Improved Multistage Decoding of Multilevel Codes for Digital Radio Mondiale (DRM)

Volker Fischer, Alexander Kurpiers and Florian Kulla

***Abstract*— The new digital radio standard Digital Radio Mondiale (DRM) uses multilevel coding as channel coding. A very effective decoding algorithm is the iterative multistage decoder (MSD). In our paper we analyze different implementations of the MSD utilizing hard-decision with different metrics. The performance of these implementations is evaluated by computer simulations. Additionally, an MSD with soft-decision (Max-Log-MAP) is investigated and serves as a benchmark for the hard-decision algorithms.**

***Index Terms*— Digital Radio Mondiale, multilevel code, multistage decoding**

I. Introduction

DIGITAL Radio Mondiale (DRM) is a new OFDM-based digital radio standard for the long-, medium- and short-wave ranges which was formed by an international consortium [1], [2]. It is designed to use the same frequency allocation as the current analog systems to offer a high degree of compatibility. The aim is to replace the analog system since the digital system has a lot of advantages. The audio quality is much better and additional digital information can be transmitted. Also, it is designed to cope perfectly with the strong channel impairments on the desired frequency bands. Long interleaving in combination with a multilevel channel code (MLC) [3] and various pilot cells make the signal robust against severe fading.

In our paper, we want to focus on the decoding of the MLC which is a powerful technique for constructing modulation codes systematically and was devised by Imai and Hirakawa [4] in 1977. Since the optimum maximum likelihood decoding is not feasible because of the interleavers used in the different levels of the MLC, suboptimal approaches have to be used. A commonly applied decoding strategy for MLC is the multistage decoding (MSD) [4–7].

The authors are with the Institute for Communication Technology, Darmstadt University of Technology, Germany (e-mail: {v.fischer, a.kurpiers}@nt.tu-darmstadt.de, florian.kulla@web.de)

0-7803-8526-8/04/$20.00 ©2004 IEEE.

For the MSD, soft- and hard-decision decoders in the different levels can be used. In case of hard decisions, only one Viterbi-Algorithm is needed which is much less computationally demanding than using a soft-in, soft-out (SISO) decoder which is usually a symbol-wise MAP decoder. In the following sections we introduce improved metrics for use with hard-decision decoders.

The paper is structured as follows: Sect. II gives a brief introduction in MLC and MSD. The SISO decoder needed for MSD using soft-information is described in Sect.III. The derivation of improved metrics for using hard-decisions is given in Sect.IV and a metric using erasures is presented in Sect. V. In Sect. VI, BER simulations with different channels are shown as well as measurements of real DRM transmissions.

II. Multilevel codes and Multistage decoding

The idea of multilevel coding is to jointly optimize coding and modulation, i.e., optimize the code in Euclidean space rather than dealing with Hamming distance. The signal set $\mathbf{A} = \{a_0, a_1, \cdots, a_{M-1}\}$ of an $M = 2^l$-ary modulation scheme is successively binary partitioned in l steps defining a mapping of binary addresses $\mathbf{x} = (x^{(0)}, x^{(1)}, \cdots, x^{(l-1)})$ to signal points a_m. DRM defines three types of mapping for the main data channel: a standard mapping, a symmetrical hierarchical mapping and a mixture of the previous two mappings. In this paper, we only analyze the 64-QAM standard mapping since it is the most common type. This mapping defines the following signal point addresses $a_m = (x^{(0)}, x^{(1)}, x^{(2)})$ for each quadrature component of the QAM scheme: $a_0 = (000)$, $a_1 = (001)$, $a_2 = (010)$, $a_3 = (011)$, $a_4 = (100)$, $a_5 = (101)$, $a_6 = (110)$, $a_7 = (111)$, thus a sequential binary mapping instead of a Gray mapping is applied. If one or two bits of the address are

Authorized licensed use limited to: University of Texas at Austin. Downloaded on March 03,2010 at 21:38:46 EST from IEEE Xplore. Restrictions apply.

