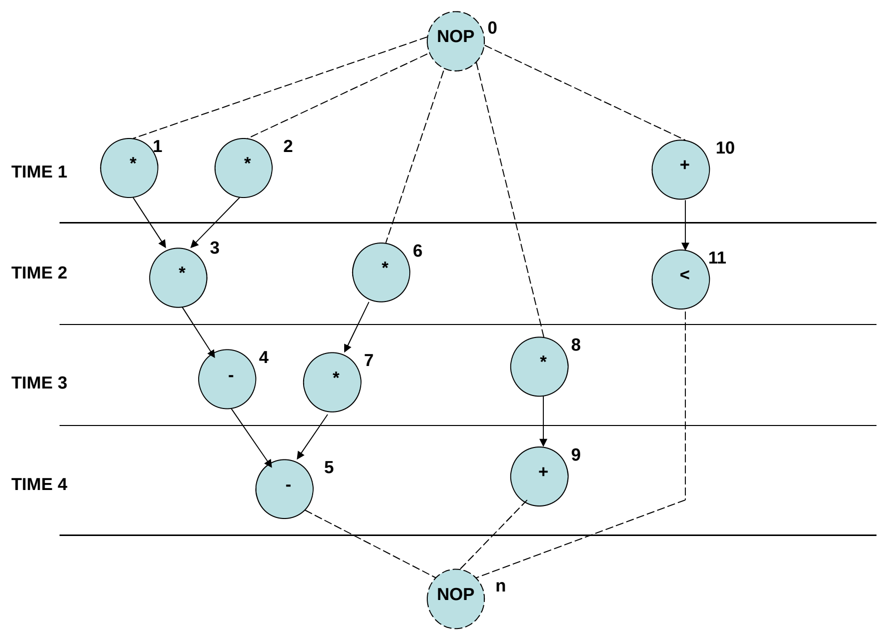
$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq 2$$
$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \leq 2$$
$$x_{7,3} + x_{8,3} \leq 2$$
$$x_{10,1} \leq 2$$
$$x_{9,2} + x_{10,2} + x_{11,2} \leq 2$$
$$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \leq 2$$
$$x_{5,4} + x_{9,4} + x_{11,4} \leq 2$$

- **Objective:**
  - Since $\lambda$=4 and sink has no mobility, any feasible solution is optimum, but we can use the following anyway:

$$Min \quad x_{n,1} + 2.x_{n,2} + 3.x_{n,3} + 4.x_{n,4}$$



## ILP Example: Solution





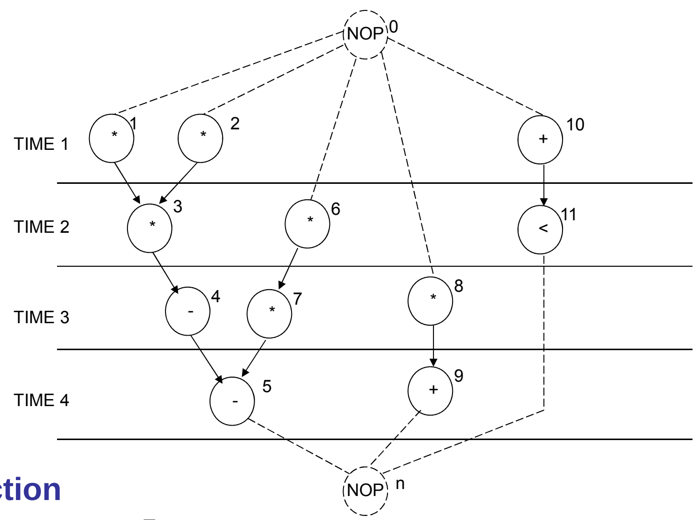
## MR-LCS Dual ILP formulation

- **Minimize resource usage under latency constraint**
- **Additional constraint**
  - Latency bound must be satisfied
  - $\Sigma_l \, l \, x_{nl} \leq \lambda + 1$

➢ **Resource usage is unknown in the constraints**
  - Resource usage is the objective to minimize

EE382V: SoC Design, Lecture 16 © G. De Micheli 27

## MR-LCS ILP Example




- **Cost function**
  - Multiplier area = 5
  - ALU area = 1
  - Objective function: $5a_1 + a_2$

