Transmission capacity of CDMA ad hoc networks employing successive interference cancellation

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***Abstract*— In this paper, upper and lower bounds on the *transmission capacity* of direct-sequence CDMA wireless ad hoc networks are derived. The transmission capacity is a stochastic measure of the allowable number of transmissions per unit area, and is a generalization of previous measures of ad hoc network capacity. Successive interference cancellation (SIC) is attractive for DS-CDMA ad hoc networks since the dominant nearby interferers can be cancelled. Our closed-form results cleanly summarize the dependence of ad hoc network capacity on pathloss, spreading, outage probability, and interference cancellation accuracy. Other multiple access schemes such as CSMA and DS-CDMA without SIC are special cases. Perfect interference cancellation increases transmission capacity by nearly two orders of magnitude. Furthermore, cancelling just the strongest interferer generally gives the majority of the capacity gain, so the latency and complexity cost of SIC should be negligible.**

I. Introduction

Exploring the capacity of wireless ad hoc networks is a current topic of great interest. Ad hoc networks are distinguished by the absence of centralized wired infrastructure, and by the requirement that the nodes of the network spontaneously route traffic from source to destination, which often requires multiple hops. Since many nodes are distributed spatially and wish to share the same frequency spectrum, the choice of multiple access technique has a large impact on the capacity. In this paper, we use stochastic geometry [2], [3], [4] to study the *transmission capacity* of ad hoc networks, defined as the maximum density of simultaneous transmissions that the network can support while ensuring that a certain target signal-to-interference-plus-noise ratio (SINR) is maintained at each receiver (with some specified outage probability).

In this paper, we derive upper and lower bounds on the transmission capacity of direct sequence CDMA (DS-CDMA) ad hoc networks with imperfect successive interference cancellation (SIC) at the receivers. The common CSMA-type scenario is a special case of our analysis, where the spreading factor is 1, and conventional DS-CDMA without interference cancellation is also the special case where the interference cancellation error is 100%. Closed-form expressions are provided in terms of important quantities such as spreading factor, pathloss exponent, outage probability, and interference cancellation proficiency. Although the results presented in this paper required some compromising assumptions such as fixed transmission distances, and the neglect of short and long-term fading as well as network-level scheduling and routing, they still illuminate many of the key dependencies of wireless network capacity. Future research should be capable of generalizing the results to more realistic propagation and network-level models.

II. Related Work

Research on CDMA ad hoc networks was active in the mid to late 1980s [9], [12], but was somewhat subdued in the 1990s, before picking up again recently. Early work by Pursley and Taipale [8] studied error probabilities for spread spectrum ad hoc networks and found that frequency hopping was generally preferable to direct sequence due to the near-far effect, a result that will be reinforced in this paper from a network capacity perspective. Spread spectrum has often been considered a desirable means of rejecting the inevitable interference experienced in ad hoc networks [11], and recent results on capacity regions [14] has suggested that SIC is a powerful technique in wireless ad hoc networks. Our recent work has shown that frequency hopped spread spectrum (FHSS) typically has a large advantage over DS-CDMA in wireless ad hoc networks due to the near-far effect [16]; SIC's ability to cancel strongly interfering nearby nodes is hence a major motivation of this research.

More recently, Gupta and Kumar [6] established that the *transport capacity* of an ad hoc network, defined as the number of bit-meters pumped over a given time interval for a network of nodes occupying a unit area, is $O(\sqrt{\lambda})$, where λ is the density of transmitting nodes. Their physical model takes the form of an SINR requirement, i.e., the ratio of signal power over interference plus noise power must not exceed some threshold, with powers measured at the receiver. Our model is a stochastic SINR requirement, i.e., the probability of the SINR ratio being inadequate for successful reception must be below some ϵ, which we call the outage probability requirement.

The work by Baccelli et. al. [2] is similar in spirit and scope to our approach. They use a stochastic geometric model, as do we, to investigate MAC design for ad hoc networks. In particular, their performance metric is the *mean spatial density of progress* and is the product of the number of simultaneously successful transmissions times the average jump/hop distance per transmission. Their work identifies the optimal medium access probability (MAP) for a given transmission distance and the optimal transmission distance for a given MAP. They place no limit on the outage probability, whereas we impose such a limit as a means of ensuring QoS. We briefly discuss the relationship between our results and those in [2] and [6] in the conclusion.

