Site Specific Knowledge and Interference Measurement for Improving Frequency Allocations in Wireless Networks

Jeremy K. Chen, Member, IEEE; Gustavo de Veciana, Fellow, IEEE, and Theodore S. Rappaport, Fellow, IEEE;

Abstract—We present new frequency allocation schemes for wireless networks and show that they outperform all other published work. Two categories of schemes are presented: those based purely on measurements and those that use site-specific knowledge, which refers to knowledge of building layouts, the locations and electrical properties of APs, users, and physical objects. In our site-specific knowledge-based algorithms, a central network controller communicates with all APs and has site specific knowledge so that it can predict, a priori, the received power from any transmitter to any receiver. Optimal frequency assignments are based on predicted powers to minimize interference and maximize throughput. In our measurement-based algorithms, clients periodically report in-situ interference measurements to their associated APs; then the APs’ frequency allocations are adjusted based on the reported measurements. Unlike other work, we minimize interference seen by both users and APs; use a physical rather than binary model for interference; and mitigate the impact of rogue interference. Our algorithms consistently yield high throughput gains irrespective of network topology, AP placement level, the number of APs, and each AP is allocated one channel. When the number of channels is limited relative to the number of APs, some APs inevitably use the same channel and induce co-channel interference. The same problem exists in cellular networks. Judicious channel reuse mechanisms are necessary to reduce interference, particularly for the case of mobile users, such as in enterprise voice over IP networks or in cellular networks. Today, site specific knowledge is becoming much more available from blueprints, AutoCAD, and Google™ Maps, and Google™ Earth, for example. This work demonstrates vast improvement in overall network performance, particularly by greatly raising the throughputs of the users that suffer low throughputs. In other words, our schemes enable many more users to have acceptable throughputs for file transfer or voice-over-IP applications, as compared to prior published work.

A number of WLAN frequency allocation schemes have been proposed thus far. The work in [8] assumes each AP has a different fixed traffic load, and defines the effective channel utilization of an AP as the fraction of time the channel is used for data transmission or is sensed busy due to interference from other APs; then, the maximum effective channel utilization among all APs is minimized. AP placement and frequency allocation are jointly optimized in [9] with the same objective of minimizing the max channel utilization as in [8]. Work in [8], [9] fail to maximize throughput or minimize interference seen by clients, and hence perform poorer than our proposed work in today’s WLANs, where downlink traffic dominates. The frequency allocation problem is modeled as a minimum-sum-weight vertex-coloring problem in [10] where vertices are APs, and the weight of each edge between two APs denotes the number of clients that are associated with either one of these two APs and are interfered by the other AP. The work in [11] minimizes the number of clients whose transmissions suffer channel conflicts; a client associated with an AP suffers conflicts if other clients or other APs interfere with the client or the AP under consideration. The definition of channel conflict in [11] is more comprehensive than those in [8]–[10]. The work in [11] has been shown to outperform [8]–[10], but performs poorer than our proposed schemes in the presence of rogue interferers, i.e., intentional or unintentional RF interferers, microwave ovens, or other RF devices that also they can be implemented for real-time network management applications. This paper is the first work to analyze how site specific knowledge can improve on-going frequency allocation in wireless local area networks (WLAN). In WLANs, a number of orthogonal frequency channels are allocated, and each AP is allocated one channel. When the number of channels is limited relative to the number of APs, some APs inevitably use the same channel and induce co-channel interference. The same problem exists in cellular networks. Judicious channel reuse mechanisms are necessary to reduce interference, particularly for the case of mobile users, such as in enterprise voice over IP networks or in cellular networks. Today, site specific knowledge is becoming much more available from blueprints, AutoCAD, and Google™ Maps, and Google™ Earth, for example. This work demonstrates vast improvement in overall network performance, particularly by greatly raising the throughputs of the users that suffer low throughputs. In other words, our schemes enable many more users to have acceptable throughputs for file transfer or voice-over-IP applications, as compared to prior published work.

