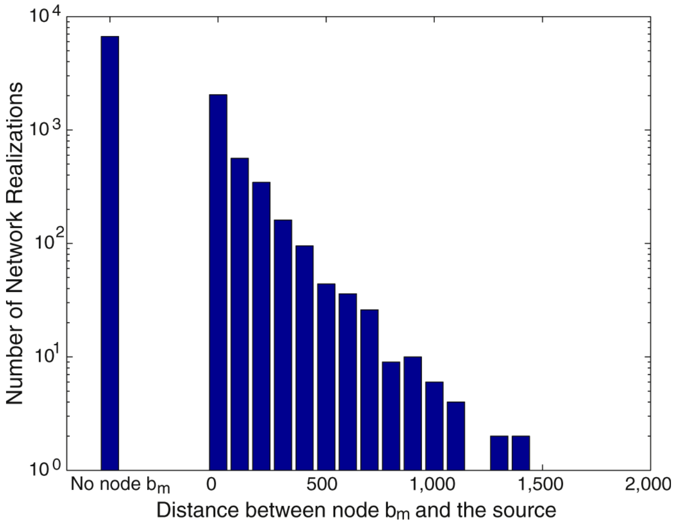

Fig. 5. Histogram of the distance between node b_m and the source node for $\lambda = 0.03$.

of existing solutions. The other centralized algorithm is linear-time and finds an approximation of the optimal solution. Furthermore, we have proposed a simple distributed range assignment algorithm for energy-efficient broadcasting. We have demonstrated that, on average, both the linear-time approximation and the distributed algorithm are almost as efficient as the optimal range assignment for networks with uniformly distributed nodes. The distributed algorithm would be of particular interest not only because of its distributed nature but also for its very low complexity (constant in network size) and the small amount of network knowledge that each node requires to perform the algorithm (only the distances to the adjacent neighbors).

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Opportunistic Feedback and Scheduling to Reduce Packet Delays in Heterogeneous Wireless Systems

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***Abstract*—We consider an opportunistic feedback and scheduling scheme for multiuser wireless systems. In opportunistic feedback schemes, a set of thresholds is assigned to the users, and only users with channel quality above the threshold transmit the feedback on channel state information (CSI) to the base station. We propose an opportunistic feedback scheme that combines techniques from queue/weight-based opportunistic scheduling to improve users' delay performance. A set of queue-based weights designed to slowly track variations in users' backlog is computed; then, each user is assigned a threshold based on the weights. The proposed scheme effectively captures heterogeneity in the offered loads, which is crucial for delay performance. To that end, we formulate a nonconcave maximization problem and propose an approximation algorithm that is numerically shown to have high accuracy. By simulation, we show that our scheme substantially reduces the mean delay, compared with conventional schemes.**

***Index Terms*—Channel state information (CSI) feedback, delay, multiuser diversity (MUD), opportunistic scheduling (OS), user heterogeneity.**

I. Introduction

Multiuser diversity (MUD) can lead to capacity gains in wireless networks by exploiting the time-varying nature of users' channels and opportunistically scheduling those with the highest capacity. Such channel-aware opportunistic scheduling (OS) for downlink transmission requires timely feedback of channel state information (CSI) from the users to the base station (BS). However, the resources available for CSI feedback are scarce; thus, a considerable amount of research has been carried out toward achieving efficient utilization of feed-

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back resources while minimally compromising the gain achievable from MUD.

Several opportunistic feedback (OF) schemes have been proposed (see, e.g., [1]–[3]). The idea is to have users *contend* for a shared resource to transmit their feedback. To reduce the likelihood of collisions, thresholds are assigned to each user, and only users having channel quality above their respective threshold transmit feedback. Such channel-aware feedback schemes attempt to reduce feedback overhead while enabling the BS to schedule users with the "best" channel quality. In the OF scheme proposed in [1], multiple *minislots* are available for feedback. Users contend for minislots using a common threshold that is chosen to maximize the sum throughput of the system. A key issue for such OF schemes is the management of *heterogeneity*, particularly when the channel distributions are "unequal" among the users. To deal with a similar problem in the context of OS, several schemes based on the cumulative distribution function (cdf) of the rates have been proposed, e.g., [4]. We refer to such schemes as *maximum quantile* (MQ) scheduling, where the quantile is defined as the cdf evaluated at the current rate. The idea behind scheduling the user with the highest quantile is to serve a user whose channel quality is the highest relative to its *own* distribution. MQ scheduling has desirable properties, e.g., it is temporally fair and maximizes throughput asymptotically in the number of users [5]. The *static splitting* scheme [2] is based on quantiles. Users are divided into k equal-sized groups, where k is the number of minislots. In each group, the users are assigned a single threshold based on *quantile* and contend for a minislot. The BS selects the user with the highest quantile among those identified in each group. In the scheme of [3], thresholds change over minislots, and users with higher quantiles are likely to transmit feedback earlier in the contention phase. A feedback protocol [6] is proposed based on the proportional fair (PF) scheduling metric, where users are assigned multiple thresholds and access probabilities, depending on their channel distributions.

In this paper, we consider OF and OS schemes to achieve low packet delays. Delay performance critically depends on heterogeneity in the *offered loads*, i.e., not only heterogeneous channel distributions but different traffic arrival rates among the users as well. To track offered loads, one should take *queue length* into account. A queue-based feedback strategy that achieves the maximum throughput under partial CSI was studied in [7]. However, their feedback model is not based on random access like ours: instead, the BS receives feedback only from certain subsets of the users.

In this paper, we propose an OF scheme integrating the principles of queue-based OS. We first compute a set of weights chosen to slowly track variations in users' queue lengths. Next, the thresholds to be assigned to the users are determined based on the weights, so that the BS schedules users to maximize the average *weighted sum of quantiles*. This is motivated by queue-based schedulers [8], [9] designed to achieve low delays, where they essentially maximize the average *weighted sum of normalized rates*. The idea behind such objective is that rates capture opportunism from MUD and weights control opportunism achieved by the users, so that users with longer queue lengths achieve higher degrees of opportunism. Rates are normalized to prevent rate starvation of the users with poor signal-to-noise ratio (SNR) distributions. The quantile is essentially a *nonlinear* normalization of the rate in a way that is *neutral* to the distribution of rates. Thus, opportunism achieved by users is controlled in a fair manner, which enables us to both exploit opportunism and achieve fairness. Moreover, since the quantile is a unified measure of channel quality, thresholds can be defined in terms of quantiles. Hence, our scheme provides a unified framework for computing thresholds. If raw rates were used, instead of quantiles, the computation would depend on the rate distribution of *all* the users and would be extremely hard under user heterogeneity. We will focus on threshold assignments and scheduling for maximizing the average sum of weighted quantiles (ASWQ). To our knowledge, this work is the first attempt to combine and jointly optimize OF and queue-based OS.

Our problem is shown to correspond to maximizing a nonconcave polynomial function, i.e., a nonconcave polynomial program (PP) for which there is no known polynomial-time solution algorithm in general. We propose an approximation algorithm that has a complexity that is *linear* in the network size as follows: We first reveal key properties of the optimal thresholds and then propose to use a set of approximate thresholds that possess those properties and are optimized to maximize the ASWQ. Furthermore, the algorithm deals with the important problem of optimally scheduling a user when the BS *cannot* recover feedback due to undesirable events such as collisions or when none of the users transmits feedback. Through such a joint optimization of threshold assignments and scheduling, the algorithm is numerically shown to achieve an ASWQ that is nearly optimal. Our simulations show that the proposed scheme can substantially reduce the mean delay, compared with other existing schemes. This paper is organized as follows: Section II introduces the system and feedback models. The proposed feedback/scheduling algorithm is discussed in Section III. Section IV presents numerical and simulation results. Section V concludes the paper. The proofs for all the theorems and lemmas in this paper are relegated to Appendices.