III. System Model

The system model assumes that locations of nodes employing spread spectrum transmission are randomly distributed in space according to a homogeneous Poisson point process (PPP) $\Pi = \{X_i\}$ on the plane $\mathbb{R}^2$. We assume a simplified path loss model where the received power $P_r = \rho r^{-\alpha}$ at a distance r from the transmitter, where ρ is the transmit power (multiplied by some constant) and α is the pathloss exponent. We make the simplifying assumption that all transmissions are over a fixed distance r_{TX} and use a fixed transmission power ρ, so the received signal power is $\rho r_{TX}^{-\alpha}$. The total noise power in the system is assumed to be $M\eta$, where M is the spreading factor used.

The transmission capacity of the network is the maximum intensity λ of the process Π such that the outage probability $p_o(\lambda) < \epsilon$, for $0 < \epsilon \ll 1$. An outage event occurs when the post-despreading SINR is below some threshold, β. Mathematically, for the case of a conventional DS-CDMA receiver located at the origin, this requirement can be stated as

$$p_0(\lambda) \equiv \mathbb{P}^0\Big(\frac{\rho r_{TX}^{-\alpha}}{M\eta + \sum_{i\in\Pi}\rho R_i^{-\alpha}} \leq \frac{\beta}{M}\Big) \leq \epsilon. \qquad (1)$$

Here, the distance of node i from the origin is $R_i = |X_i|$, and we have conservatively assumed that the PN code cross-correlation is $1/M$ [5] (sometimes $1/3M$ is used [7], which would increase the capacity bounds given in Section IV-B by $3^{\alpha/2}$). As noted previously, more general channel models are left for future work.

CDMA receivers can use successive interference cancellation (SIC) to improve the system capacity. A simple SIC receiver is shown in Fig. 1. SIC is especially promising for ad hoc networks since it is well-suited to asynchronous signals of unequal powers [1]; furthermore, only the strongest interfering users would need to be cancelled, reducing the normal latency issue with SIC. We will consider four different types of CDMA receivers, where ζ is the fractional interference left after performing interference cancellation:

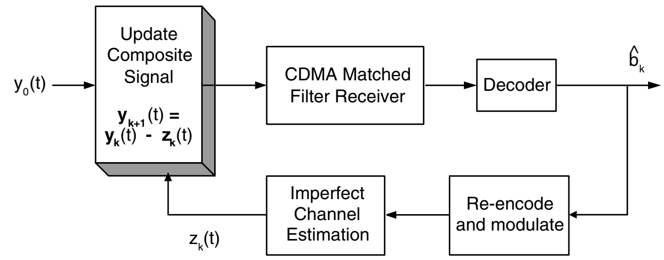

Fig. 1. Successive interference cancellation

1) Conventional DS-CDMA, $\zeta = 1$.
2) DS-CDMA with perfect interference cancellation of the strongest K users, i.e. $(K, \zeta = 0)$.
3) DS-CDMA with imperfect cancellation of all users, $(K = \infty, \zeta)$.
4) DS-CDMA with imperfect cancellation of the strongest K users, (K, ζ).

The strongest K users in our simplified pathloss model correspond simply to the K closest users, by virtue of the fact that all users employ the same transmission power. Instead of modeling SIC via the number of cancelled users, K, it is more convenient analytically to model a *cancellation radius* $r_{SIC} = \bar{r}$ with the understanding that all interfering transmitting nodes within a distance r_{SIC} of a receiver may be cancelled by that receiver. We choose r_{SIC} so that the *expected number* of cancelled users is K. From the properties of the PPP, it is straighforward to determine that this corresponds to all users within a radius of size $\bar{r} = \sqrt{\frac{K}{\pi\lambda}}$.

