Procedure for Estimating Grid Inertia $H$ from Frequency Droop Measurements

While the expressions for inertia and frequency droop are well known, it is prudent to rederive them here. Treating all the grid generators as one large unit, the stored energy is

$$ W_{ke} = \frac{1}{2} J \omega_s^2. $$

(1)

Taking the derivative,

$$ \frac{dW_{ke}}{dt} = J \omega_s \frac{d\omega_s}{dt}. $$

(2)

The rate of change of stored energy is net input mechanical power minus net electrical output power,

$$ \frac{dW_{ke}}{dt} = P_m - P. $$

(3)

There is power balance before a unit trip,

$$ P_m^{pre} = P. $$

(4)

When one of the many generators trips (i.e., loss of $\Delta P_m$), $P_e$ temporarily remains unchanged because voltage regulators maintain load voltage and load power. Frequency has not changed enough to affect load power. Thus generators begin to give up some of their stored kinetic energy (commonly called the inertial droop).

$$ \frac{dW_{ke}}{dt} = J \omega_s \frac{d\omega_s}{dt} = P_m^{pre} - \Delta P_m - P_e = -\Delta P_m, \quad J \omega_s = \frac{-\Delta P_m}{d\omega_s / dt}. $$

(5)

The post-trip $H$ constant is defined as

$$ H \equiv \frac{1}{P_{post,\text{rated}}} \frac{1}{2} J \omega_s^2 = \frac{1}{2} \left( J \omega_s \right) \cdot \omega_s = \frac{1}{2} \left[ -\frac{\Delta P_m}{d\omega_s / dt} \right] \frac{\omega_s}{P_{post,\text{rated}}}. $$

(6)

Rewriting,

$$ H = \frac{1}{2} \frac{-\Delta P_m}{P_{post,\text{rated}}} \frac{\omega_s}{(d\omega_s / dt)} = \frac{-\Delta P_m}{2P_{post,\text{rated}}} \frac{2\pi f_s}{2\pi(df_s / dt)} = \frac{-\Delta P_m f_s}{2P_{post,\text{rated}}(df_s / dt)}. $$

(7)
Net $H$ for ERCOT can be evaluated with experimental data as will be shown.

$H$ has the units of seconds. The correct interpretation is that the kinetic energy in the equivalent system machine corresponds to $H$ seconds of rated power. Thus, the machine could provide rated power for $H$ seconds, at which time it would have spun down to zero RPM.

The multiplier $f_s$ term in the numerator can be considered constant. $P_{\text{rated}}$ is post-event and should not contain the tripped generator. Post-event spinning reserve (excluding that of the tripped generator) should be included in $P_{\text{post, rated}}$.

Summarizing (7), $\Delta P_m$ is the MW tripped, $f_s$ is 60 Hz, $P_{\text{post, rated}}$ is total post-trip generation MW plus post-trip spinning reserve, and $df_s/\ dt$ is the inertial slope of the frequency droop immediately after the trip occurs.

Estimation of grid inertia $H$ is illustrated as follows. Consider the event shown in the following figure. Total generation is 41,493 MW, spinning reserve is 12,146 MW, and the tripped MW is 477. The sources of the data and important notes concerning their use are

1. MW trip values are from ERCOT Daily Grid Reports
2. Total Generation MW, Spinning Reserve MW, and Wind Generation MW are 1-minute averages
3. Frequency measurements are taken by the Texas Synchrophasor Network, with stations in Austin (two locations, U.T. Austin campus and at nearby Harris substation), McDonald Observatory, and U.T. Pan American, with 30 points per second. Some unit trips are not reported here due to internet problems.
4. Frequency measurements at all reporting stations are shown.
5. The spinning reserve of a tripped unit is estimated from total spinning reserve using its fraction of total MW generation.

The first step in determining $H$ is to draw the dashed line used to estimate inertia slope $df_s/\ dt$. Measuring the slope of the dashed line yields $df_s/\ dt = 0.039$ Hz per second (falling).
Next, the prorated share of spinning reserve of the tripped generation is estimated to be

$$12146 \cdot \frac{477}{41493} = 139.6 \text{ MW}.$$ 

Substituting into (7) yields

$$H = \frac{-\Delta P_m f_s}{2P_{post,\text{rated}} \left(df_s/dt\right)} = \frac{-477 \cdot 60}{2 \left[(41493 - 477) + (12146 - 139.6)\right] \cdot (-0.0386)},$$

$$H = 6.98 \text{ sec}.$$
Frequency Recovery Ratio (FRR).