II. System Model and Opportunistic Feedback

Consider a time-slotted system with n users sharing a time-varying channel served in the downlink by a BS. Denote the set of user indices by $J = \{1, 2, \ldots, n\}$. For user i, the rate at time slot $t \in \mathbb{Z}$ is modeled by a stationary ergodic random process $R_i(t)$, which has the marginal distribution of a random variable (RV) R_i. We assume that R_i's are independent but not necessarily identical across users. Let $F_i(\cdot)$ denote the cdf of R_i. Define the quantile of user i by $X_i := F_i(R_i)$, and let $\boldsymbol{X} = (X_1, \ldots, X_n)$. Thus, it can be shown that $\boldsymbol{X}$ consists of n i.i.d. uniform RVs on [0,1]. We assume that $F_i(\cdot)$ is known to each user. Let $\mathcal{I}(\phi, \boldsymbol{X})$ be an RV representing the index of the scheduled user under a policy ϕ when the quantiles are $\boldsymbol{X}$ in some timeslot. (The policy will be defined in the sequel.) For an arbitrary nonnegative weight vector $\boldsymbol{w} = (w_1, \ldots, w_n)$, the ASWQ denoted by $\theta(\boldsymbol{w}; \phi)$ is defined as follows:

$$\begin{aligned}\theta(\boldsymbol{w}; \phi) &:= \sum_{i \in J} w_i \mathbb{E}\left[X_i \cdot 1\left(\mathcal{I}(\phi, \boldsymbol{X}) = i\right)\right] \\ &= \sum_{i \in J} w_i \mathbb{E}\left[F_i(\Gamma_i) \cdot 1\left(\mathcal{I}(\phi, \boldsymbol{X}) = i\right)\right]. \qquad (1)\end{aligned}$$

In the uplink, we assume that there is a single minislot in which the users contend to transmit channel feedback. Each user is assigned a threshold, and only users whose quantiles exceed the threshold transmit feedback. If only one user transmits feedback, the BS successfully receives the feedback and schedules the user in that slot. If two or more users transmit feedback, a collision occurs. If no user transmits feedback, the uplink slot for feedback will be idle. We call this event an idle feedback slot (IFS). When a collision or IFS occurs, the BS cannot recover CSI from any user. We assume however that the BS can distinguish between the events of a collision and IFS. In the case of a collision or IFS, the BS selects a user, *polls* the user to receive feedback, and schedules the user in that slot. This model is similar to those of [1], [2], and [6]. We denote the vector of thresholds assigned to the users by $\boldsymbol{x} = (x_1, \ldots, x_n)$, where $\boldsymbol{x} \in [0, 1]^n$. Let $\alpha \in J$ ($\beta \in J$) be the index of the user selected in the case of an IFS (collision). A *policy* denoted by ϕ is defined as $\phi := (\boldsymbol{x}, \alpha, \beta)$. We denote the set of all possible policies by Φ. The idea is to choose a policy $\phi \in \Phi$ that maximizes (1).

III. Proposed Feedback and Scheduling Scheme

We first discuss the choice of weights as a function of queue lengths. Recently, weights that are *logarithmic* in queue lengths have been proposed in the context of queue-and-channel aware scheduling and are shown to be effective in reducing mean delay [9]. Specifically, at time slot t, the BS sets weights $w_i(t)$, $i \in J$, according to $w_i(t) = \log(1 + q_i(t))$, where $q_i(t)$ is the number of backlogged bits or the queue length of user i for downlink transmission. Such logarithmic weights slowly vary with temporary fluctuations in $q_i(t)$. Thus, graceful degradation of delay performance is possible under temporary fluctuations in offered load while maintaining the gain from MUD [9]. In our scheme, the BS sets $w_i = w_i(t)$ at each time slot, and with newly obtained $\boldsymbol{w} = (w_1, \ldots, w_n)$, the BS maximizes $\theta(\boldsymbol{w}; \phi)$ and finds optimal thresholds. New thresholds are broadcast to the users prior to their feedback transmissions. The motivation behind maximizing (1) is discussed in [10]. We show that, in case of full CSI, such a policy not only guarantees certain rates for the user with the longest queue but also serves the longest queue with highest possible rate and thus maximizes the throughput asymptotically in the number of users in the system.

Since $\boldsymbol{w}$ changes over time, threshold broadcast may frequently occur, which can be an overhead in practice. We propose the following method to reduce such overhead. Let $\tilde{\phi}$ denote the policy (i.e., thresholds) which has been most recently updated to the users. At every time slot, the BS computes the optimal policy ϕ^*, given the current weight $\boldsymbol{w}$. Users get the policy update from $\tilde{\phi}$ to ϕ^* only if $\tilde{\phi}$ significantly deviates from ϕ^* in terms of the achievable ASWQ. Specifically, denote the ASWQ achieved by $\tilde{\phi}$ and ϕ^* given the current weight $\boldsymbol{w}$ by $\tilde{\theta}$ and θ^*, respectively. Define the error $\epsilon = |\tilde{\theta} - \theta^*|/\theta^*$ and parameter δ. We propose that the update occurs only if $\epsilon > \delta$. Later, we show that, by changing δ, one can achieve a smooth tradeoff between performance and the broadcast overhead.

This is because $w_i(t)$ is logarithmic in $q_i(t)$ and thus slowly changes with $q_i(t)$, which leads to a slow change in the optimal ASWQ over time. Thus, users see a graceful degradation of performance with decreasing frequency of threshold broadcast.

Next, we formulate the problem of determining thresholds that maximize ASWQ for the given $\boldsymbol{w}$, i.e.,

$$\text{Problem 1 } (\mathbf{P1}) \qquad \begin{array}{ll} \text{Maximize} & \theta(\boldsymbol{w}; \phi) \\ \text{Subject to} & \phi \in \Phi. \end{array} \tag{2}$$

Without loss of generality, we assume that $w_1 \geq w_2 \geq \cdots \geq w_n$. We rewrite $\theta(\boldsymbol{w}; \phi)$ as follows:

$$\theta(\boldsymbol{w}; \phi) = \left\{ \sum_{i \in J} w_i r_S^i(\phi) \right\} + w_\alpha r_I(\phi) + w_\beta r_C(\phi) \tag{3}$$

where $r_S^i(\phi)$ is defined as the average quantile achieved by user i when only user i transmits feedback, and $r_I(\phi)$ ($r_C(\phi)$) is defined as the average quantile achieved by user α (user β) in case of IFS (collision). Since X_i is a uniform RV on [0,1] $\forall i \in J$, given threshold x_i, the probability of feedback transmission is $1 - x_i$. Thus

$$\begin{aligned} r_S^i(\phi) &= \mathbb{E}\left[X_i \cdot 1\left(X_i \in [x_i, 1],\ X_j \in [0, x_j)\ \forall j \in J \setminus \{i\}\right)\right] \\ &= \mathbb{E}\left[X_i \cdot 1\left(X_i \in [x_i, 1]\right)\right] \cdot \mathbb{P}\left(X_j \in [0, x_j)\ \forall j \in J \setminus \{i\}\right) \\ &= \int_{x_i}^{1} s\, ds \cdot \prod_{j \in J, j \neq i} x_j = \frac{(1 - x_i^2)}{2} \prod_{j \in J, j \neq i} x_j. \end{aligned}$$