EE382V: SoC Design, Lecture 16 © G. De Micheli 28

## ILP Solving

- **Use standard ILP packages**
- **Transform into LP problem**
- **Advantages**
  - Exact method
  - Others constraints can be incorporated
- **Disadvantages**
  - Works well up to few thousand variables



## Hu's Algorithm

- **Simple case of the scheduling problem**
  - Operations of unit delay
  - Operations (and resources) of the same type
- **Hu's algorithm**
  - Greedy, polynomial *and* optimal (exact)
    - Computes lower bound on number of resources for given latency<br>OR<br>Computes lower bound on latency subject to resource constraints
- **Basic idea**
  - Label operations based on their distances from the sink
  - Try to schedule nodes with higher labels first<br>(i.e., most "critical" operations have priority)



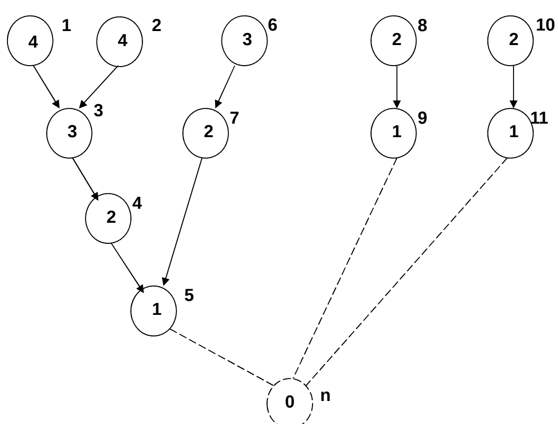
 

## Hu's Algorithm with $\bar{a}$ Resources

- **Label operations with distance to sink**
- **Set step $l = 1$**
- **Repeat until all ops are scheduled**
  - $U$ = unscheduled vertices in $V$
    - Predecessors have been scheduled (or no predecessors)
  - Select $S \subseteq U$ resources with
    - $|S| \leq \bar{a}$
    - Maximal labels
  - Schedule the $S$ operations at step $l$
  - Increment step $l = l + 1$



## Hu's Algorithm Example

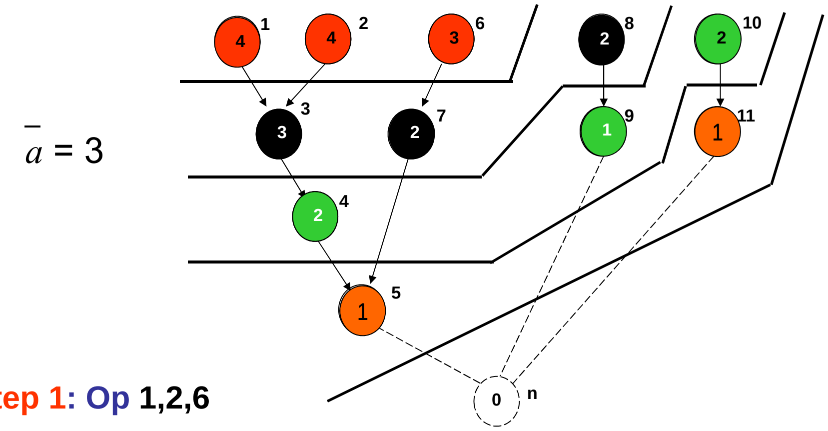



- **Assumptions**
  - One resource type only
  - All operations have unit delay
- **Labels**
  - Distance to sink



## Hu's Algorithm Example




**Step 1: Op 1,2,6**

**Step 2: Op 3,7,8**

**Step 3: Op 4,9,10**

**Step 4: Op 5,11**



## List Scheduling

- **Heuristic method for:**
  - Min *latency* subject to *resource bound* (ML-RCS)
  - Min *resource* subject to *latency bound* (MR-LCS)
- **Greedy strategy (like Hu's)**
  - Does not guarantee optimality (unlike Hu's)
- **General graphs (unlike Hu's)**
  - Resource constraints on different resource types
  - Operations of arbitrary delay
- **Priority list heuristics**
  - Priority decided by criticality (similar to Hu's)
  - Longest path to sink, longest path to timing constraint
  - $O(n)$ time complexity