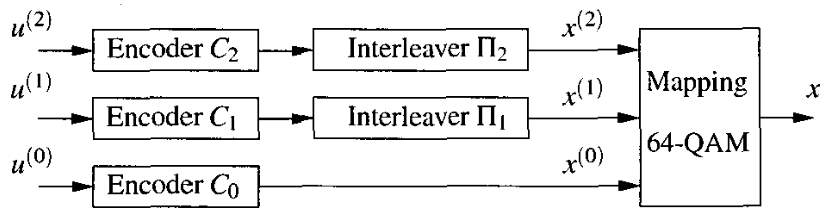

Fig. 1. Multilevel encoder for 64-QAM.

fixed, only a subset of the original signal set **A** remains, e.g., if $x^{(1)} = 1$ and $x^{(2)} = 1$, only a_6 and a_7 are valid signal points. The mapping of the signal points together with the subsets for a fixed $x^{(1)}$ and $x^{(2)}$ is shown in Fig. 2.

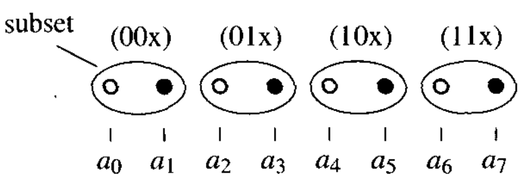

Fig. 2. Signal point mapping and definition of subsets for a fixed $x^{(1)}$ and $x^{(2)}$. Filled circles represent a 1 for $x^{(0)}$.

MLC protects each information bit $u^{(i)}$ at level i by an individual binary code C_i resulting in an address bit $x^{(i)}$ of the signal point as illustrated in Fig. 1. In the DRM system, the component codes C_i are based on punctured convolutional codes with a mother code of rate $1/4$ and constraint length 7. Additionally, interleavers Π_i based on pseudo random bit ordering are inserted on levels 1 and 2.

A simple and effective decoding strategy for MLC is the MSD which sequentially decodes the individual levels by using the decoding results of previous levels. The a-posteriori probability at level i taking into account the reliability information from the coded bits from level j and assuming that the coded bits of the i-th and j-th level are independent can be written as [7]

$$p(y_n|x_n^{(i)}) = \sum_{\hat{x}_n^{(j)} \in \{0,1\}} p(y_n|x_n^{(i)}, \hat{x}_n^{(j)}) P(\hat{x}_n^{(j)}), \quad (1)$$

where $P(\hat{x}_n^{(j)})$ is the reliability information about the decision $\hat{x}_n^{(j)}$ from level j and y_n is the received symbol at time index n. Passing hard decisions from the j-th level to the i-th level, $P(\hat{x}_n^{(j)})$ is set to either 0 or 1. In this case, the information of the previous stage is needed for choosing the correct subset of constellation points. For that, the coded bits $\hat{x}_n^{(j)}$ based on the decoded bits have to be regenerated by the corresponding encoders and interleavers. Furthermore, iterative decoding is possible by utilizing the information provided by the higher stages to improve the decoding of the lower stages. In this paper, we refer the levels 0, 1, 2 as the first iteration and the levels 3, 4, 5 as the second iteration of the decoding process. Thus, level 3 is on the same stage as level 0 but uses information about decoding results from previous levels. The complete MSD structure is shown in Fig. 3. In case of soft decisions, a SISO decoder must be used. To get an impression of the maximum gain achievable, we implemented SISO using the almost optimum Max-Log-MAP algorithm which is a simplification of the optimum symbol-by-symbol MAP algorithm known as the BCJR algorithm [8]. This algorithm is widely used for iterative Turbo decoding.

However, since the channel decoder takes most of the computational complexity in a DRM receiver and the MAP approximately doubles the decoding complexity of a Viterbi algorithm, it is very desired to use hard decision decoding. The draw-back of using hard-decisions is that the data of previous levels do not contain any information about reliability of the decision. The decoder of the following level is forced to make an assumption that this decision is known with a probability of 1. The performance of this decoder is degraded if the assumption is not true (if there was an incorrect decoded bit on the previous level). In Sect. IV, we will present new metrics which improve the decoding performance if hard decisions are used.

