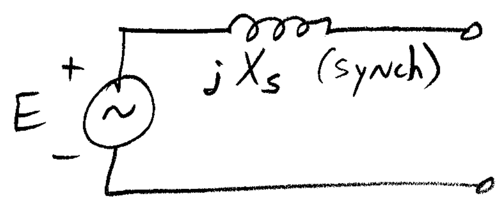
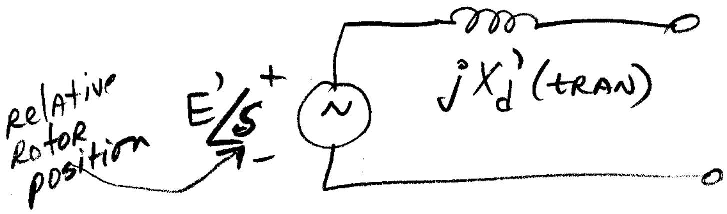
Generator Representation

Accurately modeled using transformation of variables, into a reference frame rotating at synch speed. Internal impedances are actually in operational form $X(s)$. Time constants involved.

Sometimes simplified as



Steady State



Transient
5 to 30 cycles

Note constant E, E'. Constant voltage behind synch / trans reactance

Time constants for E' ~ ½ second

Assume voltage regulator, governor, boiler controls unchanged.

S1A on back

Swing Equation

Rotating Mass with no damping

$$J\alpha = T_A = T_m - T_e \qquad (1)$$

J: Moment of inertia — $kg\text{-}m^2$

α: Angular acceleration — rad/s^2

T_A: Net accelerating torque — Newt-m

T_m: Applied mechanical torque (from external sys)

T_e: Produced electrical torque (to external sys)

$$(kg\ m^2)\frac{rad}{s^2} = N\text{-}m$$

$$kg\frac{m^2}{s^2} \rightarrow \left(\frac{kg\ m}{s^2}\right)m$$

units of Newton

Define $P = T\omega \qquad (N\text{-}m)\left(\frac{R}{s}\right) \Rightarrow \frac{N\text{-}m}{s}$ rate of doing work

$$\omega = \dot{\theta}, \quad \alpha = \dot{\omega} = \ddot{\theta}$$

$$\frac{d\theta}{dt} \qquad \frac{d\omega}{dt} \qquad \frac{d^2\theta}{dt^2}$$

$$J\frac{d^2\theta}{dt^2} = T_m - T_e, \qquad J\omega\frac{d^2\theta}{dt^2} = P_m - P_e$$

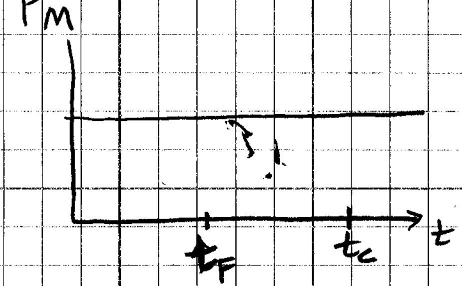
P_m: Mech input power $\qquad$ P_e: Generated Elect Power