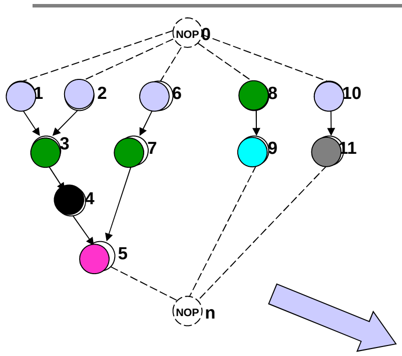
 

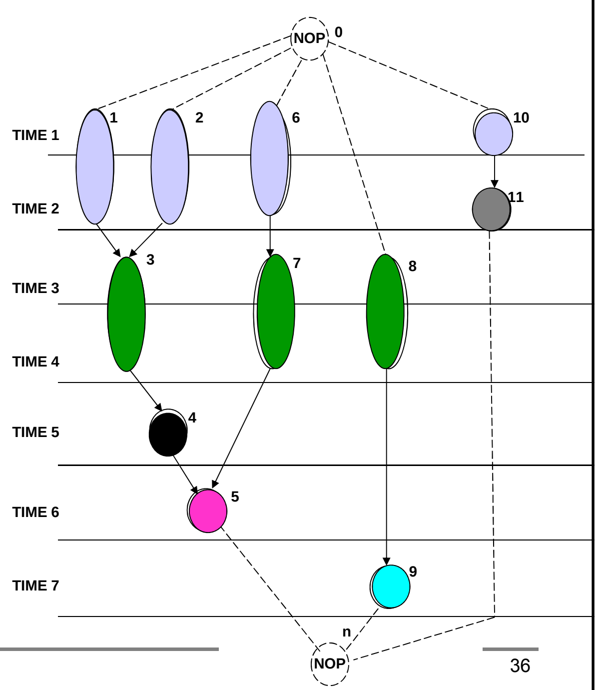
## List Scheduling for Minimum Latency

**LIST_L(** $G(V, E), a$**) {**
    $l = 1$;
    *repeat* **{**
        *for each* **resource type** $k = 1, 2, \ldots, n_{res}$ **{**
            **Determine ready operations** $U_{l,k}$;
            **Determine unfinished operations** $T_{l,k}$;
            **Select** $S_k \subseteq U_{l,k}$ **vertices, s.t.** $|S_k| + |T_{l,k}| \leq a_k$;
            **Schedule the** $S_k$ **operations at step** $l$;
            **}**
        $l = l + 1$;
    **}**
    *until* **(**$v_n$ **is scheduled) ;**
    *return* **(**$t$**);**
**}**



## List Scheduling Example




**Resource bounds**

3 multipliers with delay 2

1 ALU with delay 1

## Lecture 16: Outline

- ✓ The scheduling problem
- ✓ Unconstrained scheduling
- ✓ Resource constrained (RC) scheduling
- Time constrained (TC) scheduling
  - ✓ Exact methods
    - ✓ ILP formulations
    - ✓ Hu's algorithm
  - Heuristics
    - List scheduling
    - Force-directed scheduling
- Advanced scheduling problems



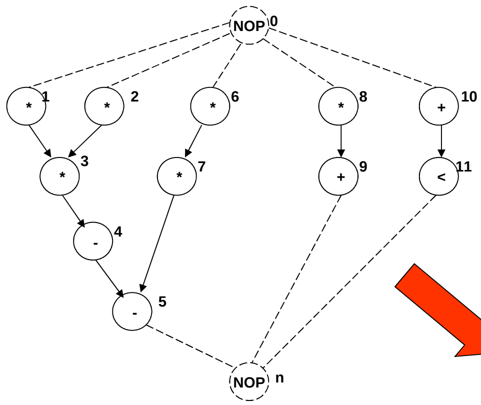
## List Scheduling for Minimum Resources