III. BCJR-Max-Log-MAP

It can be shown [9] that the BCJR-Max-Log-MAP algorithm which basically consists of two Viterbi-algorithms running forward and backward through the trellis performs almost equally well as the optimum BCJR-Log-MAP algorithm when channel estimation is considered. Another important advantage

Authorized licensed use limited to: University of Texas at Austin. Downloaded on March 03,2010 at 21:38:46 EST from IEEE Xplore. Restrictions apply.

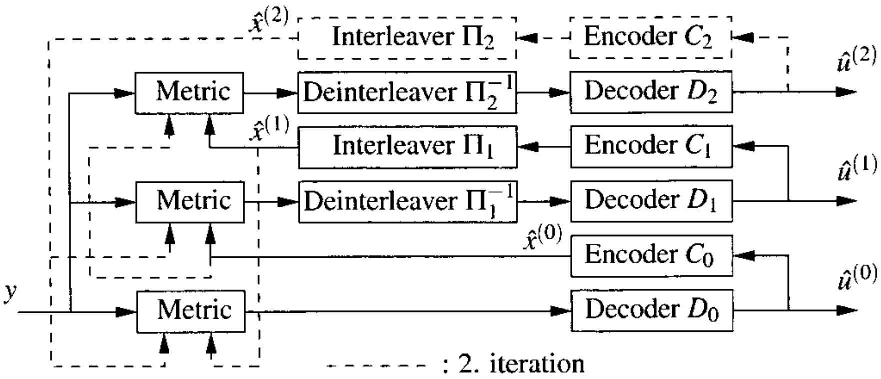

Fig. 3. Multistage decoder for 64-QAM.

of the Max-Log-MAP is the fact that no estimate of the channel SNR is needed.

Unfortunately, the MLC used in DRM is based on a non-systematic convolutional code so the soft-information has to be convolved with the code prior to passing through the interleaver. This adds to the already about two times higher computational complexity of the Max-Log-MAP algorithm as compared to the simpler hard-decision based algorithms.

We now show how soft-information of a previous level is incorporated in the metric for the Max-Log-MAP decoder. Our decoder implementation delivers binary L-values according to the following definition:

$$L(\hat{x}_n^{(j)}) = \log\left(\frac{P(\hat{x}_n^{(j)}=1)}{P(\hat{x}_n^{(j)}=0)}\right). \tag{2}$$

Using the max-log approximation [9]

$$\log(e^{\delta_1}+e^{\delta_2}) \approx \max\{\delta_1,\delta_2\} \tag{3}$$

results in

$$\log(P(\hat{x}_n^{(j)}=0)) \approx -\max\{0, L(\hat{x}_n^{(j)})\} \tag{4}$$

$$\log(P(\hat{x}_n^{(j)}=1)) \approx -\max\{0, -L(\hat{x}_n^{(j)})\}. \tag{5}$$

Inserting (4) and (5) in (1) and using (3) gives

$$\begin{aligned}\log(p(y_n|x_n^{(i)})) \approx \max\Big\{ & \log(p(y_n|x_n^{(i)},\hat{x}_n^{(j)}=0)) \\ & -\max\{0,L(\hat{x}_n^{(j)})\}, \\ & \log(p(y_n|x_n^{(i)},\hat{x}_n^{(j)}=1)) \\ & -\max\{0,-L(\hat{x}_n^{(j)})\}\Big\}. \end{aligned} \tag{6}$$

For $\log(p(y_n|x_n^{(i)},\hat{x}_n^{(j)}))$, a squared distance metric is used as presented in the following section, where the expressions are slightly modified to allow a minimum search instead of a maximization.

