Transmission Capacity of Wireless *Ad Hoc* Networks With Outage Constraints

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***Abstract*—In this paper, upper and lower bounds on the transmission capacity of spread-spectrum (SS) wireless *ad hoc* networks are derived. We define transmission capacity as the product of the maximum density of successful transmissions multiplied by their data rate, given an outage constraint. Assuming that the nodes are randomly distributed in space according to a Poisson point process, we derive upper and lower bounds for frequency hopping (FH-CDMA) and direct sequence (DS-CDMA) SS networks, which incorporate traditional modulation types (no spreading) as a special case. These bounds cleanly summarize how *ad hoc* network capacity is affected by the outage probability, spreading factor, transmission power, target signal-to-noise ratio (SNR), and other system parameters. Using these bounds, it can be shown that FH-CDMA obtains a higher transmission capacity than DS-CDMA on the order of $M^{1-\frac{2}{\alpha}}$, where M is the spreading factor and $\alpha > 2$ is the path loss exponent. A tangential contribution is an (apparently) novel technique for obtaining tight bounds on tail probabilities of additive functionals of homogeneous Poisson point processes.**

***Index Terms*—*Ad hoc* networks, capacity, stochastic geometry.**

I. Introduction

Wireless *ad hoc* networks operate without the benefit of fixed infrastructure, i.e., nodes are responsible for relaying data, as well as being sources and sinks of data. Given these additional responsibilities, it is natural to inquire about the capacity of such networks. Although analysis of *ad hoc* networks goes back 30 years or more—in the earlier work the term packet radio networks was used—closed-form and concrete expressions for *ad hoc* network capacity have only been discovered recently. This is because *ad hoc* networks are generally difficult to analyze: all users interfere with each other in a manner that is difficult to model, there is no natural duplex or multiple-access scheme, and the distributed nature of the network renders traditional analysis methodologies obsolete.

Although recent research has made great strides toward understanding wireless *ad hoc* network capacity, there are still fundamental questions that remain at least partially unanswered. In particular, how does the capacity depend on various system parameters including channel characteristics, choice of physical layer, medium access control (MAC) scheduling, and power consumption? In this paper, we approach this problem by studying two representative models for SS *ad hoc* networks, which incorporate traditional modulation types (no spreading) as a special case. To consider the capacity, we first introduce a useful notion termed the *optimal contention density*, which corresponds to the maximum spatial density of nodes that can contend for the channel subject to a constraint on the typical outage probability. We then define our metric for *ad hoc* network capacity, termed *transmission capacity*, to be the area spectral efficiency of the successful transmissions resulting from the optimal contention density. More formally, if λ^ϵ denotes the maximum contention density such that at most a fraction ϵ of the attempted transmissions are permitted to fail, then the transmission capacity $c^\epsilon = \lambda^\epsilon b(1-\epsilon)$ is the area spectral efficiency of the successful transmissions, where b bits per second per Hertz (b/s/Hz) is the average rate that a typical *successful* user achieves. As we will discuss shortly, the transmission capacity is a natural outgrowth of previous results on *ad hoc* network capacity, allowing consideration for a random distribution of nodes, their achievable data rate, *and* a constraint on outage (or, equivalently, success) probability in random channel access. A principal benefit of this approach will be the derivation of simple expressions for upper and lower bounds on transmission capacity, that clearly demonstrate the dependence of *ad hoc* network capacity on key system design parameters.

A simple path loss model for propagation is adopted, and although multihop routing is not precluded in our framework, it is not directly considered. The MAC protocol adopted in this paper is also simple in the sense that all randomly distributed transmitters are presumed to transmit in an ALOHA-type fashion for a given network configuration [1]–[3]. A potential advantage of this approach is that our system model is closer to a practical distributed *ad hoc* network, unlike other recent work [4], [5], in which scheduling is deterministic and no outages are permitted. Instead, the randomness and MAC contention in our model simply result in outages, which are included in the analytical framework. Although more accurate and complicated models for the channel, routing, and MAC are left as open topics, the key dependencies of channel capacity are exposed by the transmission capacity bounds derived in this paper.

Manuscript received November 8, 2004; revised May 17, 2005. This work was supported in part by the National Science Foundation under Grant CNS-0435307. The material in this paper was presented in part at IEEE GLOBECOM, Dallas, TX, November 2004.

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Communicated by M. Médard, Associate Editor for Communications.

Digital Object Identifier 10.1109/TIT.2005.858939

A. Background and Related Work

There have been some notable recent results on *ad hoc* network capacity [4], [6], [7], [5]. Gupta and Kumar [4], for example, established that the *transport capacity* of an *ad hoc* network, defined as the number of bit-meters pumped over a given time interval for a network of nodes occupying a unit area, is $O(\sqrt{\lambda})$, where λ is the density of transmitting nodes. Their "physical" model takes the form of a signal-to-interference-noise ratio (SINR) requirement, i.e., the ratio of signal power over interference plus noise power must exceed some threshold, with powers measured at the receiver. However, their analysis focuses on a deterministic SINR model, employs a deterministic channel access scheme and, thereby, precludes the occurrences of outages. By contrast, in order to accurately model the behavior of a distributed *ad hoc* network at the physical and MAC layer, our model includes a stochastic SINR requirement coupled with random channel access. That is, under a random distribution of transmitters, the probability of the SINR being inadequate for successful reception must be below some constant ϵ, which we call the outage constraint.

Taking this a step further, more recent work [8]–[10] has shown that the scaling of transport capacity depends on the amount of attenuation in the channel. Roughly speaking, in the low-attenuation regime with no channel absorption and small path loss, the transport capacity can be unbounded even under a fixed power constraint, by using coherent relaying and interference subtraction. On the other hand, in the high-attenuation regime with channel absorption or high path loss, the transport capacity is bounded by the total available power and thus scales as $\Theta(n)$ when the n nodes in the network are individually power constrained, regardless of channel fading [11]. Our results will also show that the capacity of code-division multiple-access (CDMA) *ad hoc* networks is sensitive to the channel path loss, but in fact the two modulation types behave quite differently.

SS transmission is considered due to its ability to gracefully cope with nontrivial levels of interference. Two different types of SS modulation are considered. In direct-sequence SS, also known as DS-CDMA, users' signals are multiplied by a "spreading sequence" that has a bandwidth M times larger than than the original signal. This is the familiar type of CDMA [12] that is used in IS-95 [13] and third generation (3G) cellular networks [14], [15] and also in 802.11b wireless local-area networks (LANs) [16], albeit in a slightly different form. The other type of SS modulation is frequency hopping (FH-CDMA), where the code sequence now controls a hopping sequence for the user's narrowband signal, causing it to hop, typically in a pseudorandom fashion, to one of M narrowband frequency slots on a periodic basis. These traditional types of SS modulation and their properties are thoroughly discussed in [17]. So, although the systems require the same total bandwidth, they use it quite differently. One popular current FH-CDMA adherent is the Bluetooth system for *ad hoc* networking.

Research on CDMA *ad hoc* networks was very active in the mid to late 1980s [18]–[21], but was somewhat subdued in the 1990s, before once again becoming an active area of research. Early work by Pursley and Taipale [22] studied error probabilities for SS *ad hoc* networks and found that frequency hopping was generally preferable to direct sequence due to the near–far effect, a result that will be reinforced in this paper from a network capacity perspective. Sousa and Silvester [23] focused on choosing the optimal transmission range to optimize successful relay progress per time slot, but their closed-form result requires the path loss exponent to be 4. SS has often been considered a desirable means of rejecting the inevitable interference experienced in *ad hoc* networks [1], although it has not been clear that the gain in robustness from SS techniques is worth the extra bandwidth that is required. Although recent papers [24]–[26] have utilized SS at the physical layer and focus on performance of MAC layer designs, there have not been, prior to this paper, analytical results that show how *ad hoc* network capacity is affected by the spreading factor or other relevant factors, e.g., path loss. See [27] for a discussion of how the physical layer affects the MAC layer.

The work of Baccelli *et al.* [2] is similar in spirit and scope to our approach. They use a stochastic geometric model, as do we, to investigate MAC design for *ad hoc* networks. In particular, their performance metric is the *mean spatial density of progress* and is the product of the number of simultaneously successful transmissions times the average jump/hop distance per transmission. Their work identifies the optimal medium access probability (MAP) for a given transmission distance and the optimal transmission distance for a given MAP. They place no limit on the outage probability, whereas we impose such a limit as a means of ensuring quality of service (QoS) and reducing power consumption. The findings in [2] include a guideline to selecting the optimal density of transmitting nodes for a fixed transmission distance so as to maximize the spatial density of successful transmissions. Their findings show that the probability of outage under the contention density that maximizes the density of successful transmissions is $1 - \frac{1}{e} \approx 63\%$. The high occurrence of outages means that the scarce energy budget of most mobile devices will be used on unsuccessful transmissions most of the time. Our inclusion of the parameter ϵ, bounding the acceptable outage probability, ensures our capacity regions are not achieved at this expense. Furthermore, our results are valid for generalized power levels, path loss exponents, and other system parameters, whereas the few prior analytical results typically relied on special cases, such as an exponential distribution for transmit power [2] or a path loss exponent of $\alpha = 4$ [23].