IV. Transmission Capacity Analysis

In this, the main section of the paper, upper and lower bounds on the transmission capacity λ in the form $\lambda_l \leq \lambda \leq \lambda_u$ are presented. The lower bound λ_l is such that $\lambda < \lambda_l$ ensures $p_o(\lambda) < \epsilon$, i.e., the outage probability requirement is definitely met, and the upper bound λ_u is such that $\lambda > \lambda_u$ ensures $p_o(\lambda) > \epsilon$, i.e., the outage requirement is definitely violated. Due to space constraints, the majority of the proofs are in a journal paper [17], which is presently accessible as a technical report [15].

A. General Analysis

A general analytical framework is now developed that can be used to determine upper and lower bounds on transmission capacity for all the subcases of interest. Then, in order to maintain the readability and brevity of the paper, the results for each subcase will simply be given along with a brief explanation.

First, note that (1) may be rewritten as

$$\mathbb{P}^0\Big(\sum_{i\in\Pi}(R_i)^{-\alpha} \geq M\kappa\Big) \leq \epsilon. \tag{2}$$

for $\kappa = \frac{r_{TX}^{-\alpha}}{\beta} - \frac{\eta}{\rho}$. We assume throughout this paper that $M\kappa < 1$, which can be forced to be true by appropriately choosing units (in metric units, $r_{TX} > \frac{M}{\beta}$ meters is generally sufficient).

In order to bound the transmission capacity, we define the following three events

$$\begin{aligned} F &= \Big\{\omega\Big| \sum_{i\in\Pi(\omega)} R_i^{-\alpha}(\omega) \geq M\kappa\Big\} && (3)\\ F_u(s) &= \Big\{\omega\Big| \Pi(\omega)\cap b(0,s) \neq \emptyset\Big\} && (4)\\ F_l(s) &= \Big\{\omega\Big| \sum_{i\in\Pi(\omega)\cap\bar{b}(0,s)} R_i^{-\alpha}(\omega) \geq M\kappa\Big\} && (5)\end{aligned}$$

which correspond, respectively, to not achieving the cutoff SINR, to a realization of the PPP where there are one or more nodes within a distance s of the origin, and to a realization of the PPP where there is sufficient interference from *outside* of the same distance s to cause an outage.

Lemma 4.1: The following relationships hold among the events $F_u(s), F_l(s), F$:

i) $F_u(s) \subset F$ *for all* $s \leq \big(M\kappa\big)^{-\frac{1}{\alpha}}$.
ii) $F_l(s) \subset F$ *for all* s.
iii) $F \subset (F_l(s) \cup F_u(s))$ *for all* s.

Proof: see appendix.

The probabilities of all three events, $F, F_l(s), F_u(s)$ are increasing in λ, so as we increase the transmission density there are more and more outcomes which constitute outages and more and more outcomes with one or more nodes in $b(0,s)$. These boundary conditions can be used to compute exact upper and lower bounds on the transmission capacity, noting that condition (i) corresponds to the upper bound on capacity since an outage *must* occur in this case. Similarly, condition (iii) allows a lower bound to be computed since an outage cannot occur for the *complement* of this case, since all outage events must be the union of $F_l(s) and F_u(s)$.

Events of the form $F_u(s)$ are useful because they can be computed exactly. That is, events of the form $F_u(s)$ are the only types of events we know of through which we can obtain a *lower bound* on $\mathbb{P}^0(F)$ and hence an *upper bound* on λ. Note that a lower bound on $\mathbb{P}^0(F)$ is needed so that we can say: if $\lambda > \lambda_u$ implies $\mathbb{P}^0(F_u(s)) > \epsilon$, then it also implies $\mathbb{P}(F) > \epsilon$ since $\mathbb{P}^0(F_u(s)) < \mathbb{P}^0(F)$. In particular, we can compute

$$\mathbb{P}^0(F_u(s)) = 1 - \mathbb{P}(\Pi \cap b(0,s) = \emptyset) = 1 - e^{-\lambda\pi s^2}, \tag{6}$$

using the void probabilities for a spatial Poisson point process [13]. Solving $1 - e^{-\lambda\pi s^2} = \epsilon$ for λ yields $\lambda_u(s)$ with the property that $\lambda > \lambda_u(s)$ implies $\mathbb{P}^0(F_u(s)) > \epsilon$, which in turn implies $\mathbb{P}^0(F) > \epsilon$.