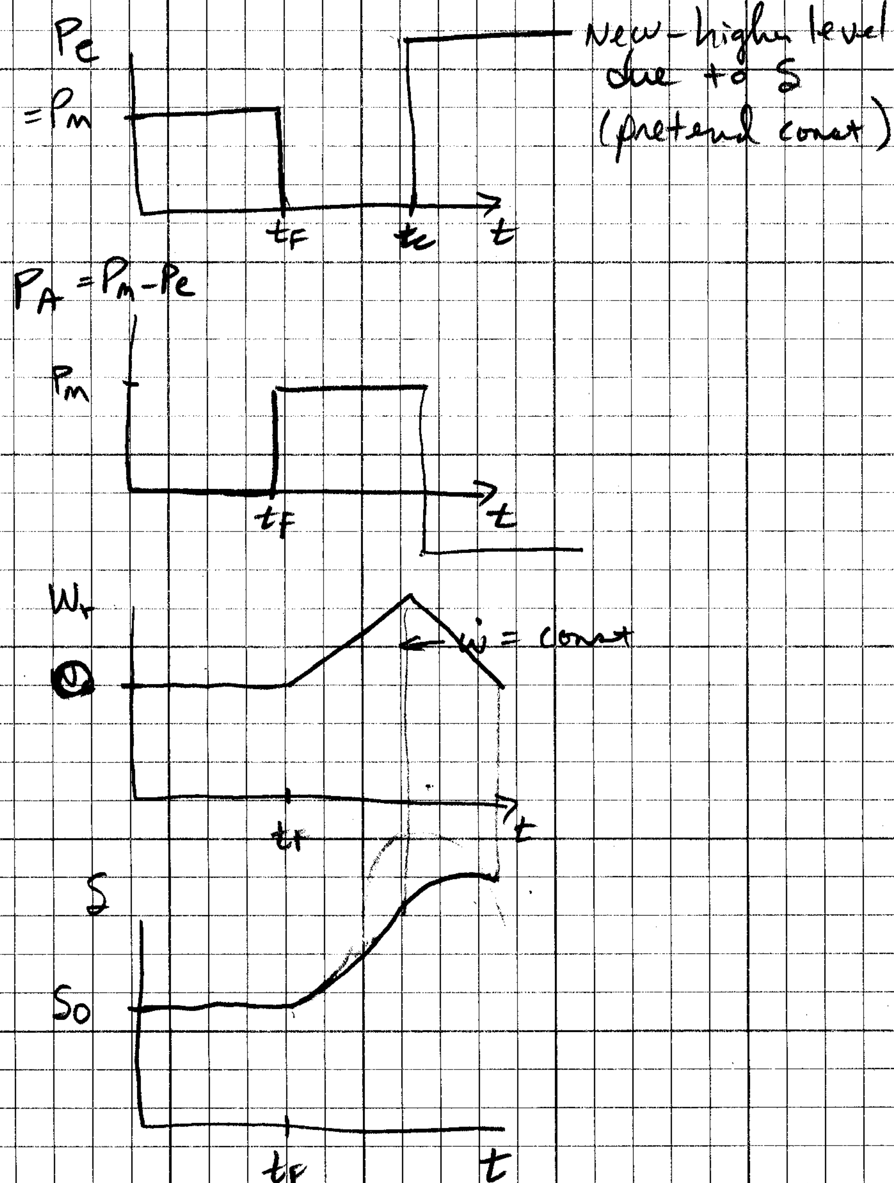
Define $H = \dfrac{\text{Stored kinetic energy at synch speed}}{\text{machine rating in MVA}}$

$$= \frac{\frac{1}{2}J\omega_s^2}{S_{MB}} = (J\omega_s)\frac{\omega_s}{2S_{MB}} = H$$

$$\frac{d^2\delta}{dt^2} = \frac{P_A \omega_s}{2H}$$

$$\ddot{\delta} = \dot{\omega} = \frac{P_A \omega_s}{2H}$$





~~PA2/2~~

Actually, $\omega \neq \omega_s$, but % variation very small.

Let $J\omega = J\omega_s \rightarrow J\omega = \frac{2HS_{MB}}{\omega_s}$

Now, $\frac{2HS_{MB}}{\omega_s}\frac{d^2\theta}{dt^2} = P_m - P_e$

$$\frac{2H}{\omega_s}\frac{d^2\theta}{dt^2} = \frac{P_M}{S_{MB}} - \frac{P_e}{S_{MB}} = P_m - P_e \Big]$$ per unit

H for turbines (on own base) 4–9 ← 9 sec @ rated power

Takes a long time to roll to a stop ~ 45 min

325 MW
93 ft long, 178 tons,
on 7 bearings,
3 RPM turning gear for 72 hours before stop

Spins in Hydrogen
↳ 7% of air density
Better cooling – Would burn up in air – red hot tips
13 ft Diam wheel @ 3600
Pressurized system – no moisture, oxygen
Not explosive if > 70% hydrogen
Check > 94% H

$\omega = 60(2\pi) = 377$ R/s
$R = 6\frac{1}{2}' = 1.98$ M
$\omega R = 746.5$ m/s
$= 1700$ MPH
> MACH 2

Of course, θ is always increasing

$$\theta(t) = \int \underset{\substack{\uparrow \\ \text{Synch}}}{\omega}\, dt + \theta_0$$

Steady state $\rightarrow P_m = P_e, \ \frac{d^2\theta}{dt} = 0$

Important quantity is movement of θ from synch θ.