IV. Improved Metrics for Hard-Decision Decoders

Since DRM uses orthogonal frequency division multiplexing (OFDM), a flat fading channel model can be applied for each symbol under the assumption of perfect synchronization and a slow fading channel. The received symbol y_n is then

$$y_n = x_n h_n + n_n, \tag{7}$$

where x_n is the transmitted symbol, h_n is the channel coefficient and n_n is an additive noise sample.

Implying statistical independent Gaussian distributed noise samples n_n, the maximum likelihood (ML) approach results in the squared distance (Euclidian distance) metric [10] which is called *quadratic metric* in this paper:

$$\lambda_n^{(\text{quadratic})} = \left|y_n - \tilde{x}_n\hat{h}_n\right|^2, \tag{8}$$

where $\hat{h}_n$ is an estimate of the channel coefficient h_n generated by a preceding channel estimation unit and $\tilde{x}_n$ is a signal point on a certain level in a subset based on the decided bits on lower levels. If one bit (on level 1) or two bits (on level 0) of the higher levels are unknown, the nearest signal point corresponding to the current bit is chosen. The decoder (usually a Viterbi decoder) decides for the message which minimizes the sum of the metrics for all symbols in a certain block.

Since the metrics on higher levels depend on hard-decided bits, the assumption of white Gaussian noise is not satisfied anymore in case of bit errors on previous levels. In Fig. 4, the simulated negative loga-

Authorized licensed use limited to: University of Texas at Austin. Downloaded on March 03,2010 at 21:38:46 EST from IEEE Xplore. Restrictions apply.

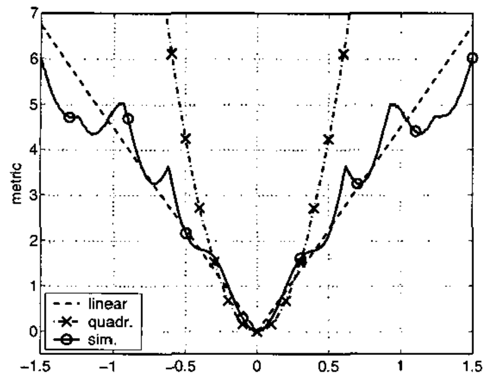

Fig. 4. Simulated negative logarithm of noise probability density function at level 3 in MLC.

rithm of the distribution of the noise at level 3 for an AWGN channel with an SNR of $\sim$ 12 dB is shown. Obviously, in this case the quadratic metric is no longer optimum since large distances are weighted too much. A better approximation of the simulated curve is a linear metric:

$$\lambda_n^{(\mathrm{linear})} = \left|y_n - \tilde{x}_n\hat{h}_n\right|. \tag{9}$$

So if, e.g., on level 3 a subset is chosen by previous levels that is quite far away from the received symbol, the metric for a 0 and a 1 will be quite high. The quadratic nonlinearity in the Euclidian distance emphasizes this behavior, whereas the linear metric compensates it.

Obviously, the linear metric cannot be the optimum, so we looked for better alternatives. The best metric so far is a Gaussian function. The idea of the Gaussian metric emerged from a simulation of the error probability of lower levels as a function of the distance of the received vector to the nearest constellation point. The simulations showed that for small distances the probability of correct decided bits on the lower levels is close to one. In this region the quadratic metric should give optimum results. But at a certain distance the probability of erroneous decisions on lower levels rapidly increases up to the point where virtually only false decisions were made on lower levels. The metric should consider this region as an erasure which can be achieved by a flat curve. The Gaussian metric defined by

$$\lambda_n^{(\mathrm{Gaussian})} = -\exp\left(-a^2\left|y_n - \tilde{x}_n\hat{h}_n\right|^2\right) \tag{10}$$

fulfills these specifications. Close to the center it can be approximated by a quadratic function and far away from the center the gradient is close to zero. Additionally, the Gaussian metric provides a smooth transition between the two regions. Unfortunately, this metric contains a parameter a which normally depends on the signal-to-noise ratio (SNR). In our simulations we set this parameter to a fixed value of 2.7 resulting in still better performance than the linear metric.