B. Overview of Main Results

In this work, the transmission capacity is derived for two models of a CDMA *ad hoc* network. Both models assume the transmitters comprising the *ad hoc* network form a homogeneous Poisson point process on the plane. The first model assumes all transmitting nodes use a fixed transmission power and that all receivers are a fixed distance away from their transmitters. The second model relaxes these assumptions and permits a variable transmission power and a variable distance between transmitter and receiver. As will be explained, we assume nodes utilize *pairwise power control*, meaning that each transmitter chooses its transmission power such that the signal power at the transmitter's intended receiver will be some designated constant.

For the bounds of both models, developed in the next two sections, respectively, some dominant trends can be observed. First, for frequency hopping, the capacity scales roughly linearly with allowable outage probability ϵ for small ϵ and linearly with the spreading ratio M. On the other hand, for direct sequence, while the capacity growth is still roughly linear with ϵ, it grows only as $M^{\frac{2}{\alpha}}$. Since $\alpha > 2$ usually, this means that DS-CDMA capacity does not grow fast enough to justify the bandwidth loss factor of M. Additionally, it can be seen that the capacity advantage of FH-CDMA relative to DS-CDMA is $M^{1-\frac{2}{\alpha}}$. This implies that for *ad hoc* networking, frequency hopping is superior to direct-sequence SS. In other words, it is preferable to try to avoid interference than to try to simply suppress it in a linear fashion using a matched filter with processing gain. Note that near–far resistant multiuser receivers or interference cancellation techniques should, in principle, improve upon the matched-filter receiver considered in this paper [28]. Additional interpretations are given later in the paper.

An additional contribution is, to the best of our knowledge, a novel technique of obtaining bounds on tail probabilities of additive functionals of homogeneous Poisson point processes. The essence of the approach is to divide the nodes comprising the point process into "near nodes" and "far nodes," bound the tail probabilities for each set of nodes individually, then optimize over all possible near–far boundaries.

The rest of the paper is organized as follows. Section II discusses the first transmission model with homogeneous transmission power and relay distance, and develops closed-form expressions for the upper and lower bounds. Although this model is not as realistic as the subsequent model, it is easier to follow and useful from a pedagogical standpoint. Section III discusses the second transmission model with heterogeneous transmission power and relay distances, and using similar methodology arrives more directly at the transmission capacity bounds. Section IV provides some numerical and simulation results and interpretations, with a comprehensive discussion of the merits of DS- and FH-CDMA given in Section V. Section VI offers the customary concluding remarks.

II. First Model: Uniform Transmission Power and Relay Distance

A. Modeling and Assumptions

For simplicity, we initially assume that all transmitters use the same transmission power ρ, and all transmission distances are over the same distance r. These assumptions will be relaxed in the subsequent model. Our channel model is also simple: we consider only path-loss attenuation effects and ignore additional channel effects such as shadowing and fast fading. In particular, if the transmitted power is ρ and the path-loss exponent is $\alpha > 2$, then the received power at a distance $d > 1$ from the transmitter is $\rho d^{-\alpha}$.

Our interference model is as follows. For FH-CDMA we assume the availability of sufficient bandwidth W that can be divided into M subchannels, where $\frac{W}{M}$ is required bandwidth per channel. A receiver attempting to decode a signal from a transmitter on subchannel $m \in \{1, \ldots, M\}$ only sees interference from other simultaneous transmissions on that subchannel. By contrast, on a DS-CDMA network, a receiver sees interference from all transmitters, it uses the spreading factor to reduce the minimum SINR required for successful reception. If the nominal SINR requirement for FH-CDMA is β, then DS-CDMA has a reduced SINR requirement of $\frac{\beta}{M}$, assuming that the interference is treated as wideband noise [29].

The ambient noise density is denoted by N_o. For FH-CDMA, the total ambient noise power is $N_o \frac{W}{M} \equiv \eta$, i.e., only the power from the frequency subband corresponding to the active subchannel contributes noise at the receiver. For DS-CDMA, the total ambient noise power is $N_o W = M\eta$, i.e., power from the entire band W contributes noise at the receiver. Note that this corresponds to the same postprocessing signal-to-noise (SNR) (*not* SINR) for FH-CDMA, DS-CDMA, and for traditional narrowband transmission $(M = 1)$.

The model employs a homogeneous Poisson point process $\Xi = \{X_i\}$ on the plane to represent the locations of all nodes transmitting at some time t. For the FH-CDMA case, we assume each transmitter chooses its subchannel independently. We let Ξ_m denote the set of transmitters which select subchannel m, for $m = 1, \ldots, M$ and they are independently sampled at random. Because of the independent sampling assumption, each process Ξ_m is a homogeneous Poisson point process with intensity $\frac{\lambda}{M}$.

To evaluate the outage probability, we will condition on a typical transmitter at the origin resulting in what is known as the Palm distribution for transmitters on the plane [30]. It follows by Slivnyak's theorem [30] that this conditional distribution also corresponds to a homogenous point process with the same intensity and an additional point at the origin. Now shifting this entire point process so that the receiver associated with the typical transmitter lies at the origin, we have that the conditional distribution of potential interferers is a homogenous Poisson point process with the same intensity. We will denote this process by Π and denote probability and expectation with respect to this distribution by $\mathbb{P}^0$ and $\mathbb{E}^0$, respectively. Similarly, for FH-CDMA, we denote the shifted locations of transmitters in subchannel m by Π_m, which is still a homogeneous Poisson point process with intensity $\frac{\lambda}{M}$. Also denote $|X_i|$ the distance from node $i \in \Pi$ to the origin.

B. Quantifying the Transmission Capacity

The outage constraint on λ corresponds to ensuring that the probability that the received SINR is below the appropriate threshold, is less than ϵ. For the above model these are given by

$$\text{FH} \quad \mathbb{P}^0\left(\frac{\rho r^{-\alpha}}{\eta + \sum_{i \in \Pi_m} \rho |X_i|^{-\alpha}} \le \beta\right) \le \epsilon \tag{1}$$

$$\text{DS} \quad \mathbb{P}^0\left(\frac{\rho r^{-\alpha}}{M\eta + \sum_{i \in \Pi} \rho |X_i|^{-\alpha}} \le \frac{\beta}{M}\right) \le \epsilon. \tag{2}$$

We let λ^ϵ denote the optimal contention density, i.e., the maximum density λ for Π such that outage probability at a typical receiver is less than ϵ, where $\epsilon \in (0,1)$. We will obtain upper and lower bounds on λ^ϵ denoted by $\lambda_u^{\epsilon,\text{DS}}, \lambda_l^{\epsilon,\text{DS}}$ and $\lambda_u^{\epsilon,\text{FH}}, \lambda_l^{\epsilon,\text{FH}}$ for the DS-CDMA and FH-CDMA cases, respectively. In the context of the above outage constraints (1) and (2), b is the average rate that a typical user achieves given that the SINR constraint is met. For this work, we assume that the transmitted spectral efficiency is held constant at b for all users, which is reasonable since the goal of SS techniques is to increase the number of users relative to narrowband transmission for a fixed data rate. In future work, it may be fruitful to allow different users to have different rates, and hence, the typical achieved data rate b could change. With these assumptions, the bounds on transmission capacity $c_u^{\epsilon,\,\mathrm{DS}}, c_l^{\epsilon,\,\mathrm{DS}}$ and $c_u^{\epsilon,\,\mathrm{FH}}, c_l^{\epsilon,\,\mathrm{FH}}$ correspond to bounds on the optimal contention density multiplied by $b(1-\epsilon)$, where $b \equiv 1$. These bounds and the transmission capacity ratio of FH-CDMA over DS-CDMA are given in the following theorem.

Theorem 1: Let $\epsilon \in (0,1), \kappa = \frac{r^{-\alpha}}{\beta} - \frac{\eta}{\rho}$, and $h(\alpha) = \frac{\alpha-1}{\alpha}$. The lower and upper bounds on the transmission capacity subject to the outage constraint ϵ for FH-CDMA when transmitters employ a fixed transmission power ρ for receivers that are a fixed distance r away are

$$c_l^{\epsilon,\mathrm{FH}} = (1-\epsilon)\lambda_l^{\epsilon,\mathrm{FH}}, \qquad c_u^{\epsilon,\,\mathrm{FH}} = (1-\epsilon)\lambda_u^{\epsilon,\,\mathrm{FH}}$$

where

$$\lambda_l^{\epsilon,\,\mathrm{FH}} \geq h(\alpha)\frac{M}{\pi}\kappa^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)$$
$$\lambda_u^{\epsilon,\,\mathrm{FH}} = \frac{M}{\pi}\kappa^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)$$

as $\epsilon \to 0$.

The lower and upper bounds on the transmission capacity subject to the outage constraint ϵ for DS-CDMA when transmitters employ a fixed transmission power ρ for receivers that are a fixed distance r away are

$$c_l^{\epsilon,\,\mathrm{DS}} = (1-\epsilon)\lambda_l^{\epsilon,\,\mathrm{DS}}, \qquad c_u^{\epsilon,\,\mathrm{DS}} = (1-\epsilon)\lambda_u^{\epsilon,\,\mathrm{DS}}$$

where

$$\lambda_l^{\epsilon,\,\mathrm{DS}} \geq h(\alpha)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)$$
$$\lambda_u^{\epsilon,\,\mathrm{DS}} = \frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)$$

as $\epsilon \to 0$.