Let $\theta(t) = \underbrace{\omega_s t + \theta_0}_{\text{steady state}} + \delta(t)$

$$\frac{d\theta(t)}{dt} = \omega_s + \frac{d\delta(t)}{dt}, \quad \frac{d^2\theta(t)}{dt^2} = \frac{d^2\delta(t)}{dt^2}$$

$$P_{Mech} = P_{Elec} + \frac{d}{dt} W_{kinetic} + \cancel{P_{Friction}}^{\text{SMALL}} \quad (1)$$

$$W_{kinetic} = \frac{1}{2} J \omega^2 = \frac{1}{2} J (\dot{\theta})^2$$

$$\frac{dW_{kinetic}}{dt} = J\omega\dot{\omega} = J\omega\ddot{\delta} \simeq J\omega_0\ddot{\delta}$$

So $P_{Mech} = P_{Elec} + J\omega_0 \ddot{\delta}$

$$\cancel{\ddot{\delta} = \frac{P_{Mech} - P_{Elec}}{J\omega_0}}$$

$$\frac{P_{Mech}}{S_{B3\phi}} = \frac{P_{Elec}}{S_{B3\phi}} + \frac{J\omega_0\ddot{\delta}}{S_{B3\phi}}$$

$P_{M,pu}$ $\quad P_{E,pu}$

$$\frac{2\left(\frac{1}{2}J\omega_0^2\right)}{\omega_0 S_{B3\phi}} = \frac{2 W_{kinetic}}{\omega_0 S_{B3\phi}} \leftarrow H \text{ sec.}$$

$$= \frac{2H}{\omega_0} = \frac{H}{\omega_0/2}$$

$$= \frac{H}{2\pi f_0/2} = \frac{H}{\pi f_0} = M$$

So

$$P_{Mech\,pu} - P_{Elec\,pu} = M\ddot{\delta}$$

$$\ddot{\delta} = \frac{P_{Mech\,pu} - P_{Elec\,pu}}{M}$$ The swing Equation

$\omega_{REL} = \omega - \omega_0$

$\dot{\omega} = \dot{\omega}_{REL}$

Before a disturbance, ω_{REL} is zero.

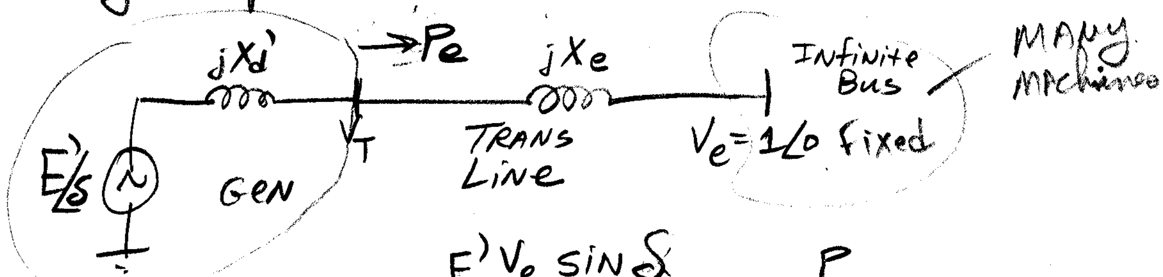
After a disturbance, ω_{REL} becomes ZERO again if stable (see curve)

$$\int_0^{t_{\delta MAX}} \omega_{REL}\, dt = 0$$

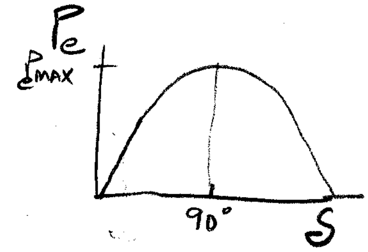
Then $$\boxed{\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_M - P_e}$$ Per Unit swing eq. (No damping)

δ is departure from synch reference value

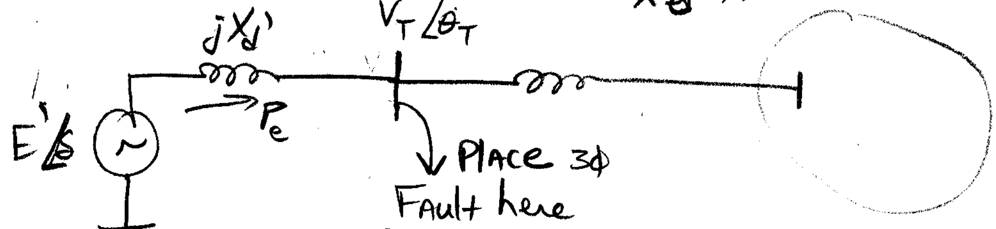
Why important?