V. Erasures

As most errors stem from the fact that the information passed between the stages sometimes is useless if the set chosen from lower levels passed back to the next level is far away from the received symbol. Then these symbols should not be used for the decoding (they can be treated as erasures).

If the previous level passes information that would select the symbol the hard decision decoder would have chosen anyway, the squared distance metric is used. If a different symbol is chosen, the metric will be quite large as the decoder without further information from previous levels would have made a decision in favor of a nearer symbol.

If the decision of the previous level is correct, than the large metric is in fact right and the decoding of the current level is improved. On the other hand the decision of the previous level may be wrong leading to additional decoding errors in the current stage. If there is enough redundancy in the code and these large metrics do not happen too often, it may be advantageous to just erase the respective decision.

The analyzed algorithm has a threshold for the metric (not taking the channel power into account). If both metrics for a 1 and a 0 are above this threshold, the decision is erased.

The optimum threshold was sought by BER simulations with the AWGN channel. A value of 0.35^2 (with the constellation amplitudes as given in the DRM standard) turned out to be optimum with a slight dependency on the SNR.

VI. Simulations and Measurements

The different metrics and algorithms are evaluated by BER simulations. These simulations utilize our open-source DRM implementation "Dream" [11]. The first simulations are done with the AWGN channel (DRM channel 1). Results can be seen in Fig. 5[1]. With one iteration (there is no feedback

[1]The parameters for our simulations are 64-QAM modulation, code rate $R = 0.6$, long cell interleaving (approx. 2 s), DRM robustness

Authorized licensed use limited to: University of Texas at Austin. Downloaded on March 03,2010 at 21:38:46 EST from IEEE Xplore. Restrictions apply.

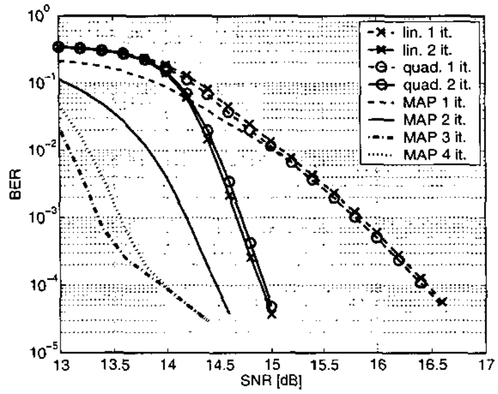

Fig. 5. BER Simulation results on AWGN channel (DRM channel 1).

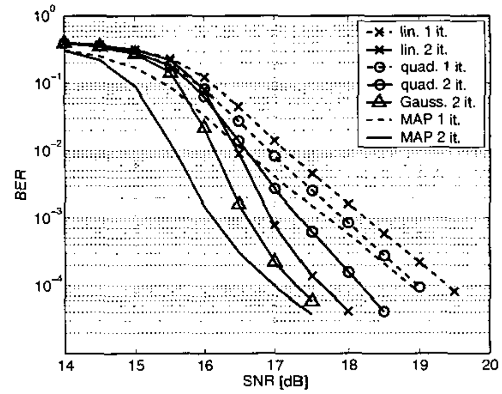

Fig. 6. BER simulation results on static frequency-selective LTI channel based on DRM channel 3.

from the higher stages back to the first stage), the quadratic metric is slightly better than the linear metric which is expected as the quadratic metric is optimum at level 0 where no decisions from other levels contribute to the decoding process. The results are different for two iterations. Now the linear metric is clearly better than the quadratic metric but the gain is only a fraction of a dB. Gaussian metric and Erasure are again slightly better than the linear metric but the gain is negligible so they have been omitted in Fig. 5. The gain achieved with two iterations is about 1.4 dB at a BER of 10^{-4} compared to the case with one iteration. Further iterations do not improve the BER significantly.

The MAP algorithm with one iteration does not show an advantage compared to the linear and quadratic metric decoder. But with two iterations the gain is already about 0.4 dB higher (again at a BER of 10^{-4}). As shown in Fig. 5, a third iteration improves by another 0.5 dB. The fourth iteration also lowers the BER at low SNR, but at a BER of 10^{-4} the BER seems to saturate so that additional iterations do not enhance the performance.