Overview of the Proof of Theorem 1. Let us briefly consider how these results are obtained. The outage constraints in (1) and (2) can be rewritten as

$$\text{FH} \quad \mathbb{P}^0\left(\sum_{i\in\Pi_m}|X_i|^{-\alpha} \geq \kappa\right) \leq \epsilon \tag{3}$$

$$\text{DS} \quad \mathbb{P}^0\left(\sum_{i\in\Pi}|X_i|^{-\alpha} \geq M\kappa\right) \leq \epsilon \tag{4}$$

which usefully shows that for any set of parameter selections (encapsulated in κ) outage can be thought of as simply determined by the positions of the interfering nodes. Since these expressions are complex functions of the contention density λ we will resort to obtaining careful bounds with regards to the possible positions of the interferers.

Consider the DS-CDMA case. Specifically, according to (4), the outage event is given by

$$E(\lambda) = \left\{\sum_{i\in\Pi}|X_i|^{-\alpha} \geq M\kappa\right\}.$$

We will define events $E_u(\lambda,s)$ and $E_l(\lambda,s)$ such that $E_u(\lambda,s) \subset E(\lambda) \subseteq E_l(\lambda,s)$ and the probabilities of all events $E(\lambda), E_l(\lambda,s)$ and $E_u(\lambda,s)$ increase in λ. Here s is a parameter that will be discussed in the sequel. More intuitively, these two events are represented and explained in Fig. 1.

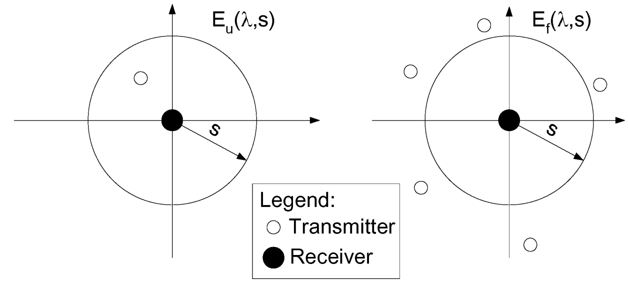

Fig. 1. On the left, the outage event $E_u(\lambda,s)$ corresponds to the scenario where just one interferer is close enough to cause an outage, i.e., it is within a distance of s. On the right, the outage event $E_f(\lambda,s)$ corresponds to an outage caused by the accumulation of far-field interference; the aggregate interference level is high enough to cause an outage even though there are no nodes inside of $b(O,s)$. Note that for DS-CDMA, s will usually be less than the transmission range but for FH-CDMA and narrowband transmission, s will usually be larger [31].

If we solve for the largest possible λ such that $\mathbb{P}^0(E_u(\lambda,s)) \leq \epsilon$, we obtain an upper bound $\lambda_u^{\epsilon,\,\mathrm{DS}}$ on the optimal contention density, i.e., if $\lambda > \lambda_u^{\epsilon,\,\mathrm{DS}}$ the outage probability must exceed our outage constraint ϵ. If we solve for the smallest λ such that $\mathbb{P}^0(E_l(\lambda,s)) \geq \epsilon$, we obtain a lower bound on the optimal contention density $\lambda_l^{\epsilon,\,\mathrm{DS}}$, i.e., if $\lambda < \lambda_l^{\epsilon,\,\mathrm{DS}}$ the outage probability must not exceed ϵ, so we definitely have a legal value for contention density.

In order to define $E_l(\lambda,s)$ and $E_u(\lambda,s)$, we consider the overall interference a receiver sees from both the "near field" and "far field." As shown in Fig. 1, the near and far fields are the regions inside and outside a circle of radius s around the typical receiver at the origin, denoted $b(O,s)$ and $\bar{b}(O,s)$, respectively; the use of these terms is similar to but distinct from their use in antenna design and channel modeling. The radius s is selected to be small enough such that one or more nodes within distance s would cause an outage. According to (4), this means $s^{-\alpha} \geq M\kappa$, which limits $s \leq (M\kappa)^{-\frac{1}{\alpha}}$. In the sequel, we shall consider optimizing over s subject to this constraint so as to get the tightest bounds. The rationale for separating the near- and far-field interference is that near-field nodes contribute a major part of the interference, as will be shown later.

Proof of Theorem 1: We first consider the DS-CDMA case. The outage events associated with the near- and far-field interference are defined as follows.

Definition 1

$$E_u(\lambda,s) = \{\Pi \cap b(O,s) \neq \emptyset\}$$
$$E_f(\lambda,s) = \left\{\sum_{i\in\Pi\cap\bar{b}(O,s)}|X_i|^{-\alpha} \geq M\kappa\right\}$$
$$E_l(\lambda,s) = E_u(\lambda,s) \cup E_f(\lambda,s).$$

According to Definition 1, $E_u(\lambda,s)$ consists of all outcomes where there are one or more transmitters within distance s of the origin O, and the event $E_f(\lambda,s)$ consists of all outcomes where the set of transmitters outside the ball $b(O,s)$ generate enough interference power to cause an outage at the origin.

These events satisfy the following properties.

a) For $s \le (M\kappa)^{-\frac{1}{\alpha}}$, $E_u(\lambda,s) \subset E(\lambda) \subseteq E_l(\lambda,s)$.
b) Each of $\mathbb{P}^0(E(\lambda)), \mathbb{P}^0(E_u(\lambda,s))$, and $\mathbb{P}^0(E_f(\lambda,s))$ increases in λ for fixed s.
c) For fixed λ, $\mathbb{P}^0(E_u(\lambda,s))$ is increasing in s, while $\mathbb{P}^0(E_f(\lambda,s))$ is decreasing in s.
d) $E_u(\lambda,s)$ and $E_f(\lambda,s)$ are independent events.

To prove Property a), consider the following facts: i) If $s \le (M\kappa)^{-\frac{1}{\alpha}}$ then even if there is only one node in $b(O,s)$, and even if that one node is as far away from the origin as possible, i.e., $|X_i| = s$, then the normalized interference generated by that node $s^{-\alpha} \ge ((M\kappa)^{-\frac{1}{\alpha}})^{-\alpha} = M\kappa$, is still sufficient to cause an outage. This proves that $E_u(\lambda,s) \subset E(\lambda)$. ii) Consider an outage outcome $\omega \in E(\lambda)$. Suppose $\omega \in E(\lambda)$ and $\omega \notin E_u(\lambda,s)$. Then ω constitutes an outage but there are no nodes in $b(O,s)$. Then clearly the interference is caused by nodes in $\bar{b}(O,s)$, which means $\omega \in E_f(\lambda,s)$. Suppose $\omega \in E(\lambda)$ and $\omega \notin E_f(\lambda,s)$. Then ω constitutes an outage but the external interference generated by nodes in $\bar{b}(O,s)$ is insufficient to cause outage. Then this means there are one or more transmitters in $b(O,s)$, which means $\omega \in E_u(\lambda,s)$. Thus, $\omega \in E(\lambda)$ implies either $\omega \in E_u(\lambda,s)$ or $\omega \in E_f(\lambda,s)$, which is equivalent to saying $\omega \in (E_u(\lambda,s) \cup E_f(\lambda,s))$. Properties b) and c) are straightforward by considering the number of interfering transmitters considered in each event. To prove Property d), recall that $E_u(\lambda,s)$ only depends upon the space $b(O,s)$ and $E_f(\lambda,s)$ only depends upon the space $\bar{b}(O,s)$. The independence property of the Poisson point process states that the number of points $N(A)$ and $N(B)$ in disjoint regions A and B are independent random variables, hence, $E_u(\lambda,s), E_f(\lambda,s)$ are independent events.

The upper Bound λ_u^ϵ. We calculate $\mathbb{P}^0(E_u(\lambda,s))$, set it equal to ϵ, and solve for λ. Consider the probability that there are no transmitters in $b(O,s)$, which is simply the void probability for $b(O,s)$ [30]. For a Poisson point process in the plane with intensity λ, the void probability for a set $A \subset \mathbb{R}^2$ is $e^{-\lambda\nu(A)}$, where $\nu(\cdot)$ denotes the area of the set contained in the argument. Thus,

$$\mathbb{P}^0(E_u(\lambda,s)) = 1 - \mathbb{P}^0(\Pi \cap b(O,s) = \emptyset) = 1 - e^{-\lambda\pi s^2} \quad (5)$$

for $s \in (0, (M\kappa)^{-\frac{1}{\alpha}})$.

Now given the outage constraint $\mathbb{P}^0(E_u(\lambda,s)) \le \epsilon$, we obtain the parameterized upper bound

$$\lambda_u^\epsilon(s) = -\frac{1}{\pi}s^{-2}\ln(1-\epsilon)$$

for all $s \in (0, (M\kappa)^{-\frac{1}{\alpha}})$ and all $\epsilon \in (0,1)$. We further optimize this bound over s and find that the tightest (smallest) upper bound is obtained by choosing the largest possible $s = (M\kappa)^{-\frac{1}{\alpha}}$. Thus, the final upper bound on the optimal contention density is

$$\begin{aligned}\lambda_u^\epsilon &= -\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\ln(1-\epsilon)\\ &= \frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)\end{aligned}$$

for all $\epsilon \in (0,1)$.