$$P_e = \frac{E' V_e \sin\delta}{X_d' + X_e}$$



If E', V_e const, $P_{eMAX} = \dfrac{E' V_e}{X_d' + X}$

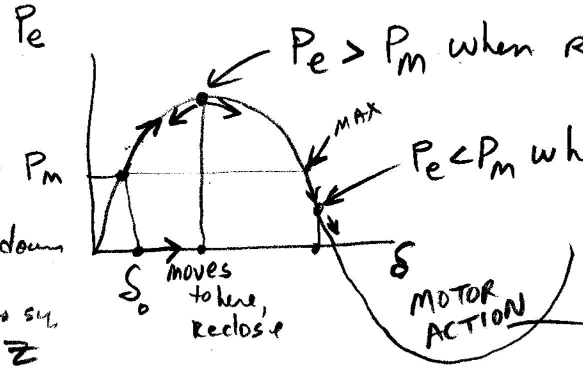


$$P_e = \frac{E' \overset{0}{\cancel{V_T}} \sin(\delta - \theta_T)}{X_d'} = 0$$

[Note – very large current, but no real power during fault]

Swing Equation

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}\left[P_M - \overset{0}{\cancel{P_e}}\right] = \frac{\omega_s P_M}{2H}$$ Large pos number



show perturbation...

$P_e > P_M$ when reclose, $\dot{\omega} < 0$, $\frac{d^2\delta}{dt^2} < 0$, slows down, overshoots, but

$P_e < P_M$ when reclose, $\dot{\omega}$, $\frac{d^2\delta}{dt^2} > 0$, keeps accelerating, too late

trip 108% overspeed

$P_e > P_M$ slows down

$P_M > P_e$ speeds up

Freq $\simeq$ 1 Hz

Machine trips off to avoid serious over currents

what is the max reclose δ for stability?

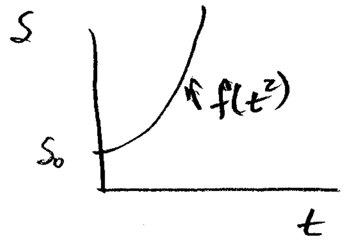
ω_r: dep from ω_s

$\omega = \omega_s + \omega_r$ (const)

During Fault t ~~M3/2~~ 55

$$\omega_r = \frac{d\delta}{dt} = \int_0^t \frac{d^2\delta}{dt^2} = \frac{\omega_s P_m t}{2H} \quad \text{(Above synch)}$$

$$\delta = \int \omega\, dt = \frac{\omega_s P_m t^2}{4H} + \delta_0 \quad \text{(Above synch)}$$



Rapid increase

$$\frac{d\omega_r}{dt} = \frac{\omega_s}{2H} P_A$$

$$\cancel{\omega_r = \frac{\omega_s}{2H}\int P_A\, dt}$$

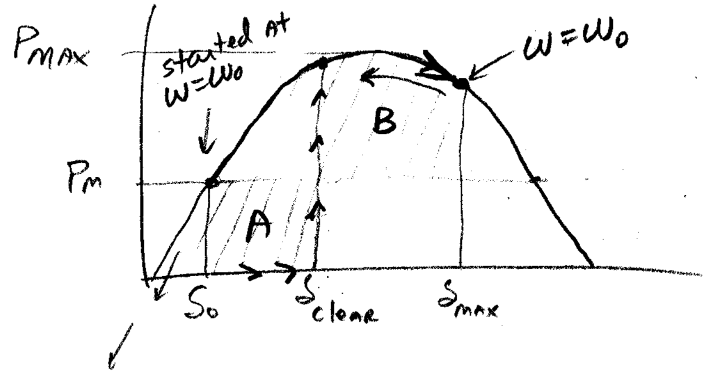
Starts & stops where $\omega_r = 0$

$$\int P_A\, dt = 0$$

$$d\delta = \omega_r\, dt$$

$$\int \frac{P_A\, d\delta}{\omega_r} =$$

EQUAL AREA criteria



$$\frac{d\omega_r}{dt} = \frac{\omega_s P_A}{2H} \rightarrow d\omega_r = \frac{\omega_s P_A}{2H} dt$$

accel power

Also, $d\delta = \omega_r\, dt \rightarrow dt = \dfrac{d\delta}{\omega_r} \rightarrow d\omega_r = \dfrac{\omega_s P_A}{2H\omega_r} d\delta$

start & finish at

$$\omega_r\, d\omega_r = \frac{\omega_s P_A}{2H} d\delta \qquad \text{Integrate both sides}$$

$\omega_r = 0$

$\int \omega_r\, d\omega_r = 0$ since ω_r same integration limits

$\omega_r = 0$

$$\therefore \int_{\delta_0}^{\delta_{MAX}} P_A\, d\delta = 0 \qquad \text{Equal area criteria}$$

$$A = B$$

$$A = P_M(\delta_{CLEAR} - \delta_0)$$

$$B = \int_{\delta_{CLEAR}}^{\delta_{MAX}} (P_e - P_m)\, d\delta$$

But $P_e = P_{MAX} \sin\delta$

$$B = P_{MAX}\int_{\delta_{CLEAR}}^{\delta_{MAX}} \sin\delta \, d\delta - P_M(\delta_{MAX} - \delta_{CLEAR})$$

$$= P_{MAX}\left[\cos(\delta_{CLEAR}) - \cos(\delta_{MAX})\right] - P_M\left[\delta_{MAX} - \delta_{CLEAR}\right]$$