I. INTRODUCTION

Radio propagation characteristics are fundamentally site specific, since radio propagation mechanisms (e.g. penetration, reflection, and diffraction) are directly related to the locations, sizes, and electrical properties of physical objects in the surroundings. Site-specific channel prediction algorithms are well understood [3]–[7]. These site specific prediction techniques use a building layout or a satellite map and compute path losses between any two locations in indoor or outdoor environments. The complexity of these prediction tools has been reduced, and computing power has increased, so that
operate on the same unlicensed bands as WLAN.

Only [12] presents mechanisms to detect and reduce the negative impact from rogue interferers. In [12], each AP senses interference and independently selects a channel whose measured interference power is below a predefined threshold, without coordinating with other APs. In networks with high interference, it may not be feasible to find a channel allocation so that every AP senses interference below the threshold; in this case, the algorithm in [12] does not converge. One could in principle set a higher threshold for the algorithm in [12] to work in high-interference regimes, but [12] does not mention methods to adapt the threshold. It is not trivial to adapt this threshold, since a high threshold will degrade network performance, but a low threshold will yield no feasible solution. By contrast, four of our proposed algorithms converge irrespective of the overall interference level. The non-convergence result of [12] in the high interference regime is due to the binary model for interference, which is also used in [8]–[11]. Our work considers a physical model for interference; that is, we assume that interference power is a continuous quantity, which properly represents the real world. Therefore, our work performs better than [12].

Most traffic in WLANs is downlink [6], [13]; hence, maximizing downlink throughputs and signal-to-interference-and-noise ratio (SINR) seen by users are key to proper network design. The work in [8], [9], [12] focuses on minimizing the interference at APs rather than that seen by users, as is done in [10], [11], and thus often perform poorer than [10], [11].

A. Main Contribution

The main contribution of this work is our two categories of new algorithms for channel allocations that outperform all other published work, i.e., those in [8]–[12]. In the remainder of this paper, we mainly present the gains of our proposed algorithms against the works in [11], [12], since the work in [11] has been shown to outperform [8]–[10]. The proposed algorithms perform well mainly because they: (1) minimize interference seen by users rather than that seen by APs; (2) use a physical model rather than a binary model for interference; and (3) have the ability to deal with rogue interferers. The first category is based on interference measurements at APs and users, and the second category based on site specific knowledge. We describe our ideas and contribution below.

1) Measurement-Based Algorithms: We propose that all or a subset of clients measure the in-situ interference power on all frequency channels periodically when their associated APs are idle, and report the average measured power to their associated APs. This technique is used in mobile-assisted hand-off (MAHO) in the cellular field [14], and results in this paper may also be applied to cellular networks.

APs also measure in-situ interference power. Since the measurements at APs or clients are performed during their idle time, the overhead is negligible. Each AP then computes a metric called weighted interference which captures the overall interference as seen by itself and its clients, by placing different weights on its and the clients’ in-situ measurements according to the clients’ traffic loads, signal strengths, and uplink and downlink traffic volume. Section V shows by simulation that our measurement-based algorithms substantially outperform [11], [12]. Since the work in [11] has been shown to outperform [8]–[10], we conclude that our algorithms outperform [8]–[12].

2) Site-Specific Knowledge-Based Algorithms: The measurement-based algorithms mentioned above can still be improved if we assume that a central network controller has and uses site specific knowledge to optimize frequency allocation of each AP and each user. The advantage of using site specific knowledge is to predict a priori path loss between any pair of AP and user, when the user’s location is obtained via GPS (Global Positioning System) or other known position location technologies. The predicted path losses can help formulate a global optimization problem, thereby maximizing throughputs and saving power, etc. Though the environment affects the path losses, empirical results show that by modeling large fixed partitions and items in the environment (such as walls, book shelves, and cubicles), the predicted and the measured path losses show high agreement (e.g., mean error is less than 4 dB) [3]–[6].

Distributed measurement-based algorithms with the knowledge of APs’ transmit powers via message exchanges can learn over time the path loss or received power between every transmitter and every receiver; examples of such algorithms are described in [17]. Nevertheless, the time needed for learning may be too long, when the number of interfering APs is large. The overall learning time could be shortened if each client learns the interference power from only the APs that are in the range of causing non-negligible interference at the client. In order to know which base stations are in the interference range, site specific knowledge (such as the environments and the locations of APs and clients) is needed. Saving the learning time for measurement-based algorithms is a topic for ongoing and future work. In this paper, we choose to use site specific knowledge to predict a priori the interference power between any transmitter and any receiver.