B. Upper and Lower Bounds

Define the function $h(\alpha) = \frac{1}{2}(\alpha-1)^{\frac{1}{\alpha}}$. Note that $0.5 < h(\alpha) < 0.7$ for $2 \leq \alpha \leq 6$, which constitutes the usual accepted range for path loss exponents [10]. Deriving the upper and lower bounds for the transmission capacity, which are presented in [17], [15] results in:

Conventional CDMA, $(arbitrary K, \zeta = 1)$:

$$\begin{aligned} \lambda_l &= h(\alpha)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon && (7)\\ \lambda_u &= \frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon && (8)\end{aligned}$$

Perfect Interference Cancellation, $(K > 0, \zeta = 0)$:

$$\begin{aligned} \lambda_l &= \big(\frac{2K}{\epsilon}\big)^{1-\frac{1}{\alpha}} h(\alpha)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon && (9)\\ \lambda_u &= (1+\frac{K}{\epsilon})\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon. && (10)\end{aligned}$$

Partial Cancellation of All Nodes, $(K = \infty, \zeta)$:

$$\begin{aligned} \lambda_l &= \zeta^{-\frac{2}{\alpha}} h(\alpha)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon && (11)\\ \lambda_u &= \zeta^{-\frac{2}{\alpha}}\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon && (12)\end{aligned}$$

Partial Cancellation of Nearby Nodes $(K > 0, \zeta)$:

$$\begin{aligned} \lambda_l &= \Big((1-\zeta^2)\big(\frac{\epsilon}{2K}\big)^{\alpha-1} + \zeta^2\Big)^{-\frac{1}{\alpha}} h(\alpha)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon \\ \lambda_u &= \min\Big(\frac{1+\frac{K}{\epsilon}}{1+\zeta^{\frac{2}{\alpha}}}, \zeta^{-\frac{2}{\alpha}}\Big)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon && (13)\end{aligned}$$

Note that the bounds for the (K, ζ) case become the bounds for the other three cases under the appropriate substitutions. Hence, the last case can be considered to be a generalized result for the transmission capacity of DS-CDMA ad hoc networks.

V. Discussion and Comparisons

The most valuable aspect of the bounds derived in this paper is the ease they allow in judging the impact of various system parameters on the overall network capacity. Before comparing the bounds derived in the preceding section, we first review the result from [16] on the transmission capacity of *frequency-hopped* (FH) spread spectrum:

$$\lambda_l = \tfrac{1}{2}\tfrac{\epsilon M}{\pi}\kappa^{\frac{2}{\alpha}} \quad \lambda_u = \tfrac{\epsilon M}{\pi}\kappa^{\frac{2}{\alpha}} \tag{14}$$

In order to see how the transmission capacity varies across the different techniques (FH, DS, DS with SIC) and also as parameters vary, four plots are presented, Fig. 2-5. Unless otherwise noted, in these plots the received signal-to-thermal-noise ratio is $SNR = 20$dB (this determines η and ρ), the communication distance is $r = 10$m, the pathloss exponent is $\alpha = 4$, the fractional cancellation error is $\zeta = .1$, the outage probability is $\epsilon = 0.1$, and the number of cancellable users is $K = 3$.

The plot captions describe the features of each plot. Key results can be summarized as:

- Overall transmission capacity decreases as the spreading factor M increases (since the available bandwidth decreases by M) for all techniques except frequency hopping.
- Perfect SIC increases capacity by approximately *two orders of magnitude* relative to conventional DS-CDMA.
- SIC capacity is very sensitive to the interference cancellation accuracy. As the residual interference ζ increases, the near-far problem dominates the outage events.
- Capacity decreases in severe propagation environments (large α), except for FH.
- It is only necessary to cancel just a couple strong nodes to get nearly all of the capacity gain from SIC. Hence, the complexity and latency of adding SIC may be negligible.

It should be mentioned that although the capacity of DS-CDMA ad hoc networks decay as the spreading increases, this does not mean that the best solution is to avoid spreading. Indeed, straightforward capacity analysis of DS-CDMA cellular systems reached similar conclusions, with implementation details such as frequency reuse and voice activity eventually giving CDMA an advantage in cellular [5]. When considering delay constraints, robustness, security (anti-jamming and low probability of detection/intercept), and the need for strong error-correction codes, we suspect that spread spectrum systems will prove very attractive for wireless ad hoc networks.