User α is served when no one transmits feedback; thus, $r_I(\phi) = \mathbb{E}[X_\alpha \cdot 1(X_i \in [0, x_i)\ \forall i \in J)] = (1/2x_\alpha^2) \cdot (\prod_{i \in J, i \neq \alpha} x_i)$. A collision occurs either when user β and one or more other users transmit feedback or when user β does not transmit feedback but at least two other users transmit feedback. Let C_m^β denote the event where at least m users from $J \setminus \{\beta\}$ transmit feedback. In the following, we give an expression for $r_C(\phi)$ in terms of $\mathbb{P}(C_1^\beta)$ and $\mathbb{P}(C_2^\beta)$, as well as that for $\theta(\boldsymbol{w}; \phi)$ accordingly from (3)

$$\begin{aligned} r_C(\phi) &= \mathbb{E}\left[X_\beta \cdot 1\left(X_\beta \in [x_\beta, 1]\right)\right] \cdot \mathbb{P}\left(C_1^\beta\right) \\ &\quad + \mathbb{E}\left[X_\beta \cdot 1\left(X_\beta \in [0, x_\beta)\right)\right] \cdot \mathbb{P}\left(C_2^\beta\right) \\ &= \frac{\left(1 - x_\beta^2\right)}{2} \mathbb{P}\left(C_1^\beta\right) + \frac{x_\beta^2}{2} \mathbb{P}\left(C_2^\beta\right) \\ \theta(\boldsymbol{w}; \phi) &= \sum_{i \in J} \left\{ w_i \frac{(1 - x_i^2)}{2} \prod_{j \in J, j \neq i} x_j \right\} + \frac{w_\alpha x_\alpha^2}{2} \left[\prod_{i \in J, i \neq \alpha} x_i \right] \\ &\quad + \frac{w_\beta \left(1 - x_\beta^2\right)}{2} \left[1 - \prod_{i \in J, i \neq \beta} x_i \right] + \frac{w_\beta x_\beta^2}{2} \\ &\quad \times \left[1 - \prod_{i \in J, i \neq \beta} x_i - \sum_{i \in J, i \neq \beta} \left\{ (1 - x_i) \prod_{j \in J, j \neq i, \beta} x_j \right\} \right] \end{aligned} \tag{4}$$

where it is easily seen that $\mathbb{P}(C_1^\beta)$ and $\mathbb{P}(C_2^\beta)$ are the last two bracketed terms of (4), respectively. Thus, $\mathbf{P1}$ is the problem of maximizing (4) subject to $\boldsymbol{x} \in [0, 1]^n$ and $\alpha, \beta \in J$. Since (4) is a nonconcave polynomial in $\boldsymbol{x}$, $\mathbf{P1}$ is a nonconcave PP. Let $\hat{\phi} = (\hat{\boldsymbol{x}}, \hat{\alpha}, \hat{\beta})$ denote the optimal policy. The globally optimal solution can be found in closed form when $n = 2$ as follows: we omit its proof for brevity.

Lemma 1: The optimal policy $\hat{\phi}$ when $n = 2$ is given by $\hat{x}_1 = (2w_2)/(3w_1)$, $\hat{x}_2 = 1/3$, and $\hat{\alpha} = \hat{\beta} = 1$.

We henceforth assume that $n \geq 3$. Recently, there has been much interest in approximately solving PP problems, notably using sum-of-squares (SOS) methods, which solve a family of semidefinite programming relaxations of the problem [11]. Such methods often have computational complexity, which is a high-order polynomial in n [11]. We present an approximation algorithm with complexity $O(n)$ as follows:

Consider the problem of finding an optimal set of thresholds that maximize the ASWQ with some fixed α and β. We temporarily assume that $\alpha \neq \beta$

$$\text{Problem 2 } (\mathbf{P2}) \qquad \begin{array}{ll} \text{Maximize} & \theta\left(\boldsymbol{w}; (\boldsymbol{x}, \alpha, \beta)\right) \\ \text{Subject to} & \boldsymbol{x} \in [0, 1]^n. \end{array}$$

We begin by characterizing the solution to $\mathbf{P2}$, showing that the optimal thresholds must be nonzero and possess a certain monotonic property, which, in turn, motivates the proposed approximation.

Lemma 2: Supposing that $\tilde{\boldsymbol{x}}$ is a solution to $\mathbf{P2}$, then $\tilde{x}_i \neq 0$ holds for all $i \in J$.

Theorem 1: Suppose that $\tilde{\boldsymbol{x}}$ is a solution to $\mathbf{P2}$. For some $i, j \in J \setminus \{\alpha, \beta\}$ such that $i \leq j$, implying that $w_i \geq w_j$, we have that $\tilde{x}_i \leq \tilde{x}_j$, i.e., the optimal thresholds are nonincreasing in the weights.

Theorem 1 is intuitive since, for $i \in J \setminus \{\alpha, \beta\}$, we have that $r_S^i(\phi) = 1/2(1 - x_i^2) \prod_{j \neq i} x_j$, which has the same form for all $i \in J \setminus \{\alpha, \beta\}$, whereas the contribution by $r_S^i(\phi)$ to (3) is proportional to w_i. Thus, to maximize (3), the user with larger w_i should have larger $r_S^i(\phi)$. Note that $r_S^i(\phi)$ is decreasing in x_i with the other variables fixed. Hence, for user i with larger w_i, x_i should be smaller, i.e., user i

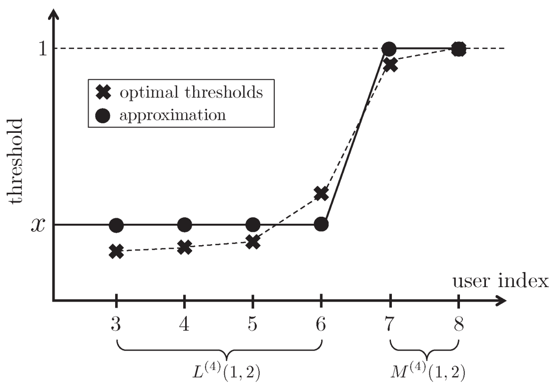

Fig. 1. Illustrated example of the optimal thresholds for $\mathbf{P2}$ and those from the proposed approximation. The parameters are $n = 8$, $\alpha = 1$, $\beta = 2$, and $k = 4$, from which we have that $L^{(4)}(1, 2) = \{3, 4, 5, 6\}$ and $M^{(4)}(1, 2) = \{7, 8\}$. Note that $L^{(4)}(1, 2)$ contains the indices of the $k = 4$ largest weights from $J \setminus \{1, 2\}$. The proposed scheme assigns the threshold x (1) to the users in $L^{(4)}(1, 2)$ ($M^{(4)}(1, 2)$). Both the optimal and approximated thresholds are shown to be nondecreasing in user index i, i.e., nonincreasing in w_i, i.e., both of them satisfy Theorem 1.

should have a *higher* probability of feedback transmission. This is the rationale for our approximation described next.

We introduce two variables $x \in [0, 1]$ and $k \in K := \{1, \ldots, n - 2\}$. We select k users that are associated with the k largest weights from $J \setminus \{\alpha, \beta\}$, and we denote the set of indices of those users by $L^{(k)}(\alpha, \beta)$. The set denoted by $M^{(k)}(\alpha, \beta)$ comprises the rest of the users from $J \setminus \{\alpha, \beta\}$. We assign threshold x to the users in $L^{(k)}(\alpha, \beta)$ and 1 to the users in $M^{(k)}(\alpha, \beta)$. (Hence, the users in $M^{(k)}(\alpha, \beta)$ will *not* transmit feedback with probability 1.) Fig. 1 shows the idea. We let $\alpha = 1$ and $\beta = 2$, and a set of the optimal thresholds $\{\tilde{x}_i\}$, $i \in J \setminus \{1, 2\}$ is shown as an example. The approximated thresholds are shown: note that they also satisfy Theorem 1, i.e., they are nonincreasing in w_i, and make a transition from x to 1 like a "step" function. From numerical experiments, we found that $\{\tilde{x}_i\}$ tend to make a transition to 1, which indeed resembles a "jump;" thus, we observe that the approximation closely mimics the optimal structure of the problem. Both x and k will be optimized for the maximum ASWQ: this, combined with the optimized selection of α and β as proposed in the sequel, turns out to yield an accurate approximation.