Note that the central controller must know the active transmitters at any point in time in order to predict correct interference at all times; this information may be too costly to obtain, but time sampling may be done. Since downlink volume presently dominates WLAN traffic, this paper considers a case where all APs are actively transmitting downlink traffic. It is reasonable to assume that frequency allocation is optimized with respect to this most active case, since in this case, frequency allocation is most crucial for interference mitigation. Simulations in Section V show that our algorithms also perform well in scenarios with both downlink and uplink traffic and with different levels of AP activity. In this paper, we consider perfect site specific knowledge; in other words, 1

1Several indoor position location approaches, based on signal strength sensing, are widely known today and used in some WLANs [15], [16]. Other triangulation methods can also be used to locate a client. Modern cellular handsets are equipped with GPS chips or other position location technologies. State-of-the-art GPS can work not only outdoors but also indoors; various vendors, e.g., Metris and SnapTrack, provide indoor GPS solutions. For example, the indoor GPS technology by Metris can be compared to the matrix of satellites that create the Global Positioning System; instead of satellites, Metris’ indoor GPS uses small infrared laser transmitters that emit laser pulses to create a measurement universe. Based on the timing of the light pulses received by photo detectors, angle and positions may be computed.
we assume that the actual path loss between any transmitter and receiver can be correctly predicted by the site specific knowledge. Work in [3], [4] shows that site specific models can achieve remarkably good predictions (zero mean error and standard deviation of 3-4 dB). The study of the effect of imperfect predictions of channel gains is also an ongoing and future work.

B. Organization

Section II introduces the system model, notation, and assumptions, and describes our notion of weighted interference in detail. The three proposed measurement-based algorithms, denoted No-Coord, Local-Coord, and Global-Coord, have different mechanisms for iteratively switching frequency channels in order to reduce the weighted interference seen in a single cell, a group of nearby cells, or all cells, respectively, where a cell means an AP (or base station) and its associated users. Section III presents the mechanisms used by the three measurement-based algorithms and their convergence. Section IV presents two site-specific knowledge-based algorithms. Section V shows by simulation that our algorithms substantially outperform [8]–[12].

II. SYSTEM MODEL, NOTATION, AND ASSUMPTIONS

We begin by describing basic notation; then the first subsection describes weighted interference, a metric used in the three proposed measurement-based algorithms to capture the overall interference of each cell. The second subsection defines notation used exclusively for the proposed measurement-based Local-Coord algorithm. The third subsection describes assumptions used in site-specific knowledge-based algorithms.

Basic Notation: Suppose $M$ APs, indexed by $\mathbb{M} = \{1, 2, \ldots, M\}$, operate on $K$ orthogonal frequency channels, indexed by $\mathbb{K} = \{1, 2, \ldots, K\}$. We index users (or clients) by $\mathbb{L} = \{1, 2, \ldots, L\}$. We denote the identity of an AP and a client by $a_m$ ($m \in \mathbb{M}$) and $c_l$ ($l \in \mathbb{L}$), respectively. We assume for this work that the locations of the APs and the clients do not vary with time, and assume that no APs or users are at the same location, although the algorithms given here also apply for mobile APs and/or clients. Let $\mathbb{Z}_m$ ($m \in \mathbb{L}_m$) denote the set of users that are associated with the AP $a_m$. We assume every user is associated with a single AP, and define a cell $\mathbb{Z}_m = \{a_m\} \cup \{c_l : l \in \mathbb{L}_m\}$. Let $f_m$ ($f_m \in \mathbb{K}$) denote the channel that $a_m$ operates on, and let $\bar{f} = (f_1, f_2, \ldots, f_M)$ denote the channels of all $M$ APs.