LIST_R( $G(V, E), \overline{\lambda}$) {
  $a = 1$;
  Compute the latest possible start times $t^L$ by ALAP ( $G(V, E), \overline{\lambda}$);
  *if* ($t_0 < 0$)
    *return* (Ø);
  $l = 1$;
  *repeat* {
    *for each* resource type $k = 1, 2, \ldots, n_{res}$ {
      Determine ready operations $U_{l,k}$;
      Compute the slacks { $s_i = t_i - l$ for all $v_i \in U_{lk}$};
      Schedule candidate operations with zero slack and update a;
      Schedule candidate operations not needing addt'l resources;
      }
    $l = l + 1$;
  }
  *until* ($v_n$ is scheduled) ;
  *return* ($t$, $a$);
}



## List Scheduling Example

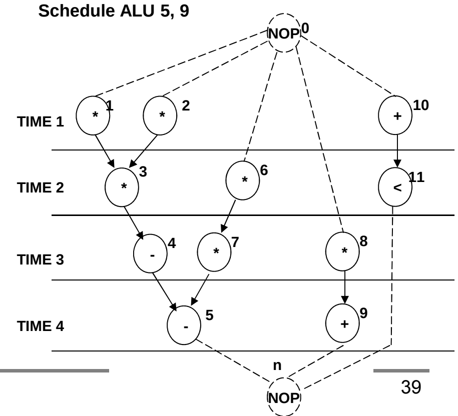



**Step 1**
Two multiplications on CP
Set $a_1 = 2$
Schedule Mult 1,2
Schedule ALU 10

**Step 2**
Schedule Mult 3, 6
Schedule ALU 11

**Step 3**
Schedule Mult 7,8
Schedule ALU 4

**Step 4**
Set $a_2$=2
Schedule ALU 5, 9




- **Assumptions**
  - Unit-delay resources
  - Maximum latency = 4
- **Start with**
  - $a_1 = 1$ multiplier
  - $a_2 = 1$ ALUs

EE382V: SoC Design, Lecture 16 39

## Force-Directed Scheduling (FDS)

- **Heuristic, similar to list scheduling**
  - Can handle ML-RCS and MR-LCS
  - For ML-RCS, schedules step-by-step
  - BUT, selection of the operations tries to find the *globally* best set of operations
- **Idea [Paulin]**
  - Find the mobility $\mu_i = t_i^L - t_i^S$ of operations (ALAP-ASAP)
  - Look at the operation type probability distributions
  - Try to flatten the operation type distributions
- **Definition: operation probability density**
  - $p_i(l) = \Pr\{v_i$ executes in step $l\}$
  - Assume uniform distribution: $p_i(l) = \dfrac{1}{\mu_i + 1} \quad \mathit{for}\; l \in [t_i^S, t_i^L]$

EE382V: SoC Design, Lecture 16 © R. Gupta 40

## Force-Directed Scheduling: Definitions

- **Operation-type distribution (sum of operation probabilities for each type)**
  - $q_k(l) = \sum_{i:T(v_i)=k} p_i(l)$
- **Operation probabilities over control steps**
  - $p_i = \{p_i(0), p_i(1), \ldots, p_i(n)\}$
- **Distribution graph of type *k* over all steps**
  - $\{q_k(0), q_k(1), \ldots, q_k(n)\}$
  - $q_k(l)$ can be thought of as *expected* operator cost for implementing operations of type $k$ at step $l$



## Force-Directed Scheduling Example

$q_{add}(1) = \frac{1}{3} = 0.33$

$q_{add}(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$q_{add}(3) = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 2$

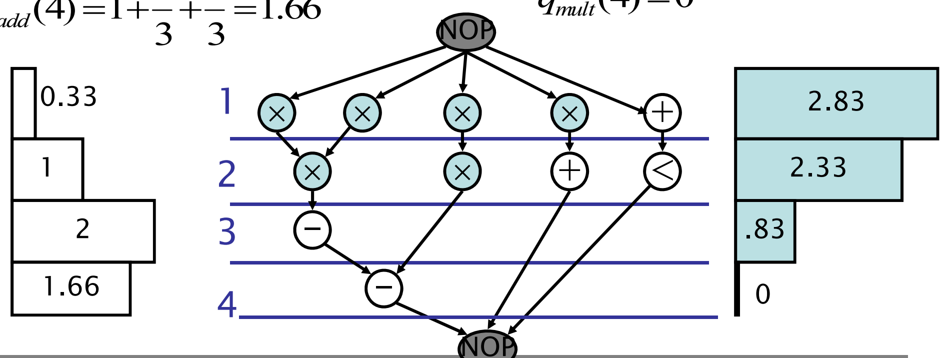
$q_{add}(4) = 1 + \frac{1}{3} + \frac{1}{3} = 1.66$