The maximization of the ASWQ under the proposed approximation is formulated as follows, where we will first consider the optimization with a fixed k: We introduce further constraints to $\mathbf{P2}$ as follows: $x_i = x \ \forall i \in L^{(k)}(\alpha, \beta)$ and $x_j = 1 \ \forall j \in M^{(k)}(\alpha, \beta)$. In addition, under the current assumptions such that $n \geq 3$ and $\alpha \neq \beta$, we have that the optimal x_α is 1 (see Case (3) in the proof of Lemma 2); thus, we may set $x_\alpha = 1$ in $\mathbf{P2}$ without affecting its solution. By adding those constraints to $\mathbf{P2}$, we obtain $\mathbf{P3}$, i.e.,

Problem 3 ($\mathbf{P3}$)

$$\begin{aligned}\text{Maximize} \quad & w_\beta \frac{(1 - x_\beta^2)}{2} x^k \\ & + \left\{ \sum_{i \in L^{(k)}(\alpha,\beta)} w_i \right\} \frac{(1 - x^2)}{2} x^{k-1} x_\beta \\ & + \frac{w_\alpha}{2} x^k x_\beta + \frac{w_\beta \left(1 - x_\beta^2\right)(1 - x^k)}{2} \\ & + \frac{w_\beta x_\beta^2}{2} \left[1 - k x^{k-1} + (k-1) x^k\right] \qquad (5)\end{aligned}$$

$$\text{Subject to} \quad 0 \leq x \leq 1, \quad 0 \leq x_\beta \leq 1. \qquad (6)$$

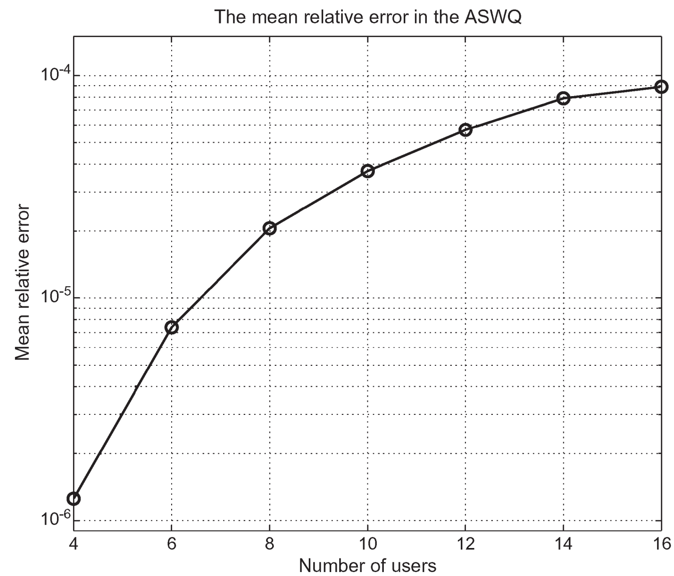

Fig. 2. Mean relative error $\mathbb{E}[\mathcal{E}]$ versus n. The relative error $\mathcal{E}$ is defined by $\mathcal{E} := 1 - (\Theta/\Theta^*)$, where Θ is the sum-of-weighted-quantiles from the proposed scheme, and Θ^* is that from the optimal solution found by the SOS method proposed in [11] computed for each instance of the randomly generated weight vectors, which are uniformly distributed on $[0, 1]^n$. When $n = 3$, the proposed scheme yields the exact solution; thus, the case is not shown here. For each n, 10^4 random vectors of weights were generated.

Note that (5) is not a concave function, yet we show that $\mathbf{P3}$ has $O(1)$ complexity, irrespective of k and n.

Lemma 3: The complexity of $\mathbf{P3}$ is $O(1)$.

Next, we discuss the optimal choices for α and β. Let us denote the value of $\mathbf{P2}$ and $\mathbf{P3}$ by $g(\alpha, \beta)$ and $h^{(k)}(\alpha, \beta)$, respectively. Note that $\mathbf{P1}$ is equivalent to maximizing $g(\alpha, \beta)$ over $\alpha, \beta \in J$. Since $\mathbf{P3}$ is obtained by adding extra constraints to $\mathbf{P2}$ for a fixed k, we will maximize $h^{(k)}(\alpha, \beta)$ over $\alpha, \beta \in J$ and $k \in K$, which is essentially solving $\mathbf{P1}$ with extra constraints. Consider maximizing $h^{(k)}(\alpha, \beta)$ over $\alpha, \beta \in J$ for a fixed k. We show that it is sufficient to consider $O(1)$ out of n^2 possible choices for (α, β) pairs.

Theorem 2: In maximizing $h^{(k)}(\alpha, \beta)$ over $\alpha, \beta \in J$, it is sufficient to consider only the following (α, β) pairs: (1,2), (2,1), $(1, k+2)$, $(k+2, 1)$, $(k+1, k+2)$, $(k+2, k+1)$, (1,1), and $(k+1, k+1)$.

In summary, our algorithm first fixes $k \in K$ and then solves $\mathbf{P3}$ for $O(1)$ number of times over the choices of α and β given by Theorem 2. From Lemma 3, for each α, β, and k, the complexity of $\mathbf{P3}$ is $O(1)$. This process is repeated for all $k \in K$; thus, the overall complexity is $O(n)$. So far, we have assumed $\alpha \neq \beta$. In the case of $\alpha = \beta$, we can use very similar arguments to those previously made to formulate $\mathbf{P3}'$ analogous to $\mathbf{P3}$, which also has $O(1)$ complexity. The overall procedure of the proposed scheme is provided in Table I.

IV. Numerical and Simulation Results

Next, we explore the accuracy of the proposed algorithm. The optimal solution to $\mathbf{P1}$ is found using the SOS method [11] and is compared with our proposed solution. Fig. 2 shows the error in the ASWQ of the proposed scheme relative to the optimal one averaged over problems with randomly generated weight vectors for each n. Over the simulated range of n, the error is observed to be below 0.01%.

Next, we demonstrate the delay performance of the proposed algorithm. In the downlink, we use the CDMA-HDR-like model: each time slot has the duration $\Delta = 1.67$ ms, the bandwidth $W = 40$ kHz, and the number of bits that can be served for user i is at most $\Delta W \log_2(1 + \Gamma_i(t))$, where $\Gamma_i(t)$ denotes user i's SNR at time t.

TABLE I
PROPOSED ALGORITHM AND FEEDBACK SCHEME

/* The overall procedure is repeated at every time slot */
Given: Queue lengths $\{q_i\}_{i \in J}$ at the BS, $\tilde{\phi} \in \Phi$, δ
/* $\tilde{\phi}$ is the policy currently updated to the users */
Variables: $\phi = (\boldsymbol{x}, \alpha, \beta) \in \Phi$
Initialize $x_i \leftarrow 1,\ \forall i \in J,\ \alpha \leftarrow 1,\ \beta \leftarrow 1$
Set the weights $w_i = \log(1+q_i)$, and $\boldsymbol{w} \leftarrow (w_1, \ldots, w_n)$
/* Assume $w_1 \geq \ldots \geq w_n$ */
$\nu \leftarrow$(the number of nonzero w_i's, or equivalently q_i's)
if $\nu = 1$ **then** /* Only one queue is nonempty */
    $x_1 \leftarrow 0$ /* Let the user transmit w.p. 1 */
else if $\nu = 2$ **then** /* Two queues are nonempty, use Lemma 1 */
    $x_1 \leftarrow (2w_2)(3w_1)^{-1}$, $x_2 \leftarrow \frac{1}{3}$, $\alpha \leftarrow 1$, $\beta \leftarrow 1$
else /* Three or more queues are nonempty */
    for each $k \in K$ **do**
        for each choice of (α, β) from Theorem 2 **do**
            if $\alpha \neq \beta$ **then**
                Solve **P3** to obtain a solution ψ
            else /* if $\alpha = \beta$ */
                Solve **P3'** to obtain a solution ψ
            end if
            /* Update ϕ if ψ is the better policy */
            if $\theta(\boldsymbol{w}; \psi) > \theta(\boldsymbol{w}; \phi)$ **then**
                $(\boldsymbol{x}, \alpha, \beta) \leftarrow \psi$
            end if
        end for
    end for
end if
$\epsilon \leftarrow \frac{|\theta(\boldsymbol{w};\phi) - \theta(\boldsymbol{w};\tilde{\phi})|}{\theta(\boldsymbol{w};\phi)}$ /* Compute the error in ASWQ */
if $\epsilon > \delta$ **then** /* Broadcast thresholds only if $\epsilon > \delta$ */
    $\tilde{\phi} \leftarrow \phi$ /* Update the current policy to the new one */
    (Action: the BS broadcasts the thresholds $\boldsymbol{x}$ to the users)
else
    $\phi \leftarrow \tilde{\phi}$ /* Adhere to the current policy */
end if
(Action: the users transmit feedback to the BS)
if a user is identified **then**
    Schedule the user
else /* A collision or IFS occurred */
    if a collision occurred **then**
        Poll and schedule User β
    else /* IFS occurred */
        Poll and schedule User α
    end if
end if