The Lower Bound λ_l^ϵ. We need to calculate $\mathbb{P}^0(E_l(\lambda,s))$. Because of Property d), we have

$$\begin{aligned}\mathbb{P}^0(E_l(\lambda,s)) &= \mathbb{P}^0(E_u(\lambda,s) \cup E_f(\lambda,s))\\ &= \mathbb{P}^0(E_u(\lambda,s)) + \mathbb{P}^0(E_f(\lambda,s))\\ &\quad - \mathbb{P}^0(E_u(\lambda,s))\mathbb{P}^0(E_f(\lambda,s)).\end{aligned}$$

The outage constraint $\mathbb{P}^0(E_l(\lambda,s)) \le \epsilon$ then can be rewritten as

$$\mathbb{P}^0(E_u(\lambda,s)) \le \epsilon_1, \qquad \mathbb{P}^0(E_f(\lambda,s)) \le \epsilon_2 \quad (6)$$

for some ϵ_1 and ϵ_2 such that $\epsilon_1 + \epsilon_2 - \epsilon_1\epsilon_2 = \epsilon$. Given the constants ϵ_1 and ϵ_2, define the contention density imposed by conditions in (6) to be

$$\begin{aligned}\lambda_u^{\epsilon_1}(s) &= \sup\{\lambda \,|\, \mathbb{P}^0(E_u(\lambda,s)) \le \epsilon_1\}\\ \lambda_f^{\epsilon_2}(s) &= \sup\{\lambda \,|\, \mathbb{P}^0(E_f(\lambda,s)) \le \epsilon_2\}.\end{aligned}$$

Thus, the lower bound of the optimal contention density can be derived as

$$\lambda_l^\epsilon = \sup_{s,(\epsilon_1,\epsilon_2)}\left\{\inf\left\{\lambda_u^{\epsilon_1}(s), \lambda_f^{\epsilon_2}(s)\right\}\right\} \quad (7)$$

for $s \in [0, (M\kappa)^{-\frac{1}{\alpha}}]$ and (ϵ_1,ϵ_2) satisfying $\epsilon_1 + \epsilon_2 - \epsilon_1\epsilon_2 = \epsilon$. To obtain the lower bound we will first consider $\lambda_u^{\epsilon_1}(s)$ and $\lambda_f^{\epsilon_2}(s)$ separately, then choose to maximize the minimum of the two for all feasible s and choices of (ϵ_1,ϵ_2) pairs. It can be seen that if we increase $\mathbb{P}^0(E_u(\lambda,s))$ then $\mathbb{P}^0(E_f(\lambda,s))$ decreases, i.e., changing s or (ϵ_1,ϵ_2) must increase one of $\lambda_u^{\epsilon_1}(s)$ and $\lambda_f^{\epsilon_2}(s)$ but decrease the other one at the same time. Thus, according to (7), a condition for the optimized lower bound is that the choice of s and (ϵ_1,ϵ_2) is such that $\lambda_u^{\epsilon_1}(s) = \lambda_f^{\epsilon_2}(s)$ if this is feasible.

Based on our calculation of the upper bound, we have that

$$\lambda_u^{\epsilon_1}(s) = -\frac{1}{\pi}s^{-2}\ln(1-\epsilon_1) = \frac{1}{\pi}s^{-2}\epsilon_1 + \Theta\left(\epsilon_1^2\right). \quad (8)$$

It is not possible to compute $\lambda_f^{\epsilon_2}(s)$ directly; we resort to finding a lower bound using Chebyshev's inequality. Let

$$Y(\lambda,s) = \sum_{i \in \Pi \cap \bar{b}(O,s)} |X_i|^{-\alpha}$$

denote the normalized far-field interference from transmitters outside the circle $b(O,s)$. In Lemma 1, found in the Appendix, we show that $\mathbb{E}[Y(\lambda,s)] = \mu(s)\lambda$ and $\mathrm{Var}(Y(\lambda,s)) = \sigma^2(s)\lambda$, and compute expressions for $\mu(s)$ and $\sigma^2(s)$. The lower bound on $\lambda_f^{\epsilon_2}(s)$ is obtained as follows:

$$\begin{aligned}\lambda_f^{\epsilon_2}(s) &= \sup\{\lambda \,|\, \mathbb{P}^0(E_f(\lambda,s)) \le \epsilon_2\}\\ &= \sup\{\lambda \,|\, \mathbb{P}^0(\sum_{i \in \Pi \cap \bar{b}(O,s)} |X_i|^{-\alpha} \ge M\kappa) \le \epsilon_2\}\\ &= \sup\{\lambda \,|\, \mathbb{P}^0(Y(\lambda,s) \ge M\kappa) \le \epsilon_2\}\\ &\ge \sup\left\{\lambda \,\middle|\, \mathbb{P}^0\left(\frac{|Y(\lambda,s) - \mu(s)\lambda|}{M\kappa - \mu(s)\lambda} \ge 1\right) \le \epsilon_2\right\}\\ &\ge \sup\left\{\lambda \,\middle|\, \frac{\sigma^2(s)\lambda}{(M\kappa - \mu(s)\lambda))^2} \le \epsilon_2\right\}\\ &= \underline{\lambda}_f^{\epsilon_2}(s).\end{aligned}$$

The first three equalities are by definition, the second inequality is obtained by using the Chebyshev bound. Clearly, $\underline{\lambda}_f^{\epsilon_2}(s)$ is achieved when

$$\frac{\sigma^2(s)\lambda}{(M\kappa - \mu(s)\lambda))^2} = \epsilon_2.$$

By solving this equation for λ and keeping the dominant term under the condition that ϵ_2 is small, we obtain

$$\begin{aligned}\underline{\lambda}_f^{\epsilon_2}(s) &= \frac{(M\kappa)^2}{\sigma^2(s)}\epsilon_2 + \Theta(\epsilon_2^2)\\ &= \frac{(\alpha-1)(M\kappa)^2}{\pi} s^{2(\alpha-1)}\epsilon_2 + \Theta\left(\epsilon_2^2\right)\end{aligned} \tag{9}$$

as $\epsilon_2 \to 0$.

We now focus on obtaining the lower bound λ_l^ϵ by solving (7) using (8), (9). As stated earlier, for a given (ϵ_1, ϵ_2) pair, the optimal s is obtained by solving $\lambda_u^{\epsilon_1}(s) = \lambda_l^{\epsilon_2}(s)$ for s, i.e., the maximum of the minimum of two functions occurs at their point of intersection, if any. Solving for s we obtain

$$s_l^{\epsilon_1,\epsilon_2} = \left(\frac{\epsilon_1}{\epsilon_2}\right)^{\frac{1}{2\alpha}} (\alpha-1)^{-\frac{1}{2\alpha}} (M\kappa)^{-\frac{1}{\alpha}}. \tag{10}$$

Substitute to find

$$\lambda_l^{\epsilon_1,\epsilon_2} = (\alpha-1)^{\frac{1}{\alpha}} \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon_2^{\frac{1}{\alpha}} \epsilon_1^{1-\frac{1}{\alpha}}. \tag{11}$$

Finally, noting $\epsilon_2 = \epsilon - \epsilon_1 + \Theta(\epsilon^2)$ find

$$\lambda_l^\epsilon = \max_{(\epsilon_1,\epsilon_2):\epsilon_1+\epsilon_2=\epsilon} \{\lambda_l^{\epsilon_1,\epsilon_2}\} = \max_{0\le\epsilon_1\le\epsilon} \left\{c(\epsilon-\epsilon_1)^{\frac{1}{\alpha}} \epsilon_1^{1-\frac{1}{\alpha}}\right\} \tag{12}$$

for $c = (\alpha-1)^{\frac{1}{\alpha}} \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}}$. The optima are $\epsilon_1^* = (1-\frac{1}{\alpha})\epsilon$ and $\epsilon_2^* = \frac{1}{\alpha}\epsilon$. Substituting this choice we obtain

$$\lambda_l^\epsilon = \left(1-\frac{1}{\alpha}\right) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon + \Theta(\epsilon^2) \tag{13}$$

as $\epsilon \to 0$.

Looking at (3) and (4), it is clear that the exact same analysis for FH-CDMA holds provided we replace $M\lambda$ with λ and $M\kappa$ with κ, i.e., if

$$\begin{aligned}Mh(\alpha)\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2) &\le M\lambda^{\epsilon,DS}\\ &\le M\frac{1}{\pi}(M\kappa)^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)\end{aligned}$$

holds for DS-CDMA, then,

$$h(\alpha)\frac{1}{\pi}M\kappa^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2) \le \lambda^{\epsilon,FH} \le \frac{1}{\pi}M\kappa^{\frac{2}{\alpha}}\epsilon + \Theta(\epsilon^2)$$

holds for FH-CDMA. □

C. Comparison of Transmission Capacity With Other Capacity Metrics

There are a number of observations that can be made from inspecting the upper and lower bounds derived in Theorem 1. We will save the bulk of such discussion until Section IV, when we will examine some plots of these expressions to enhance intuition. Before continuing, however, we would like to note some similarities between the preceding bounds on transmission capacity and other possible metrics of *ad hoc* network capacity, in particular transport capacity and network sum capacity.