A. Weighted Interference for Measurement-Based Algorithms

In brief, the weighted interference of each cell (say $\mathbb{Z}_m$) is intended to capture the overall interference seen in the cell, and is therefore defined as a weighted sum of the average in-situ measurements at $a_m$ and at all clients associated with $a_m$, i.e., at every $u \in \mathbb{Z}_m$. We propose that $a_m$ or the clients associated with $a_m$ measure their in-situ interference power when there is no traffic within Cell $\mathbb{Z}_m$, i.e., $a_m$ is neither transmitting or receiving data. The average in-situ measured interference power at $u$ (for every $u \in \mathbb{Z}_m$) on channel $k$ is denoted $I_{k}^{u}(\bar{f})$. The averaging period is a design choice and could be the same as the period that an AP switches its channel, say 1, 2, or 5 minutes. $I_{k}^{u}(\bar{f})$ is lower-bounded by the noise floor. The weighted interference function for $\mathbb{Z}_m$ on channel $k$ is defined by

$$W_{k}^{m}(\bar{f}) = \sum_{u \in \mathbb{Z}_m} B_{k}^{u}(I_{k}^{u}(\bar{f})), \quad k \in \mathbb{K},$$

where $B_{k}^{u}(\cdot)$ is a nonnegative and non-decreasing function that captures the weight of the in-situ measurement at $u$. We require that $W_{k}^{m}(\bar{f}) > 0$ to capture the existence of noise floor in the real world. $B_{k}^{u}(\cdot)$ should be designed to reflect the difference of clients’ traffic demands, signal strengths, and uplink and downlink traffic volume. In practice, clients report the measurements to $a_m$ either periodically or upon request from $a_m$; then, $W_{k}^{m}(\bar{f})$ can be computed at $a_m$.

In Section III-E we will show that two of our proposed algorithms (namely Local-Coord and Global-Coord) converge if the weighted interference function has the general form in (1). Below we introduce two simplified forms of $W_{k}^{m}(\cdot)$ representing practical metrics. The first form, denoted user-based, places different weights on the in-situ interference measurements at clients based on the traffic volume and the signal strength at each client. The user-based form captures the performance of downlink transmission, which is appropriate for WLANs since traffic measurements show that downlink traffic volume accounts for more than 84% of total (uplink plus downlink) traffic volume [6]. The second form, denoted AP-based, includes the interference measurements at APs only. The AP-based form can be viewed as a simplified version of the user-based one by considering all users have the same traffic volume and signal strength.

1) User-based: The user-based weighted interference function for $\mathbb{Z}_m$ is defined by

$$W_{k}^{(U),m}(\bar{f}) = \sum_{l \in \mathbb{Z}_m} Y_{c_l,a_m} S_{c_l,a_m} I_{k}^{l}(\bar{f}),$$

where $S_{c_l,a_m}$ denotes the average received signal power from $a_m$ to $c_l$, and $Y_{c_l,a_m}$ denotes the average traffic volume from $a_m$ to $c_l$. We incorporate the inverse of $S_{c_l,a_m}$ in (2) because a client with a stronger $S_{c_l,a_m}$ has higher tolerance to interference and thus should contribute less to the overall weighted interference. $Y_{c_l,a_m}$ is included in (2) as a scaling factor, since a client with higher traffic volume should be more important for the weighted interference. In practice, some users may be sampled to reduce the complexity of computing (2), i.e., the summation in (2) may be over a subset of $\mathbb{L}_m$.

2) AP-based: The AP-based weighted interference function for $\mathbb{Z}_m$ is defined by

$$W_{k}^{(A),m}(\bar{f}) = I_{k}^{m}(\bar{f}).$$

Note $c_l$ cannot measure $S_{c_l,a_m}$ directly but can estimate $S_{c_l,a_m}$ as follows. The average in-situ SINR at $c_l$ when $a_m$ is transmitting to $c_l$, and is denoted $\gamma_l$. We assume the interference at $c_l$ is the same whether $a_m$ is transmitting to $c_l$ or $a_m$ is idle, i.e., the interference at $c_l$ is always $I_{f_m}^{l}(\bar{f})$. Then we estimate $S_{c_l,a_m} = \gamma_l \cdot I_{f_m}^{l}(\bar{f})$. 