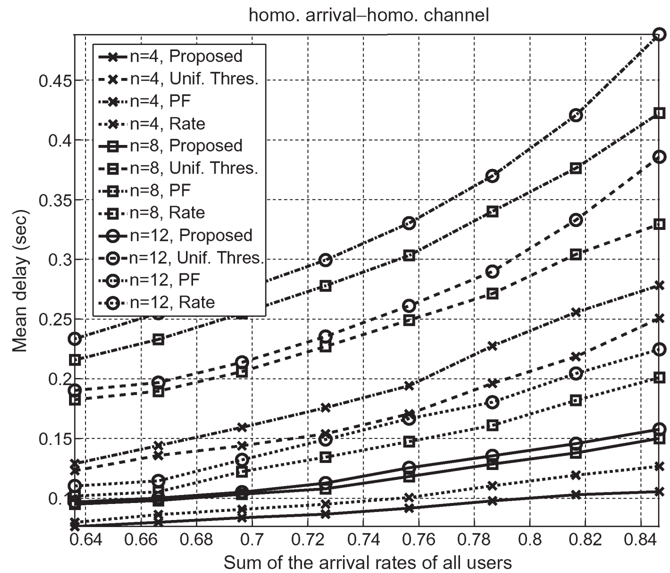

Fig. 3. Mean delay for the case of homogeneous arrival rates and homogeneous channel distributions. The horizontal axis represents the sum of arrival rates of all users, i.e., $(\lambda_1 + \lambda_2) \cdot n/2$, where we proportionally increase λ_1 and λ_2 along the axis. "Unif. Thres." represents the baseline scheme with the optimal single threshold. "PF" represents the scheme in [6]. "Rate" represents the scheme using the objective function obtained from replacing quantiles with raw rates in (1).

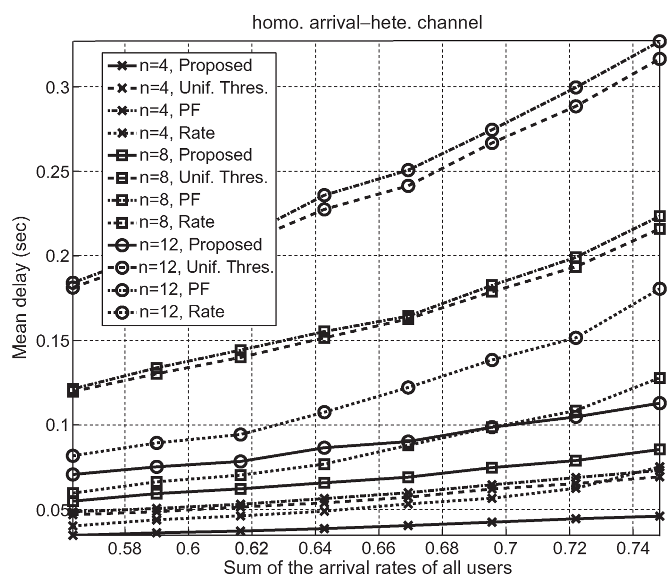

Fig. 4. Mean delay for the case of homogeneous arrival rates and heterogeneous channel distributions under Nakagami-m fading. The horizontal axis represents the sum of arrival rates of all users, i.e., $(\lambda_1 + \lambda_2) \cdot n/2$, where we proportionally increase λ_1 and λ_2 along the axis. "Unif. Thres." represents the baseline scheme with the optimal single threshold. "PF" represents the scheme in [6]. "Rate" represents the scheme using the objective function obtained from replacing quantiles with raw rates in (1).

We assume Rayleigh fading channels, which are independent across users. A packet of size 1 kb randomly arrives at each slot per user. Our baseline feedback scheme is an OF scheme, which assigns a single threshold τ in terms of quantiles to all users similar to [1] and [2]. However, in our simulation, the optimal τ is *numerically* found for each simulated set of arrival rates and channel distributions. We also compare performance with the scheme based on the PF scheduling metric in [6]. These schemes select a user at random in the case of collision or IFS. While these schemes are oblivious of queue lengths, we will make additional comparison with a queue-aware scheme, which has the objective function obtained from replacing the quantile $F_i(R_i)$ with the raw rate R_i in (1). We consider three cases, and in each case, the users are divided into equal-sized groups, i.e., Groups 1 and 2. For the users in Group i, the mean parameters for the channels are denoted by μ_i, and the probability of a packet arrival per time slot is denoted by λ_i, for $i = 1, 2$. The cases are categorized by the heterogeneity in the arrival and channel statistics. In Case 1, $\mu_1 = \mu_2$, and $\lambda_1 = \lambda_2$. In Case 2, $\mu_1 = 4\mu_2$, and $\lambda_1 = \lambda_2$. We assume Nakagami-m fading for Case 2, where $m = 2$, which represents a milder fading condition than Rayleigh channels. In Case 3, $\mu_1 = 4\mu_2$, and $3\lambda_1 = \lambda_2$.

We adjusted λ_i and μ_i to yield mean delays ranging from 100 to 300 ms for the proposed scheme. Fig. 3 shows that, in Case 1, where the system is symmetric, our scheme reduces the mean delay by 5%–69% relative to other schemes. In Case 2, our scheme reduces the mean delay by 14%–66% relative to others, as shown in Fig. 4. In Case 3, we compare 200-ms *throughputs* defined as the total arrival

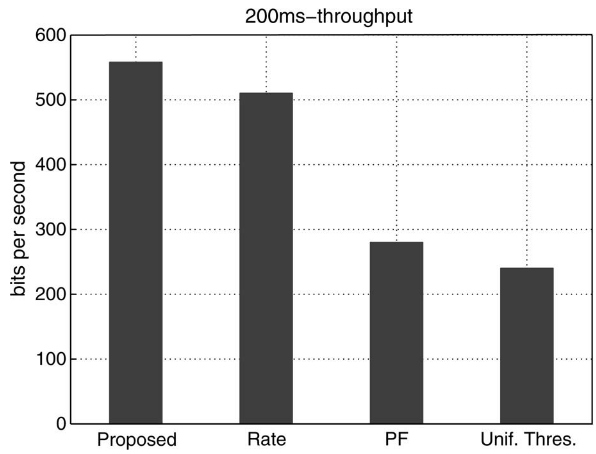

Fig. 5. Comparison of 200-ms throughputs for the case of heterogeneous arrival rates and heterogeneous channel distributions where $n = 8$. The vertical axis represents the sum of arrival rates of all users (in terms of bits per second), for which each scheme achieves the mean delay of 200 ms.