$A = B$

$$\cancel{P_M\delta_{CLEAR}} - P_M\delta_0 = P_{MAX}\left[\cos(\delta_{CLEAR}) - \cos(\delta_{MAX})\right] - P_M\delta_{MAX} + \cancel{P_M\delta_{CLEAR}}$$

$$\cos(\delta_{CLEAR}) = \frac{P_M}{P_{MAX}}\left[\delta_{MAX} - \delta_0\right] + \cos(\delta_{MAX})$$

WHAT IS δ_{MAX} allowed? (Intersection of P_M) $= \pi - \delta_0$

Also, $P_M = P_{MAX}\sin\delta_0$

$$\cos(\delta_{CLEAR}) = \sin\delta_0\left[\pi - \delta_0 - \delta_0\right] + \underbrace{\cos(\pi - \delta_0)}_{= -\cos\delta_0}$$

$$\boxed{\delta_{CLEAR} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right]}$$ critical clearing $\measuredangle$

How long?

$$\delta_{CLEAR} = \frac{\omega_s P_M t_{CLEAR}^2}{4H} + \delta_0$$

$$\boxed{t_{CLEAR} = \sqrt{\frac{4H(\delta_{CLEAR} - \delta_0)}{\omega_s P_M}}}$$ Critical clearing time

ω_s: $\frac{RAD}{SEC}$

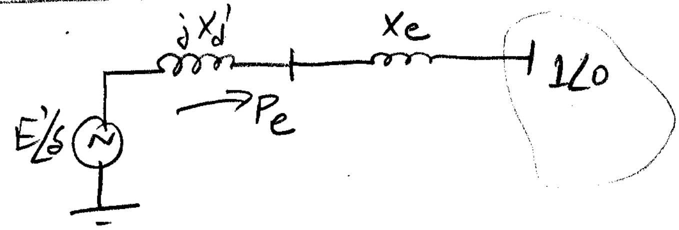
H, P_m, δ_0 important

H: $\frac{MJ}{MVA}$; P_m: pu; δ_0: RAD

≤ 10 ~~~~

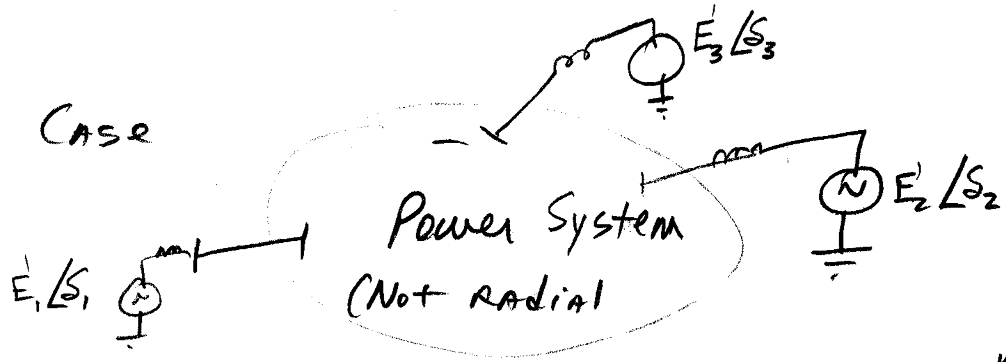
$\approx .166$ sec

Solution of Swing Equation



$$\underbrace{\frac{d^2\delta}{dt^2} = \frac{W_s}{2H}\left[P_M - P_e\right] = \frac{W_s P_M}{2H} - \frac{W_s}{2H} P_{MAX} \sin\delta}_{\text{Nonlinear D.E.}}$$

Gen Case



contolled by voltage regulator

$$\frac{2H^N}{W_s}\frac{d^2\delta_N^N}{dt^2} = P_M^N - P_e^N$$ ← Function of E, $\angle\delta$

either constant or controlled by steam/governor system

How to get P_e

If loads are represented as constant Z's to gnd, knowing E's, $\angle\delta$'s will yield all network voltages & currents ⟶ powers P_e (w/o loadflow)

Constant Power Loads

Run loadflow with E, $\angle\delta$ known. Solve for powers P_e

Constant Current Loads ?