B. Interfering Cells for the Local-Coord Algorithm

When an AP switches its channel, nearby cells may see substantial changes in their weighted interference. The notation of such cells is presented below and will be used in describing the proposed Local-Coord algorithm in Section III-B. Cell \( Z_n \) is said to be interfered by Cell \( Z_m \) (or \( Z_m \) interferes with \( Z_n \)) if and only if \( a_m \) or a user associated with \( a_m \) induces non-negligible interference (e.g., the interference power at the receiver is higher than the noise floor) at \( a_n \) or a user associated with \( a_n \). We define \( G_m \) as the set of cells interfered by \( Z_m \); i.e., \( G_m = \{ n : Z_m \text{ interferes with } Z_n \text{ given that } a_m \text{ and } a_n \text{ are on the same channel} \} \). The subset of \( G_m \) that is on channel \( k \) is denoted \( G_{m,k}(\bar{f}) \). Suppose \( a_m \) switches from channel \( k \) to \( k' \), the cells that see changes in their weighted interference are \( Z_m \) and the cells indexed by \( G_{m,k}(\bar{f}) \) and \( G_{m,k'}(\bar{f}) \). We define \( \mathbb{H}_{m,k,k'}(\bar{f}) = \{ m \} \cup G_{m,k}(\bar{f}) \cup G_{m,k'}(\bar{f}) \); we will see in Section III-B that the weighted interference of the cells indexed by \( \mathbb{H}_{m,k,k'}(\bar{f}) \) are examined by Local-Coord if \( a_m \) switches from channel \( k \) to \( k' \).

We define \( \Psi_m \) as the set of the indices of cells that interfere with \( Z_m \) or the cells indexed by \( G_m \), i.e., \( i \in \Psi_m \) if and only if there exists \( j \in \{ m \} \cup G_m \) such that \( Z_i \) interferes with \( Z_j \). The notion of \( \Psi_m \) will be used for describing a parallel protocol in Section III-B. Suppose we are given the locations of all controlled APs and possible locations of clients; then the sets of \( G_m \) and \( \Psi_m \) can be pre-computed and pre-configured in the controlled APs or a central network controller that communicates with the controlled APs, using radio propagation prediction models as described in [3]–[6], [14], [18].

C. Assumptions for Site-Specific Knowledge-Based Algorithms

We assume that the central network controller periodically (say every 5 minutes) requires the APs to stop transmitting for a short duration of time (say, one second). In this duration, APs take turns in requiring all users associated with them to perform measurements of background interference, which refers to both the noise floor and rogue interference from RF devices outside the controlled network. Note that each user needs to measure the background interference for all available frequency channels. The users then feedback to APs these measured background interference. Site specific knowledge along with measurements of background interference make the estimations of SINR at users or APs much more accurate.

We assume perfect site specific knowledge; in other words, we assume that the actual path loss between any transmitter and receiver can be correctly predicted by the site specific knowledge. A study of the effect of imperfect predictions of channel gains is an ongoing and future work.

III. THREE MEASUREMENT-BASED ALGORITHMS

The three proposed algorithms all have an iterative nature. At each point in time (presumably, randomly chosen, or determined at runtime), say every 1, 2, or 5 minutes, one iteration of channel switching takes place where one or more APs switch their frequency channels according to mechanisms that are specific to each algorithm, while other APs stay on their current channels. The channel switching time in hardware is several milliseconds and is thus negligible as compared to the interval between two iterations. APs and clients measure and average their in-situ interference between successive iterations. Iterations keep taking place on different AP(s) until the channel allocations converge; of course, when users move or the propagation environment changes, the algorithms will switch channels again until the frequency allocations reach another convergence point. Below we describe the different conditions under which each of the algorithms may switch a representative AP \( a_m \)'s channel from \( k = f_m \) to \( k = f_m' \). Throughout this paper, \( \bar{f} \in \mathbb{K} \) denotes a vector of channels selected by APs after the representative AP \( a_m \) moves from channel \( f_m \) to \( f_m' \). Hence \( \bar{f}' \) differs from \( \bar{f} \) in only the \( m \)-th element.