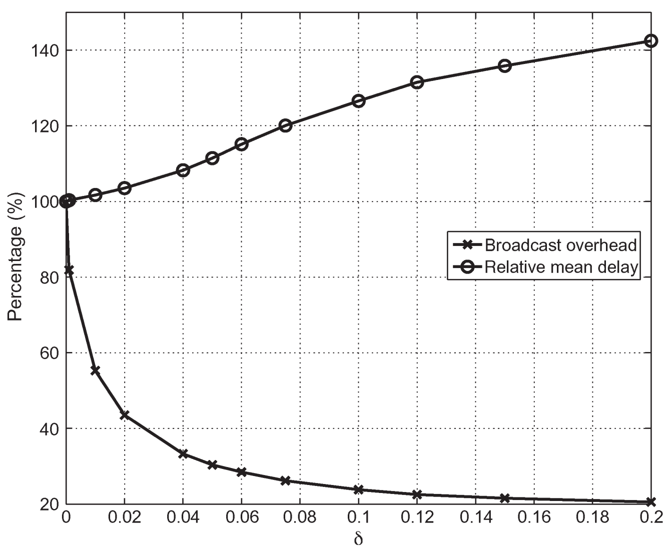

Fig. 6. Tradeoff between the delay performance and the overhead associated with threshold broadcast. One of the curves shows the mean delay relative to that in the case where $\delta = 0$. The other curve shows the average number of time slots in which thresholds were broadcast relative to that in the case where $\delta = 0$. The simulation parameters were chosen according to Case 1 with $n = 12$ and $(\lambda_1 + \lambda_2) \cdot n/2 = 0.756$.

rate to the system, which incurs the mean delay of 200 ms for a particular scheme. In Case 3, the users in Group 2 have lower service rates but higher arrival rates; thus, the imbalance in the offered loads between the groups is the worst among all the cases. We observe that, however, the proposed scheme excels all other schemes, and it achieves 1.1–2.3 times higher 200-ms throughput than other schemes for $n = 8$, as shown in Fig. 5. Overall, we observe that the queue-aware schemes significantly outperform the queue-oblivious schemes.

The simulation so far assumed $\delta = 0$, i.e., threshold broadcast occurs whenever queue lengths change. Next, we explore the tradeoff between performance and broadcast overhead. Fig. 6 shows the tradeoff with varying δ's. We observe an increase in the mean delay as the frequency of broadcast decreases. However, the tradeoff is smoothly achieved, and one can significantly reduce the overhead with a small penalty in delay, e.g., the overhead is reduced by 70% with the increase in delay by 11% when $\delta = 0.05$.

V. Conclusion

We have proposed an OF scheme aimed at achieving low packet delays in the context of a heterogeneous wireless system. Motivated by queue/weight-based OS techniques, we have assigned the user thresholds for feedback, which are determined by weights designed to capture their current backlog. We have demonstrated that it is crucial to reflect quantiles and queue lengths in the feedback design to deal with system heterogeneity, that is, providing fairness in channel opportunism and service history is shown to enhance MUD and improve delay performance. Moreover, our scheme allows a simple computation of thresholds irrespective of channel distributions, which is of practical significance.

Appendix A
Proof of Lemma 2

Denote the optimal policy by $\tilde{\phi} = (\tilde{\boldsymbol{x}}, \alpha, \beta)$. Suppose that $\tilde{x}_i = 0$ for some $i \in J$, i.e., user i transmits feedback with probability (w.p.) 1, which implies that a collision will occur if any other user transmits feedback. We consider three cases and show that $\tilde{x}_i = 0$ leads to contradictions in all cases. We assume that $\alpha \neq \beta$ in this proof: the case of $\alpha = \beta$ can be proved in a very similar way; thus, its proof is omitted.

Case 1—$i \neq \alpha$ or β: Suppose user β transmits feedback under the policy $\tilde{\phi}$, which happens w.p. $1 - \tilde{x}_\beta$. A collision occurs w.p. 1; thus, $\tilde{\phi}$ will serve user β, which yields the ASWQ $1/2w_\beta(1 + \tilde{x}_\beta)$, which is the average weighted quantile of user β conditional on user β transmitting feedback or his quantile being above $\tilde{x}_\beta$. Next, suppose that user β did not transmit feedback, which occurs w.p. $\tilde{x}_\beta$. A collision may or may not occur: if a collision did not occur, implying that user i was the only transmitter, user i will be served, which yields the ASWQ $1/2w_i$ since the quantile of user i is independent of those of the other users. If a collision occurred, user β will be served, which yields the ASWQ $1/2w_\beta\tilde{x}_\beta$, which is the average weighted quantile of user β conditional on the quantile of user β being below $\tilde{x}_\beta$. Thus, the ASWQ is no more than $\max[1/2w_i, 1/2w_\beta\tilde{x}_\beta]$ conditional on user β not transmitting feedback. Thus, $\theta(\boldsymbol{w}; \tilde{\phi})$ is at most $1/2[w_\beta(1 - \tilde{x}_\beta^2) + \tilde{x}_\beta \max\{w_i, w_\beta\tilde{x}_\beta\}]$, which we define as $\tau(\tilde{x}_\beta)$. First, assume that $w_i \geq w_\beta\tilde{x}_\beta$, i.e., $\tau(\tilde{x}_\beta) = 1/2[w_\beta(1 - \tilde{x}_\beta^2) + w_i\tilde{x}_\beta]$. We assume $\tilde{x}_\beta > 0$ for now. The case for $\tilde{x}_\beta = 0$ is discussed in Case 2. Consider a policy $\psi = (\boldsymbol{y}, \alpha, \beta)$ such that $y_\beta = \tilde{x}_\beta$ and $y_j = 1\ \forall j \in J \setminus \{\beta, i\}$. From (4), we have that

$$\begin{aligned}\theta(\boldsymbol{w}; \psi) &= \frac{w_\beta(1 - \tilde{x}_\beta^2)}{2} + \frac{w_i\tilde{x}_\beta\,(1 - y_i^2)}{2} + \frac{w_\alpha y_i \tilde{x}_\beta}{2} \\ &= \frac{w_\beta\left(1 - \tilde{x}_\beta^2\right)}{2} + \frac{w_i\tilde{x}_\beta}{2}\left[1 - y_i^2 + \frac{w_\alpha}{w_i} y_i\right]. \qquad (7)\end{aligned}$$

Define the term in the brackets of (7) by $\rho(y_i)$. Since $\rho(0) = 1$ and $\partial\rho/\partial y_i|_{y_i=0} = w_\alpha/w_i > 0$, by making y_i small enough, we can make $\rho(y_i) > 1$, implying that $\theta(\boldsymbol{w}; \psi) > 1/2[w_\beta(1 - \tilde{x}_\beta^2) + w_i\tilde{x}_\beta] = \tau(\tilde{x}_\beta) \geq \theta(\boldsymbol{w}; \tilde{\phi})$, contradicting that $\tilde{\phi}$ is optimal. Second, assume that $w_i < w_\beta\tilde{x}_\beta$, in which case, $\tau(\tilde{x}_\beta) = 1/2w_\beta$. Consider a policy $\zeta = (\boldsymbol{z}, \alpha, \beta)$ such that $z_j = 1\ \ \forall j \in J \setminus \{\beta\}$, yielding $\theta(\boldsymbol{w}; \zeta) = 1/2[w_\beta + w_\alpha z_\beta - w_\beta z_\beta^2]$. By making z_β small enough, we can make $w_\alpha z_\beta - w_\beta z_\beta^2 > 0$ or $\theta(\boldsymbol{w}; \zeta) > 1/2w_\beta$, but $1/2w_\beta \geq \theta(\boldsymbol{w}; \tilde{\phi})$, which is a contradiction.

Case 2—$i = \beta$: Since $\tilde{x}_i = \tilde{x}_\beta = 0$, user β is served at all times; thus, $\theta(\boldsymbol{w}; \tilde{\phi}) = 1/2w_\beta$. We have shown that the policy ζ defined in Case 1 satisfies that $\theta(\boldsymbol{w}; \zeta) > 1/2w_\beta = \theta(\boldsymbol{w}; \tilde{\phi})$, which is a contradiction.