In Theorem 1, we rediscover the scaling property between capacity and transmission range r to be $\Theta(r^{-2})$, the same as the result of [2]. One can understand this scaling as packing as many concurrent transmissions spatially as possible, with each occupying an area $\Theta(r^2)$. The spreading factor M allows for certain relaxations on the overlapping among concurrent transmissions, which is prohibited in narrowband systems, but eventually only provides a constant gain on the number of concurrent transmissions that can be scheduled. Thus qualitatively, the scaling of transmission capacity in r is still $\Theta(r^{-2})$. Relating this to transport capacity, the transport capacity (for a fixed bandwidth of 1 Hz and a fixed area of 1 m^2) of an *ad hoc* network is essentially the maximum number of legal transmissions (i.e., n), multiplied by the transmission range r, multiplied by the achieved spectral efficiency of each transmission, b. In the best case, this was shown to scale as $\Theta(\sqrt{n})$ in [4]. In this paper, the number of legal transmissions (i.e., have received SINR above β) is a stochastic measure λ which has been shown in Theorem 1 to be inversely proportional to r^2. Hence, it can be readily seen that our comparable metric for the transport capacity is $\lambda \cdot r \to \Theta(\sqrt{n})$ since $\lambda \propto n$. Thus, transmission capacity recovers the basic scaling result of [4], but also includes a stochastic notion of successful transmissions and allows the computation bounds for nonasymptotic n.

Although up until this point the focus of the paper has been on deriving the optimal contention density λ^ϵ, it is natural to wonder how the transmission capacity or transport capacity might relate to measurable network throughput, for example, the area spectral efficiency or sum data rate over all the nodes. The transmission capacity gives units of b/s/Hz/area, or area spectral efficiency, but in this paper we have assumed for convenience that the data rate of each transmission is fixed at $b = 1$, so the area spectral efficiency is simply the number of successful transmissions per unit area. This is sufficient for SS techniques, since these techniques do not attempt to increase the average data rate of each transmission b, but rather, the number of allowable colocated transmissions. We leave quantitative generalization for variable b to future work, while noting that techniques such as adaptive modulation and multiple-antenna transmission and reception may allow b to be substantially increased for a fixed node density λ and fixed outage probability ϵ. In summary, there is a direct relation between transmission capacity and traditional information-theoretic measures of network capacity, but it is easier to quantify the former.

III. Second Model: Variable Transmission Power and Relay Distance

In our second model, we remove the assumption that all transmitters use the same transmission power and have associated receivers at the same distance. In real *ad hoc* networks, transmission relay distances will be variable as will interference power levels. This suggests transmitters should use power control since if the signal power is too high it may cause unnecessary interference and if it is too low the signal may not be successfully received. Finding a system-wide optimal set of transmission power levels is the subject of recent work [24], [32]–[34]. In this work, we take a simple distributed approach of assuming that transmitters choose their transmission power as a function of their distance from their intended receiver but independently of the interference level at the receiver. We call this *pairwise power control* since each transmitter and receiver pair determine the transmission power independently of other pairs. Specifically, the transmitter chooses its transmission power such that the signal power at the receiver will be some fixed level ρ. Thus, if a transmitter and receiver are separated by a distance d then the transmitter will employ a transmission power ρd^α so that the received signal power is ρ. Note that $\rho > \eta\beta$ is required to keep the received signal power above the noise floor.

Devices are assumed to have a maximum transmission power of $\rho_{\max}$. Solving $\rho d^\alpha \le \rho_{\max}$ for d gives a maximum transmission distance of $d_{\max} = (\frac{\rho_{\max}}{\rho})^{\frac{1}{\alpha}}$. We assume that a transmitter is uniformly likely to choose any of the receivers within $d_{\max}$ as its intended receiver. This corresponds to a transmission distance distribution of

$$F_D(d) = \frac{d^2-1}{d_{\max}^2-1}, \qquad \text{for } 1 \le d \le d_{\max}.$$

Formally, our second model consists of a *marked* homogeneous Poisson point process $\Xi' = \{(X_i, D_i)\}$ where the points $\{X_i\}$ again denote the locations of interfering transmitters and the marks $\{D_i\}$ denote the distance between transmitter i and its intended receiver. The marks are independent and identically distributed random variables width distribution F_D, and are independent of the transmitter locations. We use $|X_i|$ to denote the distance from node i to the origin. Similar to the first model, we evaluate the outage probability using the Palm distribution $\mathbb{P}^0$ for the marked point process Φ, which is a shifted version of Ξ' and places a typical receiver at the origin. Also similar to the first model, we define the sampled subprocess Φ_m as a homogeneous marked Poisson point processes consisting of all interfering transmitters in Φ that are on subchannel m, for $m = 1, \ldots, M$.

To evaluate the interference a typical receiver sees, we define the function $l(r,d)$ as giving the signal power level at a distance r from the transmitter when the transmitter's intended recipient is at a distance d. Thus, $l(r,d) = \rho(\frac{d}{r})^\alpha$. Note in particular that $l(d,d) = \rho$, i.e., at the distance of the intended receiver the signal power is the desired level. Note that the transmission power is $l(1,d) = \rho d^\alpha$.

The appropriate outage constraints on λ are now given by

$$\text{FH} \quad \mathbb{P}^0\left(\frac{\rho}{\eta + \sum_{(X_i,D_i)\in\Phi_m} l(|X_i|, D_i)} \le \beta\right) \le \epsilon \tag{14}$$

$$\text{DS} \quad \mathbb{P}^0\left(\frac{\rho}{M\eta + \sum_{(X_i,D_i)\in\Phi} l(|X_i|, D_i)} \le \frac{\beta}{M}\right) \le \epsilon. \tag{15}$$

We can use these to obtain upper and lower bounds for the second model. These bounds on the transmission capacity of FH-CDMA and DS-CDMA, are given in the following theorem.

Theorem 2: Let $\epsilon \in (0,1), \delta = \frac{1}{\beta} - \frac{\eta}{\rho}$, and let $g(\alpha) = \frac{1}{2}((\alpha+1)(\alpha-1))^{\frac{1}{\alpha}}$. Suppose

$$F_D(d) = \frac{d^2-1}{d_{\max}^2-1}, \qquad \text{for } 1 \le d \le d_{\max}.$$

The lower and upper bounds on the transmission capacity for FH-CDMA when transmitters employ pairwise power control are given by:

$$c_l^{\epsilon,\text{FH}} = (1-\epsilon)\lambda_l^{\epsilon,\text{FH}}, \qquad c_u^{\epsilon,\text{FH}} = (1-\epsilon)\lambda_u^{\epsilon,\text{FH}}$$

where

$$\lambda_l^{\epsilon,\text{FH}} \ge g(\alpha)\frac{M\delta^{\frac{2}{\alpha}}}{\pi}\left(\frac{d_{\max}^2-1}{d_{\max}^{2\alpha+2}-1}\right)^{\frac{1}{\alpha}}\epsilon + \Theta(\epsilon^2)$$

$$\lambda_u^{\epsilon,\text{FH}} = \frac{4}{\pi}\frac{M\delta^{\frac{2}{\alpha}}(d_{\max}^2-1)}{d_{\max}^4}\epsilon + \Theta(\epsilon^2)$$

as $\epsilon \to 0$.

The lower and upper bounds on the transmission capacity for DS-CDMA when transmitters employ pairwise power control are given by

$$c_l^{\epsilon,\text{DS}} = (1-\epsilon)\lambda_l^{\epsilon,\text{DS}}, \qquad c_u^{\epsilon,\text{DS}} = (1-\epsilon)\lambda_u^{\epsilon,\text{DS}}$$

where

$$\lambda_l^{\epsilon,\text{DS}} \ge g(\alpha)\frac{(M\delta)^{\frac{2}{\alpha}}}{\pi}\left(\frac{d_{\max}^2-1}{d_{\max}^{2\alpha+2}-1}\right)^{\frac{1}{\alpha}}\epsilon + \Theta(\epsilon^2)$$

$$\lambda_u^{\epsilon,\text{DS}} = \frac{4}{\pi}\frac{(M\delta)^{\frac{2}{\alpha}}(d_{\max}^2-1)}{d_{\max}^4}\epsilon + \Theta(\epsilon^2)$$

as $\epsilon \to 0$.

Proof of Theorem 2: The Proof of Theorem 2 is similar to that of Theorem 1. The outage constraints (14) and (15) can be written as

$$\text{FH} \quad \mathbb{P}^0\left(\sum_{(X_i,D_i)\in\Phi_m}\left(\frac{|X_i|}{D_i}\right)^{-\alpha} \ge \delta\right) \le \epsilon \tag{16}$$

$$\text{DS} \quad \mathbb{P}^0\left(\sum_{(X_i,D_i)\in\Phi}\left(\frac{|X_i|}{D_i}\right)^{-\alpha} \ge M\delta\right) \le \epsilon. \tag{17}$$

Consider the FH-CDMA case. Let $m \in \{1,\ldots,M\}$ denote a particular subchannel used in FH-CDMA and let us define the following events.