Frequency drop curve with nadir slope line above has downward slope $\Delta F_{nadir} \over \Delta T_{nadir}$ Hz per second. FRR compares how far down system frequency would have dropped at $\Delta T_{nadir}$ with inertia response alone (i.e., no governor action), to how far it actually dropped with governor action. It is a relative measure for robustness.

$$FRR = \frac{\Delta F_{inertia}}{\Delta F_{nadir}} \cdot \frac{(Inertia Slope) \cdot \Delta T_{nadir}}{(Nadir Slope) \cdot \Delta T_{nadir}} = \frac{Inertia slope}{Nadir slope}$$

Now, consider the maximum frequency drop $\Delta F_{nadir}$ and the time it takes to reach the nadir of the frequency drop curve, $\Delta T_{nadir}$. The nadir slope line above has downward slope $\Delta F_{nadir} \over \Delta T_{nadir}$ Hz per second. FRR compares how far down system frequency would have dropped at $\Delta T_{nadir}$ with inertia response alone (i.e., no governor action), to how far it actually dropped with governor action. It is a relative measure for robustness.
Generalized Expression with Spinning Reserve as a Variable

The previous example included spinning reserve, but not as an explicit variable. If spinning reserve is a uniform per unit value on the base of every machine, it can be included as an explicit variable as follows. Begin with the previous equation

$$H = \frac{-\Delta P_m f_s}{2P_{post, rated}(df_s / dt)}$$

where $\Delta P_m$ is the MW generation tripped, and $P_{post, rated}$ is the post-trip MW generation plus spinning reserve SR. If SR is the same ratio at all generations, and assuming that the load MW does not change when the unit trip occurs, then the equation for H becomes

$$H = \frac{-\Delta P_m f_s}{2\left(P_{gen, pre} - \Delta P_m\right) \cdot (1 + SR) \cdot (df_s / dt)}$$

(8)

where $P_{gen, pre}$ is the pre-trip generation (excludes spinning reserve). Substituting in the numbers yields the same result as before,

$$H = \frac{-477 \cdot 60}{2 \cdot (41493 - 477) \cdot (1 + 0.293) \cdot (-0.0386)} = 6.98 \text{ sec.}$$

A reverse problem is to estimate the rating of the tripped unit, using assumptions of H and SR. In that case, from (8),

$$2H \left(P_{gen, pre} - \Delta P_m\right) \cdot (1 + SR) \cdot (df_s / dt) = -\Delta P_m f_s .$$

Rewriting,

$$\Delta P_m \left(f_s - 2H(1 + SR)(df_s / dt)\right) = -2HP_{gen, pre}(1 + SR)(df_s / dt).$$

Rewriting,

$$\Delta P_m = \frac{-2H \cdot P_{gen, pre}(1 + SR)(df_s / dt)}{f_s - 2H(1 + SR)(df_s / dt)} = \frac{P_{gen, pre} \cdot 2H(1 + SR)(df_s / dt)}{2H(1 + SR)(df_s / dt) - f_s}.$$  

Rewriting, the estimate for the tripped MW generation is

$$\Delta P_m = \frac{P_{gen, pre}}{1 - \frac{f_s}{2H(1 + SR)(df_s / dt)}}.$$  

(9)
Using the previous example,

\[ \Delta P_m = \frac{41493}{60} \left(1 - \frac{2 \cdot 6.98 \cdot (1 + 0.293) \cdot (-0.0386)}{1 + \frac{60}{1 + 0.697}} \right) = \frac{41493}{1 + 86.1} = 41493 \approx 477 \text{ MW.} \]

In most cases, the “1” in the denominator is relatively insignificant considering the accuracy of the data, so that a reasonable approximation is

\[ \Delta P_m \approx \frac{-P_{\text{gen, pre}}}{f_s} = -2 \frac{P_{\text{gen, pre}} H (1 + SR)(df_s / dt)}{2H(1 + SR)(df_s / dt)} \]

\[ = \frac{-2 \cdot 41493 \cdot 6.98 \cdot 1.293 \cdot (-0.0386)}{60} \approx 482 \text{ MW (approx.)} \quad (10) \]

The rating of the tripped unit is approximately

\[ \Delta P_{\text{rated}} = \Delta P_m \cdot (1 + SR) \approx \frac{-P_{\text{gen, pre}}}{f_s} = -2 \frac{P_{\text{gen, pre}} H (1 + SR)^2 (df_s / dt)}{2H(1 + SR)(df_s / dt)} \]

\[ \approx \frac{-2P_{\text{gen, pre}} H (1 + SR)^2 (df_s / dt)}{f_s} = 623 \text{ MW (approx.)} \quad (11) \]