As the decoding algorithms are about to be used on multi-path fading channels, more BER simulations have been performed. It turned out that a static frequency-selective channel with the same delay profile as the DRM channel 3[2] is suitable to differentiate the performance of the metrics and algorithms. The results of the BER simulations done with this channel are shown in Fig. 6. Again, the quadratic metric is slightly better than the linear metric if only one iteration is performed. With two iterations, the linear metric outperforms the quadratic metric by 0.5 dB at a BER of 10^{-4}. At the same BER, the Gaussian metric gains another 0.3 dB. The erasure algorithm is slightly better than the quadratic metric but not worth the additional effort. Obviously, this algorithm does not compete against the linear metric. So no further simulations have been done with it.

The MAP algorithm is the best solution with an improvement of another 0.3 dB compared to Gaussian metric.

After the different algorithms and metrics have been analyzed on time invariant channels, simulations have been done on the fading channels as well. We used the channel 3 from the DRM standard. The simulation results of the BER simulations are displayed in Fig. 7. All simulations were done with two iterations and the same DRM system parameters as for the time invariant channels. Quadratic and linear metric perform equally well, the optimized Gaussian metric reaches a BER of 10^{-4} already at about 0.3 dB worse SNR. The clear winner as expected is the MAP decoder which gains another 0.7 dB.

To evaluate the performance under realistic conditions, we made measurements of the reception of a real DRM transmission broadcasted from Junglinster in Luxembourg[3]. Three except for the channel decoder identical DRM software receivers located in Darmstadt, Germany, were used to receive the transmission. The distance to the transmitter is about 230 km. The measured channel conditions at the

mode B and a bandwidth of 10 kHz. The signal power includes pilots and the guard interval, the noise bandwidth equals the nominal signal bandwidth.

[2]DRM channel 3 is a four paths fading channel with a maximum Doppler spread of 2 Hz and a maximum delay of 2.2 ms.

[3]The transmission took place on the 30th of June at UTC 15.11 on 6095 kHz. The DRM parameters were DRM mode B, 10 kHz bandwidth and a bit-rate of the main audio service of ~ 21 kbps.

Authorized licensed use limited to: University of Texas at Austin. Downloaded on March 03,2010 at 21:38:46 EST from IEEE Xplore. Restrictions apply.

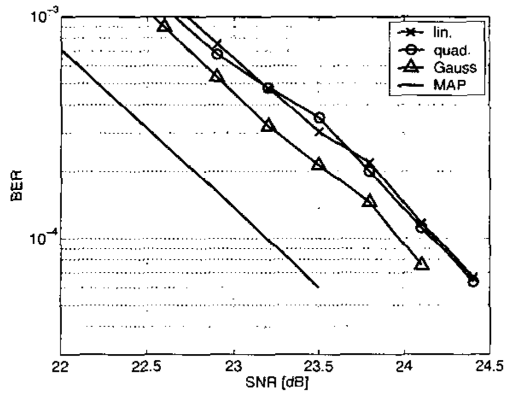

Fig. 7. BER simulation results for a fading channel (DRM channel 3).

time of recording were two dominant paths with a delay spread of 2 ms and a much weaker 3rd path after 4 ms. The Doppler spread was about 0.5 Hz. The results given in Fig.8 show the estimated SNR

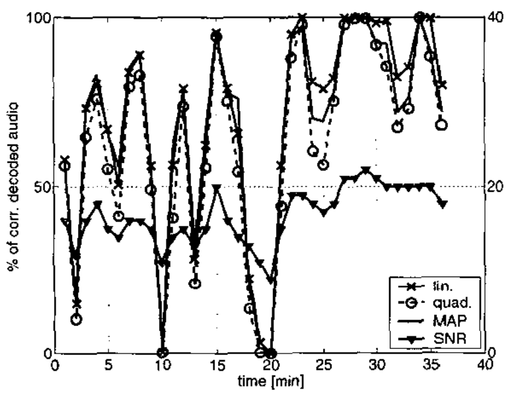

Fig. 8. Reception quality measurement of a DRM transmission broadcasted from Luxembourg and received in Darmstadt, Germany.