VI. Conclusions

We conclude with a discussion of how our findings relate to those in [6] and [2]. Using $\kappa = \frac{r_{TX}^{-\alpha}}{\beta} - \frac{\eta}{\rho}$, our result that $\lambda \propto \kappa^{\frac{2}{\alpha}}$ translates to $\lambda \propto r_{TX}^{-2}$. The transport capacity of [6] is roughly the product of the density of successful transmissions times the average distance per transmission, which in our notation is λr_{TX}. We conclude that our transport capacity is $\lambda r \propto \sqrt{\lambda}$, which recovers the result in [6]. The contribution of our analysis are the bounds on how capacity varies with a statistical guarantee on outage probability, and the generalization to spread spectrum systems.

The findings in [2] include a guideline to selecting the optimal density of transmitting nodes for a fixed transmission distance so as to maximize the spatial density of successful transmissions. Their findings show that the probability of outage under the transmission density that maximizes the density of successful transmissions is $1 - \frac{1}{e} \approx 63\%$. The high occurrence of outages means that the scarce energy budget of most mobile devices will be used on unsuccessful transmissions most of the time. Our inclusion of a QoS parameter, ϵ, bounding the acceptable outage probability, ensures our capacity regions aren't achieved at this expense.

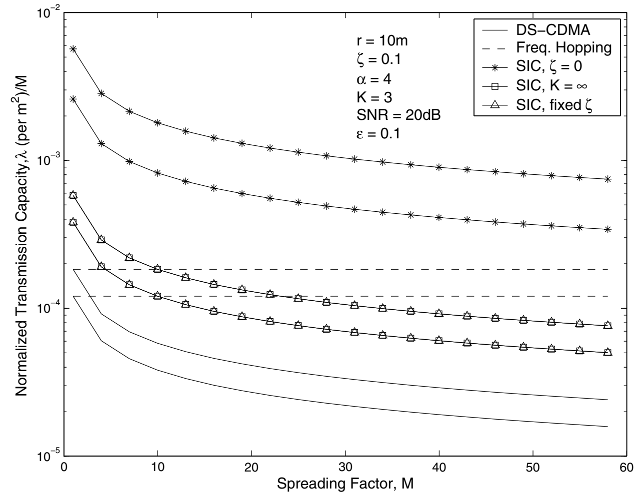

Fig. 2. Normalized transmission capacity vs. spreading factor. Since all DS-CDMA systems lose capacity as the spreading factor increases, the spreading factor should be chosen to be as small as possible while still allowing interference averaging.

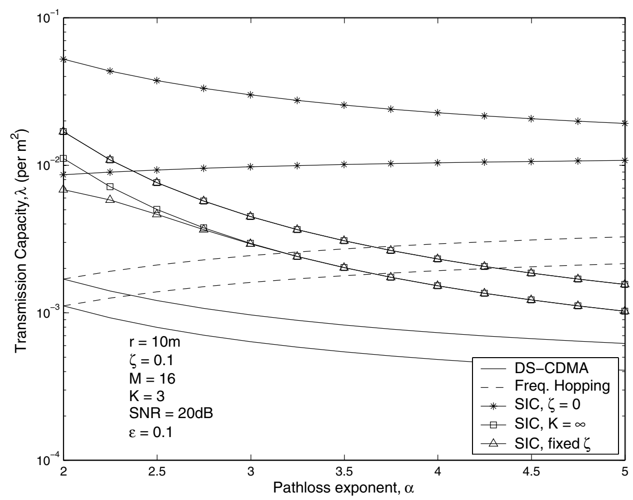

Fig. 3. Transmission capacity vs. path loss exponent. Direct sequence capacity reduces as the path loss becomes worse because much higher transmit power levels must be used, which further cripples the nearby nodes. In contrast, while frequency hopping nodes have the same problem, they only have a 1/M chance of colliding with nearby nodes, and meanwhile, the farther nodes now cause less interference than before.