Definition 2

$$E(\lambda) = \left\{\sum_{(X_i,D_i)\in\Phi_m}\left(\frac{D_i}{|X_i|}\right)^\alpha > \delta\right\}$$

$$E_u(\lambda,s) = \left\{\Phi_m \cap (b(O,s) \times [s\delta^{\frac{1}{\alpha}}, d_{\max}) \ne \emptyset\right\}$$

$$E_{l_1}(\lambda,s) = \{\Phi_m \cap (b(O,s) \times \mathbb{R}^+) \ne \emptyset\},$$

$$E_{l_2}(\lambda,s) = \left\{\sum_{(X_i,D_i)\in\Phi_m}\left(\frac{D_i}{|X_i|}\right)^\alpha \mathbf{1}(X_i \in \bar{b}(O,s)) > \delta\right\}$$

$$E_l(\lambda,s) = E_{l_1}(\lambda,s) \cup E_{l_2}(\lambda,s).$$

According to Definition 2, the event $E(\lambda)$ consists of all outage outcomes. The event $E_u(\lambda, s)$ consists of all outcomes where there are one or more transmitters within s of the origin with transmission distances exceeding $s\delta^{\frac{1}{\alpha}}$. This threshold is the smallest transmission distance such that even one transmitter in $b(O,s)$ with such a transmission distance will cause an outage at the origin. The event $E_{l_1}(\lambda, s)$ consists of all outcomes with one or more transmitters in $b(O,s)$; but note that not all outcomes in $E_{l_1}(\lambda, s)$ will cause an outage. Finally, the event $E_{l_2}(\lambda, s)$ consists of all outcomes where the interference power at the origin caused by all the transmitters outside $b(O,s)$ is adequate to cause an outage at the origin. Note that the events in Definition 2 have similar properties as the Properties a)–d) mentioned in the Proof of Theorem 1. In particular, for $s \le d_{\max}\delta^{-\frac{1}{\alpha}}$

$$E_u(\lambda, s) \subset E(\lambda) \subseteq E_l(\lambda, s) = E_{l_1}(\lambda, s) \cup E_{l_2}(\lambda, s)$$

and $E_{l_1}(\lambda, s)$ and $E_{l_2}(\lambda, s)$ are independent events.

The upper bound λ_u^ϵ. To obtain the upper bound, we need to calculate $\mathbb{P}^0(E_u(\lambda, s))$. Since the marks are independent of the point locations, the points with certain marks form a thinned process, where the thinning is proportional to the mark probability, i.e., the intensity of the thinned process is $\frac{\lambda}{M}\bar{F}_D(s\delta^{\frac{1}{\alpha}})$. Thus, the probability of event $E_u(\lambda, s)$ is given by

$$\mathbb{P}^0(E_u(\lambda, s)) = 1 - \exp\left\{-\frac{\lambda}{M}\bar{F}_D\left(s\delta^{\frac{1}{\alpha}}\right)\pi s^2\right\}$$

for $s \in (0, d_{\max}\delta^{-\frac{1}{\alpha}})$. We can derive the parameterized upper bound $\lambda_u^\epsilon(s)$

$$\lambda_u^\epsilon(s) = -\frac{M\ln(1-\epsilon)}{\pi\bar{F}_D(s\delta^{\frac{1}{\alpha}})s^2} = \frac{M\epsilon}{\pi\bar{F}_D(s\delta^{\frac{1}{\alpha}})s^2} + \Theta(\epsilon^2)$$

for all $\epsilon \in (0,1)$.

To get the tightest upper bound, we need to optimize this result over feasible s. Note that $\mathbb{P}^0(E_u(\lambda, s))$ is not monotone increasing in s as in the case of model 1. In particular, $\mathbb{P}^0(E_u(\lambda, s))$ is increasing in s for s small, and decreasing in s as s approaches $d_{\max}\delta^{-\frac{1}{\alpha}}$. Intuitively, for s small we can accept any mark but the circle $b(O,s)$ is small, while for s large the circle $b(O,s)$ is large enough but only the largest marks may be admitted. Thus, $\lambda_u^\epsilon(s)$ is also not monotone in s: $\lambda_u^\epsilon(s)$ is large for s small and s near $d_{\max}\delta^{-\frac{1}{\alpha}}$.

For the assumed distance distribution $F_D(d) = \frac{d^2-1}{d_{\max}^2-1}$ we obtain the optimal s by taking derivative of $\lambda_u^\epsilon(s)$ with respect to s and setting it to 0. This yields the minimizer $s_u^* = \frac{d_{\max}}{\sqrt{2}\delta^{\frac{1}{\alpha}}}$, which give us the tightest upper bound

$$\lambda_u^\epsilon = \frac{4M\delta^{\frac{2}{\alpha}}(d_{\max}^2-1)}{\pi d_{\max}^4}\epsilon + \Theta(\epsilon^2)$$

for all $\epsilon \in (0,1)$.

The lower bound λ_l^ϵ. Just as for the calculation of $\lambda_l^{\epsilon_u}(s)$ in Theorem 1, we have

$$\lambda_{l_1}^{\epsilon_1}(s) = -\frac{M}{\pi}s^{-2}\ln(1-\epsilon_1)$$

for all $s > 0$ and $\epsilon \in (0,1)$.

We also need to bound $\mathbb{P}^0(E_{l_2}(\lambda, s))$ and thus obtain lower bound on $\lambda_{l_2}^{\epsilon_1}(s)$. This is done in a similar way to what we did for the first model. Define the aggregate normalized far-field interference

$$Y'(\lambda, s) = \sum_{i \in \Phi_m \cap \bar{b}(O,s)} \left(\frac{D_i}{|X_i|}\right)^\alpha.$$

The mean and variance of $Y'(\lambda, s)$ are obtained in Lemma 2, see the Appendix. We apply Chebyshev's inequality and obtain the lower bound on $\lambda_{l_2}^{\epsilon_2}(s)$ as follows:

$$\begin{aligned}\lambda_{l_2}^{\epsilon_2}(s) &\ge \frac{(\alpha-1)(\alpha+1)M\delta^2 s^{2(\alpha-1)}(d_{\max}^2-1)}{\pi(d_{\max}^{2\alpha+2}-1)}\epsilon_2 + \Theta(\epsilon_2^2)\\ &= \underline{\lambda}_{l_2}^{\epsilon_2}(s).\end{aligned}$$

To obtain λ_l^ϵ, we again need to optimize

$$\lambda_l^\epsilon = \sup_{s>0,(\epsilon_1,\epsilon_2)}\{\min\{\lambda_{l_1}^{\epsilon_1}(s), \underline{\lambda}_{l_2}^{\epsilon_2}(s)\}\}$$

over s and (ϵ_1, ϵ_2) satisfying $\epsilon_1 + \epsilon_2 - \epsilon_1\epsilon_2 = \epsilon$. As previously noted, in this model, $\mathbb{P}^0(E_{l_1}(\lambda, s))$ is no longer the exact outage probability caused by near-field interference, but only a conservative estimation. Joint optimization over s and (ϵ_1, ϵ_2) is very complicated; for simplicity we instead choose to set $\epsilon_1 = \epsilon_2 = 1 - \sqrt{1-\epsilon} = \frac{\epsilon}{2} + \Theta(\epsilon^2)$ and then only optimize over s to equalize $\lambda_{l_1}^{\epsilon_1}(s)$ and $\lambda_{l_2}^{\epsilon_2}(s)$. Doing this we obtain

$$\lambda_l^\epsilon \ge g(\alpha)\frac{M\delta^{\frac{2}{\alpha}}}{\pi}\left(\frac{d_{\max}^2-1}{d_{\max}^{2\alpha+2}-1}\right)^{\frac{1}{\alpha}}\epsilon + \Theta(\epsilon^2)$$

where $g(\alpha) = \frac{1}{2}((\alpha-1)(\alpha+1))^{\frac{1}{\alpha}}$.

Looking at (16) and (17), it is clear that the exact same analysis for DS-CDMA holds provided we replace λ with $M\lambda$ and δ with $M\delta$, i.e., if

$$\begin{aligned}\lambda_l^{\epsilon,\mathrm{FH}} &\ge g(\alpha)\frac{M\delta^{\frac{2}{\alpha}}}{\pi}\left(\frac{d_{\max}^2-1}{d_{\max}^{2\alpha+2}-1}\right)^{\frac{1}{\alpha}}\epsilon + \Theta(\epsilon^2)\\ \lambda_u^{\epsilon,\mathrm{FH}} &= \frac{4M\delta^{\frac{2}{\alpha}}(d_{\max}^2-1)}{\pi d_{\max}^4}\epsilon + \Theta(\epsilon^2)\end{aligned}$$

holds for FH-CDMA, then

$$\begin{aligned}\lambda_l^{\epsilon,\mathrm{DS}} &\ge g(\alpha)\frac{M^{\frac{2}{\alpha}}\delta^{\frac{2}{\alpha}}}{\pi}\left(\frac{d_{\max}^2-1}{d_{\max}^{2\alpha+2}-1}\right)^{\frac{1}{\alpha}}\epsilon + \Theta(\epsilon^2)\\ \lambda_u^{\epsilon,\mathrm{DS}} &= \frac{4}{\pi}\frac{M^{\frac{2}{\alpha}}\delta^{\frac{2}{\alpha}}(d_{\max}^2-1)}{d_{\max}^4}\epsilon + \Theta(\epsilon^2)\end{aligned}$$

holds for DS-CDMA. □

Theorem 2 shows how the transmission capacity scales in the fundamental system parameters, e.g., transmission distance, spreading factor, and outage constraint. We see that the scaling is the same as Theorem 1 for both DS-CDMA and FH-CDMA. Power control in cellular networks solves the "near–far" problem by equalizing receiving powers at the central base station. The pairwise power control scheme in *ad hoc* networks cannot fully solve the "near–far" problem since transmitters have different intended receivers, but it offers a simple and distributed means by which to mitigate the interference across concurrent transmissions.