$q_{mult}(1) = 1 + 1 + \frac{1}{2} + \frac{1}{3} = 2.83$

$q_{mult}(2) = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} = 2.33$

$q_{mult}(3) = \frac{1}{2} + \frac{1}{3} = 0.83$

$q_{mult}(4) = 0$





## Force-Directed Scheduling Algorithm

- **Very similar to LIST_L($G(V,E)$, $a$)**
  - Compute mobility of operations using ASAP and ALAP
  - Computer operation probabilities and type distributions
  - Select and schedule operation
  - Update operation probabilities and type distributions
  - Go to next step/operation
- **Difference with list scheduling in selecting operations**
  - Select operations with least force
  - $O(n^2)$ time complexity due to pair-wise force computations

EE382V: SoC Design, Lecture 16 © R. Gupta 43

## Force

- **Used as *priority* function**
- **Force is related to concurrency**
  - Sort operations for least force
- **Mechanical analogy (spring)**
  - Force = constant × displacement
    - Constant = operation-type distribution
    - Displacement = change in probability

EE382V: SoC Design, Lecture 16 © G. De Micheli 44

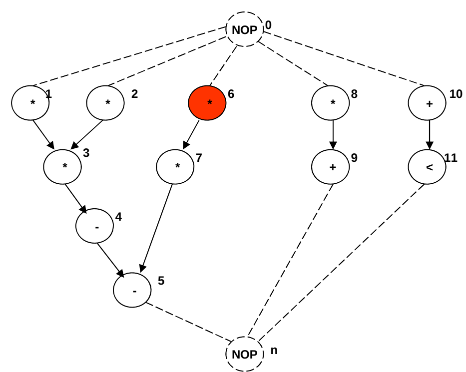
## Two Types of Forces

- **Self-force**
  - Sum of forces to feasible schedule steps
  - Self-force for operation $v_i$ in step $l$
    - Sum over type distribution × delta probability

      $\Sigma_{m \text{ in interval}}\ q_k(m)\ (\delta_{lm} - p_i(m))$
    - Higher self-force indicates higher mobility
- **Predecessor/successor-force**
  - Related to the predecessors/successors
    - Fixing an operation timeframe restricts timeframe of predecessors/successors
    - Ex: Delaying an operation implies delaying its successors
    - Computed by changes in self-forces of neighbors



## Example: Schedule Operation $v_6$




- **Distribution graphs for multiplier and ALU**


- **Operation $v_6$ can be scheduled in step 1 or step 2**



## Example: Operation $v_6$

- **Op $v_6$ can be scheduled in the first two steps**
  - $p(1) = 0.5; p(2) = 0.5; p(3) = 0; p(4) = 0$
- **Distribution**
  - $q(1) = 2.8; q(2) = 2.3$
- **Assign $v_6$ to step 1**
  - Variation in probability 1 – 0.5 = 0.5 for step 1
  - Variation in probability 0 – 0.5 = -0.5 for step 2
- **Self-force**
  - 2.8 * 0.5 – 2.3 * 0.5 = + 0.25
- **No successor force**
- **Total force = 0.25**



## Example: Operation $v_6$

- **Assign $v_6$ to step 2**
  - variation in probability 0 – 0.5 = -0.5 for step 1
  - variation in probability 1 – 0.5 = 0.5 for step 2
- **Self-force**
  - - 2.8 * 0.5 + 2.3 * 0.5 = - 0.25
- **Successor-force**
  - Operation $v_7$ assigned to step 3
  - Succ. force is 2.3 ( 0- 0.5 ) + 0.8 ( 1 – 0.5 ) = - .75
- **Total force = -1**



