Verifying Finality for Blockchain Systems

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Joint work with Brandon Moore at Runtime Verification, Inc.
Ethereum

- “a decentralized platform that runs smart contracts”
- accounts with balances instead of unspent transactions
- contracts execute in virtual machine on participating nodes
Blockchain Forks and Revisions

- “a blockchain diverges into two potential paths forward”
- accidental or intentional
- could be used by adversaries to control transactions

```
Fixpoint sprefixb (s1 s2 : seq block) :=
  if s2 is y :: s2’ then
    if s1 is x :: s1’ then (x == y) && (sprefixb s1’ s2’) else true
  else false.
```

```
Definition fork (bc1 bc2 : seq block) :=
  ~~[|| sprefixb bc1 bc2, sprefixb bc2 bc1 | bc1 == bc2].
```
Casper Finality

Buterin & Griffith, *Casper the Friendly Finality Gadget*, 2017

- overlay on top of an existing blockchain system
- “select[s] a unique chain which represents the canonical transactions of the ledger”
- protects against long-range revisions and crashes (assuming > 2/3 honest participants)
Announcing Beneficiaries of the Ethereum Foundation Grants

Posted by Ethereum Team on March 7, 2018
Background

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**Awardee List**

Here are the inaugural Ethereum Foundation grant winners:

- **L4 Research** – Scalability Grant – $1.5M. State channels research.

Ethereum’s Casper, Sharding Upgrades to Launch Together Allowing Better Scalability and Security

- Ethereum co-founder Vitalik Buterin and the platform’s development team may decide to launch Casper and Sharding upgrades together, instead of separately as planned earlier.
- New research has led Ethereum’s developers to consider launching Ethereum’s new proof-of-stake algorithm, Casper, through a shard (instead of a smart contract) to help reduce the cost of helping secure its network from 1,500 ETH to 32 ETH.
Casper Protocol Coq Formalization Goals

1. key claims in paper (following previous Isabelle/HOL models)
2. integration with blockchain model in Coq (Toychain)
Key Casper Notions

Validators and Votes

Validators deposit cryptocurrency (stake) and can then vote for blocks. With enough votes, a block becomes finalized. Validators who vote incorrectly get their deposits slashed.
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Blocks in different block tree forks cannot both be finalized if more than 2/3 of validators by deposit behave honestly.
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Plausible Liveness

Regardless of what has happened before, it is always possible to continue to finalize blocks when more than 2/3 of validators by deposit follow the protocol.
On Older Casper Designs

- DynamicValidatorSet.thy is about two-message Casper (older) with a dynamic validator set (more realistic), and proves accountable safety (not plausible liveness).
- Casper.thy is about two-message Casper (older) with a static validator set (unrealistic), and proves accountable safety (not plausible liveness).
- MinimumAlgo.thy is about two-message Casper (older) with a dynamic validator set, and proves accountable safety and plausible liveness.
On Newer Casper Designs

- DynamicValidatorSetOneMessage.thy is about one-message Casper (newer) with a dynamic validator set (more realistic), and proves accountable safety (not plausible liveness).

- CasperOneMessage.thy is about one-message Casper (newer) with a static validator set (unrealistic), and proves accountable safety (not plausible liveness).
I was able to port many [HOL Light] proofs that I did not understand: despite the huge differences between the two proof languages, it was usually possible to guess what had to be proved from the HOL Light text, along with many key reasoning steps. Isabelle’s automation was generally able to fill the gaps.