Case 3—$i = \alpha$: When $\alpha \neq \beta$ and $n \geq 3$, one can show by direct expansion of (4) that $\theta(\boldsymbol{w}; \phi)$ has the form $ax_\alpha + b$, where a and b

are functions of $\{x_i\}$, $i \in J \setminus \{\alpha\}$, only; particularly, if $x_i > 0$ for all $i \in J \setminus \{\alpha\}$, $a > 0$ holds. Consider $\theta(\boldsymbol{w}; \tilde{\phi})$, which can also be written as $\tilde{a}\tilde{x}_\alpha + \tilde{b}$. From Cases 1 and 2, $\tilde{x}_i > 0$ holds $\forall i \in J \setminus \{\alpha\}$, which implies that $\tilde{a} > 0$. Thus, $\tilde{x}_\alpha = 1$ and cannot be 0 from the optimality of $\tilde{\phi}$. ■

Appendix B
Proof of Theorem 1

Consider a policy $\psi = (\boldsymbol{y}, \alpha, \beta)$ such that $y_i = \tilde{x}_j$ and $y_j = \tilde{x}_i$, and $y_l = \tilde{x}_l$ for all $l \neq i, j$. Policies $\tilde{\phi}$ and ψ are identical, except that the thresholds for users i and j are switched. By definition, $r_S^m(\phi)$ is a symmetric function of $n-1$ variables $\{x_l\}$, $l \in J \setminus \{m\}$. In addition, $r_C(\phi)$ $(r_I(\phi))$ is a symmetric function of $\{x_l\}$, $l \in J \setminus \{\beta\}$ $[\{x_l\}, l \in J \setminus \{\alpha\}$, see (4)]. Thus, the following hold: $r_S^j(\tilde{\phi}) = r_S^i(\psi)$, $r_S^i(\tilde{\phi}) = r_S^j(\psi)$, $r_C(\tilde{\phi}) = r_C(\psi)$, $r_I(\tilde{\phi}) = r_I(\psi)$, and $r_S^l(\tilde{\phi}) = r_S^l(\psi)$, $\forall l \neq i, j$. Thus, we have that $\theta(\boldsymbol{w}; \tilde{\phi}) - \theta(\boldsymbol{w}; \psi) = (w_i - w_j)(r_S^i(\tilde{\phi}) - r_S^j(\tilde{\phi}))$, which must be nonnegative since $\tilde{\phi}$ is optimal. Since $w_i \geq w_j$, we have that $r_S^i(\tilde{\phi}) \geq r_S^j(\tilde{\phi})$ or $(1-\tilde{x}_i^2)\tilde{x}_j \prod_{l \neq i,j} \tilde{x}_l \geq (1-\tilde{x}_j^2)\tilde{x}_i \prod_{l \neq i,j} \tilde{x}_l$. From this inequality, we can cancel out the factor $\prod_{l \neq i,j} \tilde{x}_l$ since $\tilde{x}_l \neq 0$ for all $l \in J$ by Lemma 2, from which $\tilde{x}_i \leq \tilde{x}_j$ holds. ■

Appendix C
Proof of Lemma 3

We can define the Lagrangian of $\mathbf{P3}$ as $L(x, x_\beta, \lambda, \nu) := x^{k-1}\sigma(x, x_\beta) + \lambda(x-1) + \nu(x_\beta - 1)$, where $\sigma(x, x_\beta) := 1/2 \times [\{\sum_{i \in L^{(k)}(\alpha,\beta)} w_i\}(1-x^2)x_\beta + w_\alpha x_\beta x - k w_\beta x_\beta^2 + (k-1) w_\beta x_\beta^2 x]$ and λ, $\nu \geq 0$, are the Lagrange multipliers associated with the constraints $x \leq 1$ and $x_\beta \leq 1$. Note that the constraints $x \geq 0$ and $x_\beta \geq 0$ can be shown to be always inactive, i.e., the optimal x and x_β are strictly positive, similar to Lemma 2. For feasible x, x_β, λ, and ν, the Karush–Kuhn–Tucker (KKT) condition for $\mathbf{P3}$ is given by

$$\frac{\partial L(x, x_\beta, \lambda, \nu)}{\partial x} = 0 \quad \frac{\partial L(x, x_\beta, \lambda, \nu)}{\partial x_\beta} = 0$$
$$(x-1)\lambda = 0 \quad (x_\beta - 1)\nu = 0. \tag{8}$$

It can be shown that solving the KKT condition (8) has $O(1)$ complexity, e.g., when $\lambda = \nu = 0$, from (8)

$$\begin{aligned}\frac{\partial L(x, x_\beta, \lambda, \nu)}{\partial x} &= \left\{(k-1)x^{k-2} + x^{k-1}\frac{\partial}{\partial x}\right\}\sigma(x, x_\beta) = 0\\ &\Rightarrow (k-1)\sigma(x, x_\beta) + x\frac{\partial \sigma(x, x_\beta)}{\partial x} = 0 \quad (9)\\ \frac{\partial L(x, x_\beta, \lambda, \nu)}{\partial x_\beta} &= 0\\ &\Rightarrow \{2k w_\beta - 2(k-1) w_\beta x\} x_\beta\\ &= \left\{\sum_{i \in L^{(k)}(\alpha,\beta)} w_i\right\}(1-x^2) + w_\alpha x. \quad (10)\end{aligned}$$

We can solve for x_β in terms of x from (10), which renders (9) a univariate polynomial equation in x of a constant order, which has $O(1)$ complexity and can be solved exactly, e.g., by the Jenkins-Traub method. We can similarly find all the solutions to (8) and thus conclude that $\mathbf{P3}$ has $O(1)$ complexity. ■

Appendix D
Proof of Theorem 2

We will first prove the following two lemmas: Assume that $\alpha \neq \beta$.

Lemma 4: For fixed $\alpha \in J$ and $k \in K$, denote the indices associated with the maximum and $(k+1)$st largest weights from $J \setminus \{\alpha\}$ by $\bar{\beta}$ and $\underline{\beta}$, respectively. Then, $\arg\max_{\beta \in J \setminus \{\alpha\}} [h^{(k)}(\alpha, \beta)]$ is either $\bar{\beta}$ or $\underline{\beta}$.

Proof: For fixed α and β, denote the solution of $\mathbf{P3}$ by $\tilde{\boldsymbol{x}}$, and define $\phi = (\tilde{\boldsymbol{x}}, \alpha, \beta)$. Since $\tilde{\boldsymbol{x}}$ is a feasible point of $\mathbf{P3}$, we have that, for all $i \in L^{(k)}(\alpha, \beta)$, $r_S^i(\phi) = 1/2(1-\tilde{x}^2)\tilde{x}^{k-1}\tilde{x}_\beta$, which we denote by $r_S(\phi)$. Consider two cases: first assume $r_S^\beta(\phi) + r_C(\phi) \geq r_S(\phi)$. Consider a threshold vector $\bar{\boldsymbol{x}}$ such that $\bar{x}_{\bar{\beta}} = \tilde{x}_\beta$, $\bar{x}_\beta = \tilde{x}_{\bar{\beta}} = \tilde{x}$, and $\bar{x}_i = \tilde{x}$ $\forall i \in J \setminus \{\bar{\beta}, \beta\}$, i.e., $\bar{\boldsymbol{x}}$ is obtained by switching the thresholds between users β and $\bar{\beta}$ from $\tilde{\boldsymbol{x}}$. Denote the policy $(\bar{\boldsymbol{x}}, \alpha, \bar{\beta})$ by $\bar{\phi}$. The following are verifiable from the definitions: $r_S^i(\phi) = r_S^i(\bar{\phi}) = r_S(\phi)$ holds for all $i \in J \setminus \{\bar{\beta}, \beta\}$, $r_S^{\bar{\beta}}(\phi) = r_S^\beta(\bar{\phi}) = r_S(\phi)$ holds, $r_S^\beta(\phi) = r_S^{\bar{\beta}}(\bar{\phi}) = 1/2(1-\tilde{x}_\beta^2)\tilde{x}^k$, and $r_I(\phi) = r_I(\bar{\phi}) = 1/2\tilde{x}^k\tilde{x}_\beta$ holds. Finally, $r_C(\phi) = r_C(\bar{\phi}) = 1/2[(1-\tilde{x}_\beta^2)(1-\tilde{x}^k) + \tilde{x}_\beta^2\{1 - k\tilde{x}^{k-1} + (k-1)\tilde{x}^k\}]$ holds. Thus

$$\begin{aligned}&h^{(k)}(\alpha, \bar{\beta}) - h^{(k)}(\alpha, \beta)\\ &\quad = h^{(k)}(\alpha, \bar{\beta}) - \theta(\boldsymbol{w}; \phi) \geq \theta(\boldsymbol{w}; \bar{\phi}) - \theta(\boldsymbol{w}; \phi)\\ &\quad = (w_{\bar{\beta}} - w_\beta)\left[r_S^\beta(\phi) + r_C(\phi)\right]\\ &\qquad + \left[\left(\sum_{i \in L^{(k)}(\alpha,\bar{\beta})} w_i\right) - \left(\sum_{i \in L^{(k)}(\alpha,\beta)} w_i\right)\right] r_S(\phi). \quad (11)\end{aligned}$$

