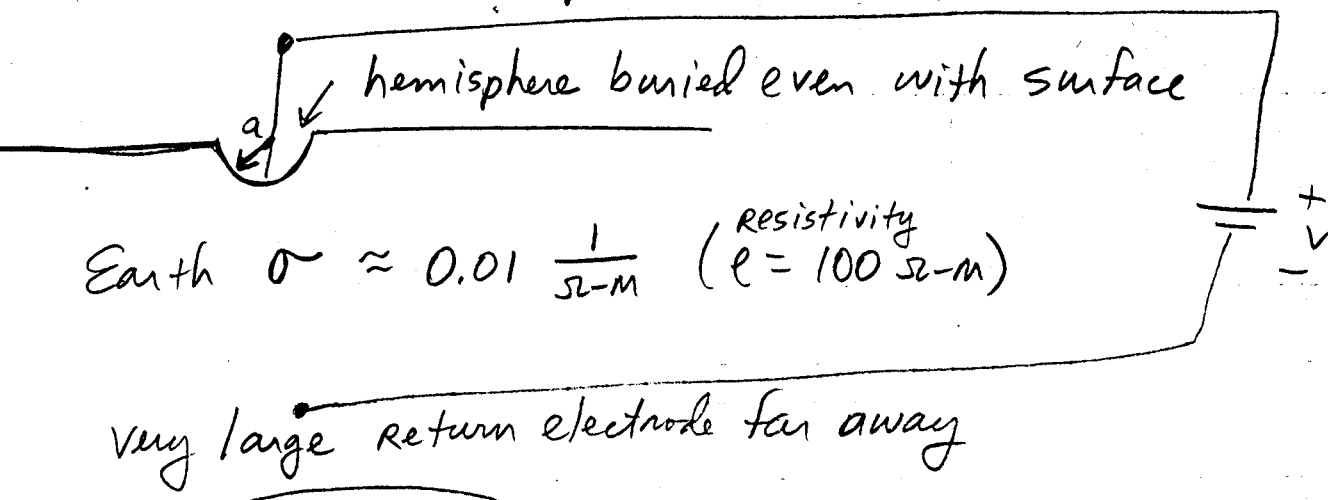
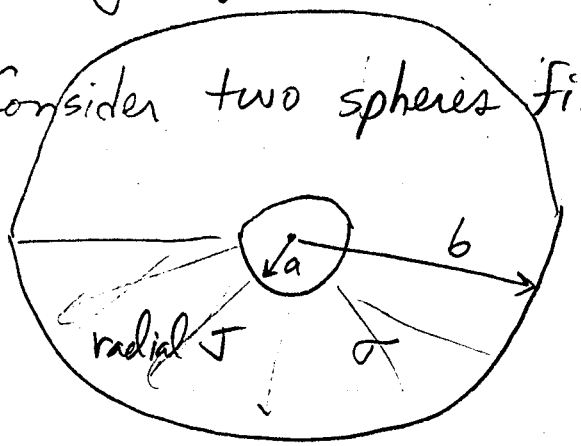
Resistance of hemisphere in Earth



Earth $\sigma \approx 0.01 \frac{1}{\Omega\text{-m}}$ (resistivity $\rho = 100\ \Omega\text{-m}$)

very large return electrode far away

Consider two spheres filled with dirt. Treat as concentric shells in series



Using $R = \frac{L}{\sigma A}$, $dR = \frac{dr}{\sigma(4\pi r^2)}$ for each shell

(dR: resistance; dr: radius)

R adds in series, so $R = \int_{r=a}^{b} dR = \frac{1}{4\pi\sigma}\int_{r=a}^{b} \frac{dr}{r^2}$

$$R = \frac{1}{4\pi\sigma}\left[-\frac{1}{r}r^{-1}\right]\Big|_{r=a}^{b} = \frac{1}{4\pi\sigma}\left[\frac{1}{a} - \frac{1}{b}\right].$$

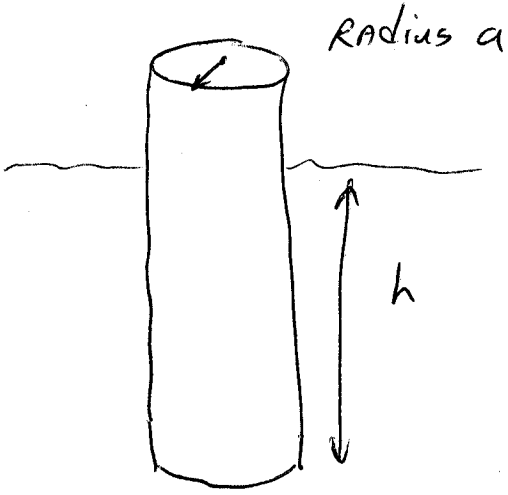
as $b \to \infty$, $R = \frac{1}{4\pi\sigma a}$.

Check $RC = \frac{\epsilon}{\sigma}$, and $C = 4\pi\epsilon_0 a$

$$RC = \frac{4\pi\epsilon_0 \cancel{a}}{4\pi\sigma\cancel{a}} = \frac{\epsilon_0}{\sigma} \quad \text{OK}$$

So a hemisphere has twice this, $\boxed{R = \frac{1}{2\pi\sigma a}}$

Ground Rods



$$R = \frac{1}{2\pi\sigma h}\left(\ln\frac{4h}{a} - 1\right)$$

σ ← dirt

$$\sigma \simeq 0.01 \frac{1}{\Omega m}$$

Let $a = 1/4$ inch, $h = 8$ feet,

$$R = \frac{1}{2\pi(0.01)(2.44m)}\left(\ln\left[\frac{4(8)(12)}{1/4}\right] - 1\right)$$

$$= 6.53(6.34) = 41\,\Omega$$

Work hard to get 20 Ω. at each tower foot.

- Perhaps not the best contact between dirt & rod
- Dirt dries out & backs away.