Appendix

A. Proof of Lemma 4.1

(i) If $s \leq \left(M\kappa\right)^{-\frac{1}{\alpha}}$ then even if there is only one node in $b(0, s)$, and even if that one node is as far away from the

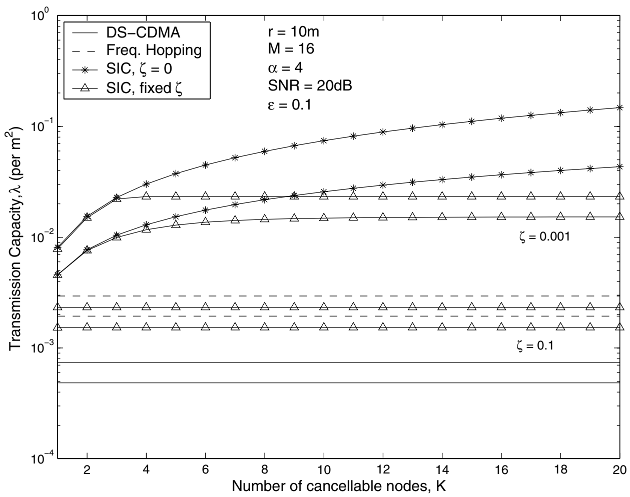

Fig. 4. Transmission capacity vs. number of cancellable users. Only a few interferers need to be cancelled to get most of the gain from SIC. This is because these closest interferers who cause most of the interference. In fact, unless the interference cancellation is extremely accurate, cancelling just one node gets most of the capacity of $K = \infty$. This outcome has favorable implications for receiver complexity and latency.

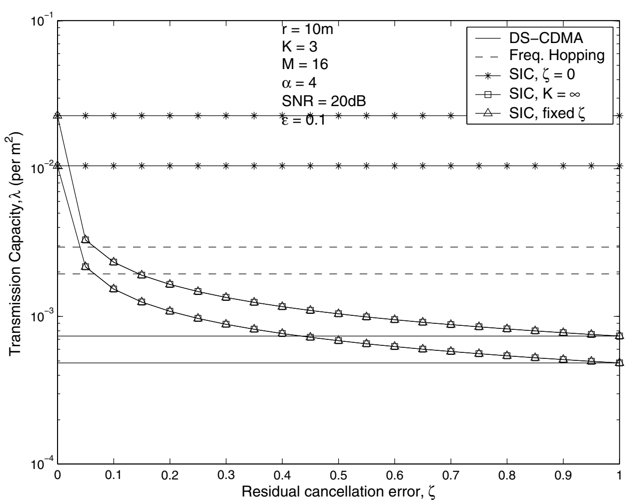

Fig. 5. Transmission capacity vs. residual cancellation error. Capacity degrades quite rapidly as interference cancellation error increases, with nearly 2 orders of magnitude difference between perfect cancellation and no cancellation ($\zeta = 1$).

origin as possible, i.e., $R_i = s$, then the normalized interference generated by that node, $s^{-\alpha} \geq \left((M\kappa)^{-\frac{1}{\alpha}} \right)^{-\alpha} = M\kappa$, is still sufficient to cause an outage, thus proving the statement.

(ii) This statement is obviously true since the set of outcomes ω which constitute outages caused by nodes in $\bar{b}(0,s)$ is clearly a subset of the set of outcomes which constitute outages with no restriction on the node locations.

(iii) Suppose $\omega \in F$ and $\omega \notin F_u(s)$. Then ω constitutes an outage but there are no nodes in $b(0,s)$. Then clearly the interference is caused by nodes in $\bar{b}(0,s)$, which means $\omega \in F_l(s)$. Suppose $\omega \in F$ and $\omega \notin F_l(s)$. Then ω constitutes an outage but the external interference generated by nodes in $\bar{b}(0,s)$ is insufficient to cause outage. Then this means there are one or more nodes in $b(0,s)$, which means $\omega \in F_u(s)$. Thus, $\omega \in F$ implies $\omega \in F_l(s)$ or $\omega \in F_u(s)$, or $\omega \in (F_l(s) \cup F_u(s))$. ■

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