By definition, $L^{(k)}(\alpha, \bar{\beta})$ is the set of indices associated with the k largest weights from $J \setminus \{\alpha, \bar{\beta}\}$. Thus, $L^{(k)}(\alpha, \bar{\beta}) \cup \{\bar{\beta}\}$ is the set of indices associated with the $k+1$ largest weights from $J \setminus \{\alpha\}$ from the definition of $\bar{\beta}$. This implies $\sum_{i \in L^{(k)}(\alpha,\bar{\beta}) \cup \{\bar{\beta}\}} w_i \geq \sum_{i \in L^{(k)}(\alpha,\beta) \cup \{\beta\}} w_i$ for any β; thus, (11) is given by

$$\begin{aligned}&(w_{\bar{\beta}} - w_\beta)\left[r_S^\beta(\phi) + r_C(\phi)\right]\\ &\quad + \left[\left\{\left(\sum_{i \in L^{(k)}(\alpha,\bar{\beta}) \cup \{\bar{\beta}\}} w_i\right) - w_{\bar{\beta}}\right\}\right.\\ &\qquad \left. - \left\{\left(\sum_{i \in L^{(k)}(\alpha,\beta) \cup \{\beta\}} w_i\right) - w_\beta\right\}\right] r_S(\phi)\\ &= (w_{\bar{\beta}} - w_\beta)\left(r_S^\beta(\phi) + r_C(\phi) - r_S(\phi)\right)\\ &\quad + \left\{\left(\sum_{i \in L^{(k)}(\alpha,\bar{\beta}) \cup \{\bar{\beta}\}} w_i\right) - \left(\sum_{i \in L^{(k)}(\alpha,\beta) \cup \{\beta\}} w_i\right)\right\} r_S(\phi)\\ &\geq (w_{\bar{\beta}} - w_\beta)\left(r_S^\beta(\phi) + r_C(\phi) - r_S(\phi)\right) \geq 0.\end{aligned}$$

Second, assume that $r_S^\beta(\phi) + r_C(\phi) < r_S(\phi)$. Consider $\underline{\phi} = (\tilde{\boldsymbol{x}}, \alpha, \underline{\beta})$; then, using similar arguments as previously given, one can show that $h^{(k)}(\alpha, \underline{\beta}) - h^{(k)}(\alpha, \beta) \geq (w_\beta - w_{\underline{\beta}})(r_S(\phi) - r_S^\beta(\phi) - r_C(\phi)) \geq 0$. ■

Lemma 5: For fixed $\beta \in J$ and $k \in K$, denote the indices associated with the maximum and $(k+1)$st largest weights from $J \setminus \{\beta\}$ by $\bar{\alpha}$ and $\underline{\alpha}$, respectively. Then, $\arg\max_{\alpha \in J \setminus \{\beta\}} [h^{(k)}(\alpha, \beta)]$ is either $\bar{\alpha}$ or $\underline{\alpha}$.

The proof of Lemma 5 is omitted since it is almost identical to that of Lemma 4. The optimal choices for (α, β) must satisfy *both* Lemmas 4 and 5. One can verify that such (α, β) pairs are the following: (1, 2), (2, 1), $(1, k+2)$, $(k+2, 1)$, $(k+1, k+2)$, and $(k+2, k+1)$. Note that we assumed that $\alpha \neq \beta$. When $\alpha = \beta$, one can make a similar argument as previously given and show that (α, β) pairs that satisfy the lemmas are (1,1) and $(k+1, k+1)$.

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Efficient Implementation of the MIMO Sphere Detector: Architecture and Complexity Analysis

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***Abstract*—An efficient implementation strategy for the multiple-input–multiple-output (MIMO) sphere detector (SD) is proposed and analyzed in this paper. The proposed method incorporates a dynamic information storage-and-retrieval mechanism to avoid repetitive computation of previously processed results. A memory-access architecture is devised to facilitate the implementation. The proposed method remarkably benefits the depth-first SD and its variants, as verified by a detailed complexity analysis and experimental results.**

***Index Terms*—Implementation architecture, maximum-likelihood (ML) detection, multiple-input–multiple-output (MIMO) systems, sphere detector (SD).**

I. Introduction

The maximum-likelihood (ML) detection problem in wireless multiple-input–multiple-output (MIMO) systems is mathematically equivalent to finding the closest lattice point in the multidimensional space, which is a problem known to be NP-hard. Factorizing the MIMO channel in upper triangular form (see Section II), detecting the lattice inputs can be represented as a tree-search process. Since ML detection employing an exhaustive search over the entire tree is computationally infeasible in practice, various reduced-complexity detection algorithms have been proposed, including (in tree-search terms) the best-first search (e.g., the stack algorithm [1]), the depth-first search (e.g., the sphere detector (SD) [2], [3]), the breadth-first search (e.g., [4]), and the hybrid tree-search algorithms (e.g., [5]). These contributions propose algorithmic improvements on previous schemes. The implementation aspect of various detection schemes has also been studied for the breadth-first K-best detector [6]–[8], the successive-interference-cancellation-based detector [9], [10], and the SD [11]–[13]. These contributions report on the details and modifications of the detection algorithm from the perspective of implementation and/or suggest an efficient architecture to realize the hardware implementation of the algorithm.

The SD originally introduced in [2] and [3] essentially conducts a constrained lattice-point search within a hypersphere defined by its sphere radius to reduce the number of decision candidates searched. Viewing the SD detection process as the depth-first tree traversal, it is observed that different nodes in the tree naturally have various lengths of shared paths to the root; therefore, metric computations for previously visited nodes can be reused to avoid redundant computations in executing the SD. This paper explicitly explores this concept, which is known as *memoization* in the computing parlance, and proposes a computationally efficient implementation through a register-based architecture and a register input/output (I/O) mechanism. The main contributions of this paper are given here.

1) The proposed implementation strategy underlies the detection algorithm itself and can be implemented in any class of the depth-first SD—optimal or suboptimal, hard-output or soft-output, with or without tree pruning—to enhance its computational efficiency.
2) A detailed complexity analysis is conducted to quantify the computational saving yielded by the proposed implementation. Our analysis shows that, if no tree pruning is enforced, the computational cost of the proposed implementation is approximately $2/(2N_T+1)$ times as much as that of the conventional implementation for an $N_T \times N_T$ system (real-valued processing). The computational advantage of the proposed method persists for any tree-pruning strategy employed, although the computational saving dwindles as the pruning becomes more extensive, as indicated by the derived upper and lower bounds on the complexity.

This paper is organized as follows: Section II defines the system model and reviews SD. Section III describes the proposed implementation. Complexity analysis is conducted in Section IV, and experimental results are presented in Section V. Finally, concluding remarks are given in Section VI.

II. Multiple-Input–Multiple-Output Sphere Detector

We consider an uncoded MIMO transmission system with N_T transmit antennas and N_R receive antennas (denoted by an $N_T \times N_R$ system). We are particularly interested in the case where $N_R = N_T$, although this is not a constraint on either the SD algorithm or the proposed implementation strategy for SD. The baseband signal model is given by

$$\mathbf{y}_c = \mathbf{H}_c\tilde{\mathbf{x}}_c + \mathbf{v}_c \tag{1}$$

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