value averaged over one minute and the percentage of correctly decoded audio blocks of 40 ms duration. It can be seen that the quadratic metric always performs worse than the others which is in consistence to the BER simulations. Surprisingly, sometimes the linear metric gives better results than the MAP decoder (see Fig. 8 at time 25 and 33 minutes). This can be explained by taking into account the analog interferers which are often present in the HF-radio channel due to the long distance propagation. This causes a non-Gaussian noise and under these conditions the linear metric seems to be better.

VII. Summary and Conclusion

We showed that an iterative multistage decoder with hard decision and the well known squared distance metric is not the optimum solution. A simple change of the metric to a linear distance metric can improve the performance significantly. This could be verified by BER simulations with the AWGN channel and with a static frequency-selective channel were 0.5 dB are gained. Further improvement can be achieved by other changes to the metric. Introducing soft decision MAP decoders gives an additional gain but at the expense of an increase of the computational complexity.

Tests with real DRM transmissions emphasize the potential of the linear metric, owing to the robustness of this metric and the hard decision decoding it sometimes outperformed the MAP decoder, probably because of the presence of in-channel interferers during the reception.

We conclude that the hard decision multistage decoder with a modified metric is a good compromise between computational complexity and decoder performance. The simple linear metric is a very good choice.

References

[1] European Telecommunications Standards Institute: Digital Radio Mondiale (DRM), *System Specification ETSI ES 201980*, 2004.

[2] F. Hofmann, C. Hansen, and W. Schäfer: Digital Radio Mondiale (DRM) digital sound broadcasting in the AM bands, *IEEE Trans. on Broadc., vol. 49, iss. 3*, pp. 319 - 328, Sept. 2003.

[3] F. Hofmann: Multilevel Codierung (MLC) in Digital Radio Mondiale (DRM), *Kleinheubacher Berichte 2000, vol. 44*, pp. 274-278, 2000.

[4] H. Imai and S. Hirakawa: A new multilevel coding method using error-correcting codes, *IEEE Trans. Inform. Theory, vol. IT-23*, pp. 371-377, May 1977.

[5] U. Wachsmann, R. F. H. Fischer and J. B. Huber: Multilevel Codes: Theoretical Concepts and Practical Design Rules, *IEEE Trans. Inform. Theory, vol. 45, iss. 5*, pp. 1361-1391, July 1999.

[6] M. Isaka and H. Imai: On the iterative decoding of multilevel codes, *IEEE Journal on Selected Areas in Comm., vol. 19, iss. 5*, pp. 935-943, May 2001.

[7] T. Woerz and J. Hagenauer: Decoding of M-PSK-multilevel codes, *Eur. Trans. on Telecommun. and Rel. Techn.: ETT4*, pp. 229-308, 1993.

[8] L. Bahl, J. Cocke, F. Jelinek and J. Raviv: Optimal decoding of linear codes for minimizing symbol error rate, *IEEE Trans. Inform. Theory, vol. IT-20* pp. 284-287, 1974.

[9] G. Bauch and V. Franz: A comparison of soft-in/soft-out algorithms for "turbo-detection", *Intern. Conf. on Telecomm. (ICT '98)* pp. 259-263, 1998.

[10] H. L. Van Trees: Detection, Estimation, and Modulation Theory, Part I., *New York: John Wiley & Sons*, 1971.

[11] A. Kurpiers, V. Fischer: Open-Source Implementation of a Digital Radio Mondiale (DRM) Receiver, *9th IEE Intern. Conf. on HF Radio Systems and Techniques*, Bath, United Kingdom, 2003.

Authorized licensed use limited to: University of Texas at Austin. Downloaded on March 03,2010 at 21:38:46 EST from IEEE Xplore. Restrictions apply.