TABLE I
SIMULATION PARAMETERS (UNLESS OTHERWISE NOTED)

Symbol	Description	Value
M	Spreading factor	16
r	Transmission radius	10m
ϵ	Target outage probability	0.1
β	Target $SINR$	$3 = 4.77$dB
ρ	Transmit Power	1
α	Path loss exponent	3

TABLE II
TRANSMISSION CAPACITY SCALINGS

	FH-CDMA	DS-CDMA	Narrowband
Spreading factor (M)	M	$M^{\frac{2}{\alpha}}$	1
Target SINR (β)	$\frac{1}{\beta^{\frac{2}{\alpha}}}$	$\frac{1}{\beta^{\frac{2}{\alpha}}}$	$\frac{1}{\beta^{\frac{2}{\alpha}}}$
Outage Constraint (ϵ)	ϵ	ϵ	ϵ
Transmission range (r)	r^{-2}	r^{-2}	r^{-2}

IV. NUMERICAL RESULTS AND INTERPRETATIONS

The derived transmission capacity results are evaluated in this section for some typical parameters in order to show how *ad hoc*network capacity can be expected to scale with path loss and spreading, and to compare frequency hopping and direct sequence SS. Additionally, a simulated *ad hoc* network where nodes are spatially distributed according to a Poisson point process is used to show how the derived bounds perform relative to simulated performance. The simulations are carried out using the parameter values enumerated in Table I. All simulation results shown in the plots are confidence intervals, although the intervals are too small to be visually distinguished from a point. Note that we only show the numerical results for our first model with fixed transmission power and relay distance because the numerical results of the second model is basically the same as the first one. For reference, we also include Table II that shows analytically how transmission capacity scales for FH, DS, and narrowband $(M = 1)$ modulation, and it can be noted that they only differ in terms of their scaling with regards to M.

A. Outage Probability Versus Transmission Density

The first investigation is to study the outage probability $p_o(\lambda) = \mathbb{P}^0(E(\lambda))$ versus the transmission density λ. The outage lower bound $\mathbb{P}^0(E_u(\lambda, s))$ is given by (5). Let

$$Y(\lambda, s) = \sum_{i \in \Pi \cap \bar{b}(O,s)} |X_i|^{-\alpha}$$

denote the normalized far-field interference from transmitters outside the circle $b(O, s)$, as shown in Fig. 1. Let $y = \kappa$ for FH-CDMA and $y = M\kappa$ for DS-CDMA denote the normalized SINR requirement as expressed in (3) and (4), respectively. We obtain an upper bound of the outage probability by applying the same technique used in the proof of the lower bound in Theorem 1

$$\begin{aligned} \mathbb{P}^0(E(\lambda, s)) &= \mathbb{P}^0(E_u(\lambda, s) \cup E_f(\lambda, s)) \\ &\leq \mathbb{P}^0(E_u(\lambda, s)) + \mathbb{P}^0(E_f(\lambda, s)) \\ &= \mathbb{P}^0(E_u(\lambda, s)) + \mathbb{P}^0(Y(\lambda, s) > y) \end{aligned}$$

where $\mathbb{P}^0(E_u(\lambda, s))$ is the same as above and $\mathbb{P}^0(Y(\lambda, s) > y)$ can be upper-bounded using the Chebyshev inequality.

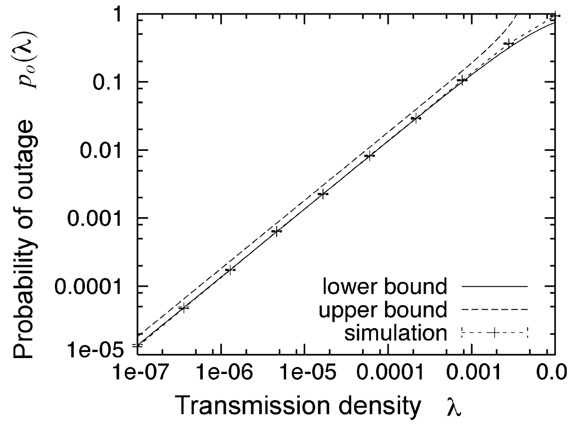

Fig. 2. Numerical and simulation results for the probability of outage $p_o(\lambda)$ versus the transmission density λ. The numerical bounds are the upper and lower bounds on $p_o(\lambda)$. The simulation results (with confidence intervals) are seen to fall between the lower and upper bounds.

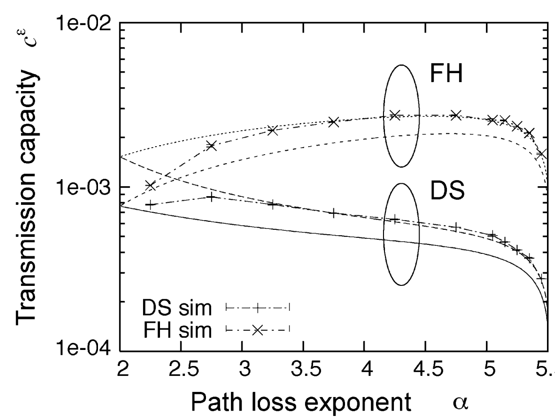

Fig. 3. Numerical and simulation results for the transmission capacity c^{ϵ} versus the path loss exponent α. The upper bound appears relatively tight relative to the simulation results. The decay in transmission capacity as $\alpha \rightarrow 5.5$ is a consequence of the received power approaching the ambient noise floor.

Fig. 2 plots numerical and simulation results of $p_o(\lambda)$ versus λ; the simulated outage probability falls between the lower and upper bounds as predicted. The plot illustrates that the lower bound is reasonably tight with respect to the simulated performance, and that as expected from our analytical expressions in Theorem 1, outage probability increases about linearly in the transmission density in the low outage regime.

B. Transmission Capacity Versus Path Loss Exponent

Fig. 3 shows the transmission capacity $c^{\epsilon} = \lambda^{\epsilon} b(1-\epsilon)$ versus the path loss exponent α for both FH-CDMA and DS-CDMA systems, with $b = 1$. The bounds given in Theorem 1 are plotted along with the simulation results, and as expected, the simulated transmission density falls between the lower and upper bounds. The plot illustrates that the upper bound is fairly tight. Recall that the lower and upper bounds are given up to an asymptotic order $\Theta(\epsilon^2)$. Here these expressions are plotted assuming that this constant is zero. So, the tightness of the upper bound is due partly to the neglect of this small term, and also due to the fact the close-in interfering nodes—which are what determine the upper bound—appear to dominate the transmission capacity.

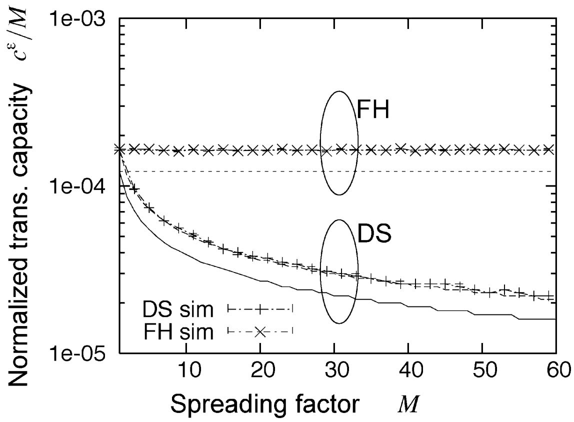

Fig. 4. Numerical and simulation results for the transmission capacity c^ϵ/M versus the spreading factor M. Frequency hopping's advantage over direct sequence is increased as M increases.

As can be seen in this plot, frequency hopping is increasingly superior to direct sequence as the path loss becomes worse. The interpretation for this is that as the path loss worsens, dramatically more power is needed to reach the desired transmitter, so dramatically more interference is caused to neighbors. FH-CDMA systems typically avoid the interference by hopping with an occasional collision, so their performance is improved by this effect since now the aggregate interference from far-away nodes in the utilized frequency slot is decreased. On the other hand, DS-CDMA receivers, which must suppress with interference from other transmitters that are closer to it than the desired transmitter, are at a distinct disadvantage since the desired power decreases more quickly than the interference power of close-in nodes as the path loss exponent increases.

The decay in transmission capacity as $\alpha \rightarrow 5.5$ is a consequence of the SNR (absent any interference) being below the SINR requirement: solving $\frac{\rho r^{-\alpha}}{\eta + 0} = \beta$ for α yields $\alpha = 5.5$ for the parameters given in Table I. This is the value of α such that even absent any interference, the SNR ratio at the receiver is below the SINR requirement β. In other words, the received power is very close to or below the noise floor.

C. Transmission Capacity Versus Spreading Factor

The final investigation studies the transmission capacity c^ϵ versus the spreading factor M for both FH-CDMA and DS-CDMA systems. Fig. 4 plots the numerical and simulation results, with the simulated random network falling between the lower and upper bounds as predicted, with again the upper bound relatively accurately approximating the actual transmission capacity. Note that we plot $\frac{c^\epsilon}{M}$ versus M. The transmission capacity is normalized by M to account for the fact that increasing M requires a commensurate increase in bandwidth. Thus, c^ϵ/M is a rough measure of the spectral efficiency.

The key insight from this plot is that FH-CDMA capacity is unaffected by the spreading gain, whereas DS-CDMA capacity grows steadily worse as more spreading is employed. The interpretation is that the amount of interference that can be suppressed with the spreading factor does not compensate for the fact the bandwidth had to be increased (or the data rate decreased) by a factor of M.