## Example: Operation $v_6$

- **Total force in step 1 = + 0.25**
- **Total force in step 2 = -1**

➢ **Conclusion:**
  - Least force is for step 2
  - Assigning $v_6$ to step 2 reduces concurrency



## FDS for Minimum Resources

**FDS ( G ( V, E ), $\overline{\lambda}$ )**
**{**
*repeat* {
**Compute/update the time-frames;**
**Compute the operation and type probabilities;**
**Compute the self-forces, p/s-forces and total forces;**
**Schedule the op. with least force;**
}
*until* (**all operations are scheduled**)
*return* (***t***);
**}**



## Scheduling Generalizations

- **Detailed timing constraints**
  - Protocol and interface synthesis
    - Bounds on start time differences
    - Forward & backward edges for min/max constraints
- **Operation generalizations**
  - Unbounded delay operations (e.g. synchronization)
    - Relative scheduling w.r. to anchors and combine
  - Conditional operations
- **Resource generalizations**
  - Multi-cycling and chaining
  - Pipelined resources
- **Model generalizations**
  - Hierarchy
  - Pipelining
  - Loops

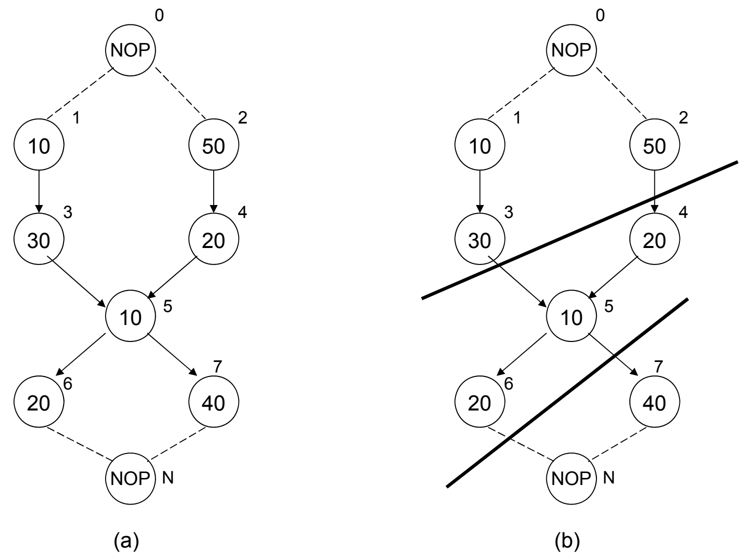
EE382V: SoC Design, Lecture 16 © R. Gupta 51

## Multi-Cycling and Chaining

- **Consider delays of resources not in terms of cycles**
  - Use scheduling to *chain* multiple operations in the same control step
  - Use scheduling to *multi-cycle* an operation across more than one control step
- **Useful techniques to explore effect of *cycle-time* on area/latency trade-off**
- **Algorithms**
  - ILP
  - ALAP/ASAP
  - List scheduling

EE382V: SoC Design, Lecture 16 © G. De Micheli 52

## Chaining Example




- **Cycle-time: 50**



## Pipelining

- **Two levels of data pipelining**
  - Structural pipelining
    - Pipelined resources
    - Non-pipelined model
  - Functional pipelining
    - Non-pipelined resources
    - Pipelined model
- **Control pipelining**
  - Pipelined control logic