—L.C. Paulson, *Formalising Mathematics In Simple Type Theory*
“[J]ust like the hammers for other systems, [CoqHammer] works very well for essentially first-order logic goals and becomes much less effective with other features of the logics […]”

—L. Czajka & C. Kaliszyk, *Hammer for Coq*, 2018
From CasperOneMessage.thy:

text {* We use first-order modeling as much as possible. This allows to reduce the size of the model, and also the size of the proofs [...] *}

locale byz_quorums =
  fixes member_1 :: "'n ⇒ 'q1 ⇒ bool" (infix "∈_1" 50)
  -- "Membership in 2/3 set"
  and member_2 :: "'n ⇒ 'q2 ⇒ bool" (infix "∈_2" 50)
  -- "Membership in 1/3 set"
  assumes "∀ q1 q2 . ∃ q3 . ∀ n . n ∈_2 q3 → n ∈_1 q1 ∧ n ∈_1 q2"

Variables quorum_1 quorum_2 : {set {set V}}.
Hypothesis qs : ∀ q1 q2, q1 ∈ quorum_1 → q2 ∈ quorum_1 →
  ∃ q3, q3 ∈ quorum_2 ∧ q3 ⊆ q1 ∧ q3 ⊆ q2.
Definitions

record ('n,'h)st = vote_msg :: "'n ⇒ 'h ⇒ nat ⇒ nat ⇒ bool"

locale casper = byz_quorums +
  fixes
    hash_parent :: "'h ⇒ 'h ⇒ bool" (infix "←" 50)
  fixes
    genesis :: 'h
  assumes
    "⟨ h₁ ← h₁; h₂ ← h₂ ⟩ ⇒ h₂ = h₂"
    and "⟨ h₁ ← h₂; h₂ ← h₃ ⟩ ⇒ h₂ = h₃"

Record st := { vote_msg : Validator → Hash → nat → nat → bool }.

Variable hash_parent : rel Hash.
Notation "h₁ ← h₂" := (hash_parent h₁ h₂) (at level 50).
Variable genesis : Hash.
Hypothesis hash_at_most_one_parent : ∀ h₁ h₂ h₃,
  (h₂ ← h₁) → (h₃ ← h₁) → h₂ = h₃.
definition justified_link
where
"justified_link s q parent pre new now ≡
(∀ n. n ∈₁ q → vote_msg s n new now pre) ∧
nth_ancestor (now - pre) parent new ∧
now > pre"

Definition justified_link s q parent pre new now :=
q ∈ quorum_1 ∧ (∀ n, n ∈ q → vote_msg s n new now pre) ∧
nth_ancestor (now - pre) parent new ∧
now > pre.
lemma non_equal_case_ind:
  assumes "justified s h1 v1"
  assumes "finalized s q2 h2 v2 xa"
  assumes "¬ h2 ←* h1"
  assumes "h1 ≠ h2"
  assumes "v1 > v2"
  shows "one_third_slashed s"

using assms proof
  (induct "v1 - v2" arbitrary: h1 v1 rule:less_induct)

\textbf{Lemma} \ non_equal_case_ind : ∀ s h1 v1 q2 h2 v2 xa,
  justified s h1 v1 →
  finalized s q2 h2 v2 xa →
  h2 </* h1 →
  h1 ≠ h2 →
  v1 > v2 →
  one_third_slashed s.
From mathcomp Require Import all_ssreflect.

Section StrongInductionLtn.

Variable P : nat → Prop.
Hypothesis IH : ∀ m, (∀ n, n < m → P n) → P m.

Lemma P0 : P 0.

Lemma pred_increasing : ∀ (n m : nat), n ≤ m → n.-1 ≤ m.-1.

Local Lemma strong_induction_all : ∀ n, (∀ m, m ≤ n → P m).

Theorem strong_induction_ltn : ∀ n, P n.

End StrongInductionLtn.
**Accountable Safety**

**Definition** finalization_fork s :=

\[ \exists \ h_1 \ h_2 \ q_1 \ q_2 \ v_1 \ v_2 \ c_1 \ c_2, \]

finalized s q_1 h_1 v_1 c_1 \land

finalized s q_2 h_2 v_2 c_2 \land

h_2 \not<~^* h_1 \land h_1 \not<~^* h_2 \land h_1 \neq h_2.\]

(* validators mustn’t double vote or vote in same span *)

**Definition** slashed s n : Prop :=

slashed_dbl_vote s n \lor slashed_surround s n.