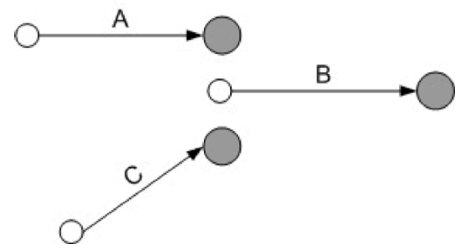

Fig. 5. Illustration of the near–far problem in *ad hoc* networks. Transmission B destroys reception of A and C unless an enormous spreading factor is used, or the interference is avoided altogether by frequency hopping or scheduling.

V. Frequency Hopping Versus Direct Sequence Versus Narrowband

A recurring theme throughout the discussion thus far has been the apparent superiority of FH-CDMA to DS-CDMA, whenever the path loss exponent $\alpha > 2$, as can be directly observed from the bounds in Theorems 1 and 2. In this section, we discuss the significance and meaning behind this result. First, we note that although in some environments (notably indoor or urban canyons) the path loss exponent is sometimes modeled to be less than 2 due to reflections, these environments usually have significant shadowing and fading, bringing the "effective" path loss exponent to much greater than 2 if all these effects were lumped into just the path loss model. So, in general, it is reasonable to assume that the effective $\alpha > 2$, and the common assumption in terrestrial environments of $\alpha = 4$ results in both FH-CDMA and narrowband having a higher normalized transmission capacity than DS-CDMA by a factor of $\sqrt{M}$.

Those experienced with CDMA may recall that FH and DS perform identically or within a small constant of each other assuming perfect power control [17], [35] in a cellular environment, with no dependence on the spreading factor. So why is FH-CDMA better than DS-CDMA by such a wide margin in *ad hoc* networks? The reason is that "perfect" power control is impossible to achieve in an *ad hoc* network due to the random locations of the transmitters and receivers, so the near–far problem is impossible to reconcile with power control. As a result, it is better to *avoid* interference than to attempt to *suppress* interference. As a simple example, consider three transmit–receive pairs shown in Fig. 5. In DS-CDMA, transmission B will continually overwhelm the receivers for transmissions A and C unless an enormous spreading factor is employed. By contrast, in FH-CDMA this situation is only a problem when transmission B is in the same frequency slot as A or C, each of which occurs only with probability $1/M$.

Other readers may have noted that throughout this paper we have assumed a CDMA matched-filter (MF) receiver, which is known to be highly suboptimal in the multiuser CDMA environment, particularly when receive powers are widely varied. We have made the MF assumption since this is still the predominant CDMA receiver used in practice, and also the easiest to analyze, since the interference suppression is simply $1/M$ (or similar, depending on the exact codes used). However, there is no question that interference-aware CDMA receivers will, at least in theory, significantly outperform the MF. An enormous number of such receivers have been proposed over the past

20 years, ranging from the maximum-likelihood detector (best performance, highest complexity) to linear multiuser detectors (lowest complexity, but questionable robustness), as described in [28], [36], [37], and the references therein. The transmission capacity framework can be adapted to analyze the improvement resulting from such receivers, and we leave this as a subject for future work. One such analysis has been undertaken in [38] for successive interference cancellation (SIC), with the conclusion that ideal SIC has large gains over DS and even FH, but more realistic SIC has far more modest gains, and might not exceed the capacity of FH in many cases. We specifically would like to caution readers that idealistic assumptions like perfect interference cancellation/suppression are especially dubious in *ad hoc* networks, since even a small fraction of residual interference from nearby nodes can constitute a very large amount of interference in the absence of centralized power control.

Finally, we would like to close our discussion by acknowledging that this paper has focused on just the physical layer of network design, and assumed a trivial MAC (ALOHA, essentially). In practice, a more sophisticated MAC and various scheduling techniques can be employed, which may change the overall calculus of network utility considerably. Additionally, SS (both DS and FH) is expected to have a number of important advantages over narrowband transmission from the point of view of security, end-to-end delay, and energy efficiency [39]. In future *ad hoc* networks, physical layer capacity is just one of a large number of metrics that designers will need to consider.

VI. CONCLUSION

The overall contribution of this paper is a new framework, coined "transmission capacity," for analyzing the capacity of outage-constrained *ad hoc* networks. A specific contribution is closed-form asymptotic upper and lower bounds on the transmission capacity for a simple network model. Even though the model is simple, these bounds provide new quantitative insights into the dependence of transmission capacity on the fundamental parameters of the network, for example, path loss exponent, spreading factor, outage requirement, and so on. As argued in the Introduction, understanding the impact of outage constraints on transmission density is valuable since outage constraints permit efficient energy utilization and low medium access contention delays. An important insight obtained from the transmission capacity bounds for FH-CDMA versus DS-CDMA is that FH-CDMA offers an increased capacity (for path loss exponent $\alpha > 2$) on the order of $M^{1-\frac{2}{\alpha}}$. The intuition is that the primary contributors to interference are nearby transmitting nodes and FH-CDMA is more efficient in mitigating this hindrance than is DS-CDMA. Put another way, due to the geographical properties of *ad hoc* networks, it is more effective to attempt to avoid interference than to suppress it, since the interference is generally too strong to suppress.

APPENDIX

Lemma 1: Assume the transmitters' locations are modeled by a homogeneous Poisson point process Π with intensity λ and transmissions are using fixed transmission power to a distance d. Let

$$Y(\lambda, s) = \sum_{i \in \Pi \cap \bar{b}(O,s)} |X_i|^{-\alpha}$$

denote the normalized far-field interference from transmitters outside the circle $b(O,s)$. The mean and variance of $Y(\lambda, s)$ are

$$\begin{aligned} \mathbb{E}[Y(\lambda, s)] &= \frac{2\pi}{\alpha - 2} s^{2-\alpha} \lambda = \mu(s)\lambda \\ \mathrm{Var}(Y(\lambda, s)) &= \frac{\pi}{\alpha - 1} s^{2(1-\alpha)} \lambda = \sigma^2(s)\lambda. \end{aligned}$$

Proof of Lemma 1: Recall we assume $\alpha > 2$. We compute the mean and variance using Campbell's theorem [30] as follows:

$$\begin{aligned} \mathbb{E}\left[\sum_{i \in \Pi \cap \bar{b}(O,s)} |X_i|^{-\alpha}\right] &= 2\pi\lambda \int_s^\infty r^{-\alpha} r\,dr \\ &= \frac{2\pi}{\alpha - 2} s^{2-\alpha} \lambda. \\ \mathrm{Var}\left(\sum_{i \in \Pi \cap \bar{b}(O,s)} |X_i|^{-\alpha}\right) &= 2\pi\lambda \int_s^\infty r^{-2\alpha} r\,dr \\ &= \frac{\pi}{\alpha - 1} s^{2(1-\alpha)} \lambda. \end{aligned}$$ □

Lemma 2: In the pairwise power control model let

$$Y'(\lambda, s) = \sum_{i \in \Phi_m \cap \bar{b}(O,s)} \left(\frac{D_i}{|X_i|}\right)^\alpha$$

denote the normalized far field interference. The mean and variance of the random variable $Y'(\lambda, s)$ are

$$\begin{aligned} \mathbb{E}[Y'(\lambda, s)] &= \frac{4\pi(d_{\max}^{\alpha+2} - 1)s^{2-\alpha}}{M(\alpha-2)(\alpha+2)(d_{\max}^2 - 1)} \lambda = \mu(s)\lambda \\ \mathrm{Var}(Y'(\lambda, s)) &= \frac{\pi(d_{\max}^{2\alpha+2} - 1)s^{2(1-\alpha)}}{M(\alpha-1)(\alpha+1)(d_{\max}^2 - 1)} \lambda = \sigma^2(s)\lambda. \end{aligned}$$

Proof of Lemma 2: Recall we assume $\alpha > 2$. We compute the mean and variance using Campbell's theorem as follows:

$$\begin{aligned} \mathbb{E}[Y'(\lambda, s)] &= 2\pi \frac{\lambda}{M} \int_s^\infty r^{-\alpha} r\,dr \int_1^{d_{\max}} x^\alpha dF_D(x) \\ &= 2\pi \frac{\lambda}{M} \int_s^\infty r^{1-\alpha} dr \int_1^{d_{\max}} \frac{2x^{1+\alpha}}{d_{\max}^2 - 1} dx \\ &= 2\pi \frac{\lambda}{M} \frac{s^{2-\alpha}}{\alpha - 2} \frac{2}{d_{\max}^2 - 1} \frac{d_{\max}^{2+\alpha} - 1}{2 + \alpha} \\ &= \frac{4\pi \left(d_{\max}^{\alpha+2} - 1\right) s^{2-\alpha}}{M(\alpha-2)(\alpha+2)\left(d_{\max}^2 - 1\right)} \lambda \\ \mathrm{Var}(Y'(\lambda, s)) &= 2\pi \frac{\lambda}{M} \int_s^\infty r^{-2\alpha} r\,dr \int_1^{d_{\max}} x^{2\alpha} dF_D(x) \\ &= 2\pi \frac{\lambda}{M} \frac{s^{2-2\alpha}}{2(\alpha-1)} \frac{2}{d_{\max}^2 - 1} \frac{d_{\max}^{2+2\alpha} - 1}{2(\alpha+1)} \\ &= \frac{\pi \left(d_{\max}^{2\alpha+2} - 1\right) s^{2(1-\alpha)}}{M(\alpha-1)(\alpha+1)\left(d_{\max}^2 - 1\right)} \lambda. \end{aligned}$$ □

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