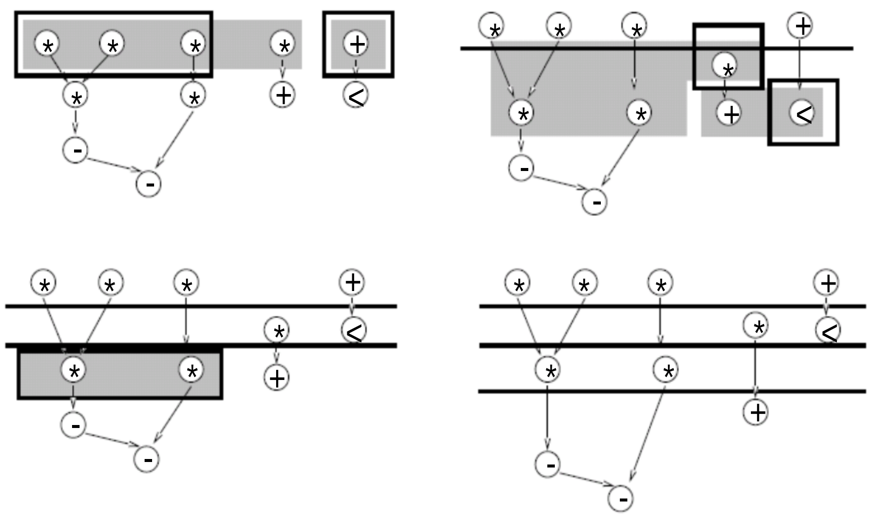
 

## Structural Pipelining

- **Non-pipelined model using pipelined resources**
- **Resources characterized by**
  - Execution delay
  - Data introduction interval: *DII*
- **Implications**
  - Operations sharing a pipelined resource are serialized (always)
  - Operations do not have data dependency
- ➢ **Solution using list scheduling**
  - Relax criteria for selection of vertices



## Structural Pipelining Example



- **3 multipliers w/ 2 cycle delay and $DII = 1$**

## Functional (Loop) Pipelining

- **Pipelined model, non-pipelined resources**
- **Assume non-hierarchical graphs**
- **Model characterized by**
  - Latency
  - Initiation interval, $II$
- **Restart source before completing sink**
  - Implicit loop
  - Limited by loop-carried dependencies

➢ **Solutions using ILP or heuristics**
  - ILP resource constraints modified to include increased concurrency
  - List or force-directed methods



## Pipelining and Concurrency

- ***II* determines resource usage**
  - Smaller $II$ leads to larger overlaps, higher resource requirements

    $\min\{a_k\} = n_k$, for $II$=1 (all $n_k$ operations are concurrent)
  - In general, $\bar{a}_k = \left\lceil \frac{n_k}{II} \right\rceil$
- **Concurrent operations**
  - Operations $v_i$ and $v_j$ are executing concurrently at control step $l$, if

    $\mathrm{rem}\{\, t_i / II \,\} = \mathrm{rem}\{\, t_j / II \,\} = l$
  - Affects the design of the controller circuitry

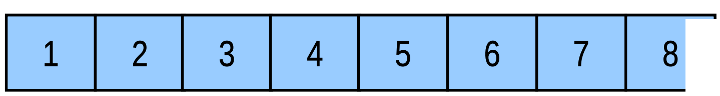
## Loop Scheduling

- **Potential parallelism *across* loop invocations**
- **Single loop executions**
  - Sequential execution
  - Loop unrolling (known iteration count)
    - Merge multiple iterations into one to provide scheduling opportunities
  - Loop pipelining (iteration count might be unknown)
    - Start next iteration while current one is still running
    - Depends on dependencies across iterations
    - ➢ Functional pipelining
- **Merging of multiple loops**
  - Run different loops in parallel (no dependencies)

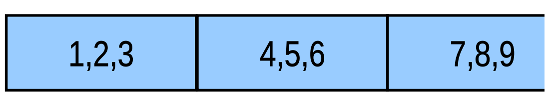


## Loop Scheduling Example

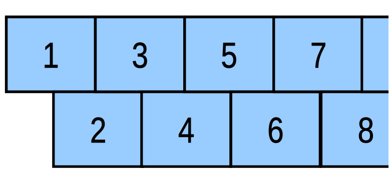
- **Sequential**




- **Unrolled**




- **Pipelined**




- Iteration count = $N$
- Loop latency = $N \cdot \lambda$
- Pipeline loop iterations with $II < \lambda$
- Latency of the pipelined loop
  - $N \cdot II$ + overhead
  - Overhead = $\lceil \lambda / II \rceil - 1$



## Lecture 16: Summary

- **Scheduling determines *area/latency* trade-off**
- **Intractable problem in general**
  - Heuristic algorithms
  - ILP formulation (small-case problems)
- **Several heuristic formulations**
  - List scheduling is the fastest and most used
  - Force-directed scheduling tends to yield good results
- **Several extensions**
  - Chaining and multi-cycling
  - Pipelining