**Definition** quorum_slashed s :=

\[ \exists \ q, q \in \text{quorum}_2 \land \forall n, n \in q \rightarrow \text{slashed} \ s \ n. \]

**Theorem** accountable_safety : \forall s, 

finalization_fork s \rightarrow \text{quorum_slashed} \ s.
Plausible Liveness

- Isabelle/HOL proofs only for old Casper (two message types)
- recent Casper removed all slashing conditions which depended on the state of the chain when vote was made
- one of these conditions was essential to the proof
- details in our tech report!
Connecting Model to Paper Claims

Variables \((T : \text{finType}) (d : T \to \text{nat}) (x y z : \text{nat})\).

Definition \(\text{gdset } n : \{\text{set } \{\text{set } T\}\} := \) 
\([\text{set } s \text{ in powerset } \{\text{set } : T\} \mid \sum_{t \in s} (d t) \geq n]\).

Lemma \(\text{gt_dset_in} : \forall n (s : \{\text{set } T\}), \) 
\(\sum_{t \in s} (d t) \geq n = (s \in \text{gdset } n)\).

Local Notation \(\text{bot} := (((x \ast \sum_{t : T} (d t)) \%\ y).+1)\). 
Local Notation \(\text{top} := (((z \ast \sum_{t : T} (d t)) \%\ y).+1)\).

Hypothesis \(\text{constr} : \text{bot} + \sum_{t : T} (d t) \leq 2 \ast \text{top}\).

Lemma \(\text{d_bot_top_intersection} : \) 
\(\forall q1 q2, q1 \in \text{gdset top} \to q2 \in \text{gdset top} \to \) 
\(\exists q3, q3 \in \text{gdset bot} \land q3 \subseteq q1 \land q3 \subseteq q2\).
Lemma constr_thirds : \( \forall n, (n \% 3).+1 + n \leq 2 \times (2 \times n \% 3).+1 \).

Variables (Validator : finType) (deposit : Validator \to \text{nat}).

Definition deposits := \( \sum_{v : \text{Validator}} (\text{deposit } v) \).
Definition deposit_bot := gdset deposit (deposits \% 3).+1.
Definition deposit_top := gdset deposit ((2 \times deposits) \% 3).+1.

Lemma Validators_deposit_constr_thirds :
\( ((1 \times deposits) \% 3).+1 + deposits \leq 2 \times ((2 \times deposits) \% 3).+1 \).
Proof. by rewrite mul1n; apply: constr_thirds. Qed.

Lemma deposit_bot_top_validator_intersection :
\( \forall q_1 q_2, q_1 \in \text{deposit_top} \to q_2 \in \text{deposit_top} \to \exists q_3, q_3 \in \text{deposit_bot} \land q_3 \subseteq q_1 \land q_3 \subseteq q_2 \).
Definition Blocktree := union_map Hash Block.

Definition hash_parent (bt : Blocktree) : rel Hash :=
    [rel x y | (x ∈ dom bt) && if find y bt is Some b
        then parent_hash b == x else false].
Current and Future Work

- dynamic validator sets
- validator deposits and slashes
- capturing beacon chain and shards chains explicitly
All existing proof translation techniques work by emulating one calculus within another at the level of primitive inferences. Could proofs instead be translated at the level of a mathematical argument?

—L.C. Paulson, *Formalising Mathematics In Simple Type Theory*
Coq/Ssreflect and MathComp Experience

- definitions more important than proof language
- library of blockchain data structures would be useful
- missed omega tactic, but see MathComp issue #251
- using bigops was hard at first, but paid off
Conclusion

Casper verification is WIP; future depends on Ethereum foundation goals and decisions

- Contact me: palmskog@utexas.edu, https://setoid.com
- Coq proofs and tech report: https://github.com/runtimeverification/casper-proofs
- Isabelle/HOL proofs: https://github.com/palmskog/pos
- Tech report has more details, e.g., on plausible liveness