A. The No-Coord Algorithm

A representative AP \( a_m \) switches from its current channel \( f_m \) to \( f_m' \) only if the weighted interference on the new channel \( f_m' \) is lower, i.e., the following No-Coord condition holds:

\[
W_{f_m}(\bar{f}) > W_{f_m'}(\bar{f}).
\]

This algorithm is denoted No-Coord, because \( a_m \) makes a greedy channel selection without coordination with other APs.

B. The Local-Coord Algorithm

If \( a_m \) switches from channel \( k \) to \( k' \), only \( Z_m \) and the cells indexed by \( G_{m,k}(\bar{f}) \) and \( G_{m,k'}(\bar{f}) \) see changes in their weighted interference. AP \( a_m \) switches from channel \( k \) to \( k' \) if the max weighted interference seen by these cells decreases after the channel switching, i.e., the following Local-Coord condition holds:

\[
\max_{i \in \mathbb{H}_{m,k,k'}(\bar{f})} W_{f_i}(\bar{f}) > \max_{i \in \mathbb{H}_{m,k,k'}(\bar{f})} W_{f_i}(\bar{f}').
\]

where \( \mathbb{H}_{m,k,k'}(\bar{f}) \) has been defined in Section II-B. This algorithm is denoted Local-Coord, since \( a_m \) needs to locally coordinate with the APs indexed by \( G_{m,k}(\bar{f}) \) and \( G_{m,k'}(\bar{f}) \) via a wired backbone network for the channel switching.

For example, Fig. 1 depicts the cells that see changes in weighted interference before and after AP-1 switches its channel.
(a) Suppose a timer triggers \(a_m\) to consider initiating a channel switching. Then \(a_m\) will do the following procedure.

1. if \(\psi_m = 0\) then
2. Phase 1: Set \(\psi_m = -1\) and send requests to lock all APs indexed by \(\mathcal{V}_m\), i.e., \(\{a_n : n \in \mathcal{V}_m\}\).
3. Phase 2: Wait for replies from \(\{a_n : n \in \mathcal{V}_m\}\).
4. if If the replies indicate that \(\{a_n : n \in \mathcal{V}_m\}\) were all successfully locked by \(a_m\) then
5. \(a_m\) switches its channel from \(k\) to \(k'\), and stays at \(k'\) if (5) is satisfied; otherwise, \(a_m\) switches back to channel \(k\).
6. Send messages to unlock \(\{a_n : n \in \mathcal{V}_m\}\).
7. else
8. Send messages to unlock the APs among \(\{a_n : n \in \mathcal{V}_m\}\) that were just successfully locked by \(a_m\). (Do not need to unlock the APs that could not be locked by \(a_m\).)
9. end if
10. Set \(\psi_m = 0\).
11. end if
(b) Upon receiving a locking request from \(a_m\), \(a_n\) will do the following procedure.
1. if \(\psi_n \neq -1\) then
2. Increase \(\psi_n\) by one.
3. Reply to \(a_m\) that \(a_n\) was successfully locked by \(a_m\).
4. else
5. Reply to \(a_m\) that \(a_n\) could not be locked.
6. end if
(c) Upon receiving an unlocking request from \(a_m\), \(a_n\) will decrease \(\psi_n\) by one.

Fig. 2. A protocol for the distributed implementation of Local-Coord.

<table>
<thead>
<tr>
<th>(\psi_m)</th>
<th>Channel switching at (a_m)</th>
<th>Can (a_m) be locked?</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>(a_m) is in the process of switching its channel</td>
<td>No</td>
</tr>
<tr>
<td>0</td>
<td>(a_m) can initiate the process of channel switching</td>
<td>Yes</td>
</tr>
<tr>
<td>1 or more</td>
<td>(a_m) cannot initiate the process of channel switching</td>
<td>Yes</td>
</tr>
</tbody>
</table>

channels simultaneously if a proper inter-AP protocol is employed. In general, the number of APs that can simultaneously change channels grows with the number of total APs; hence, Local-Coord is scalable. Fig. 2 presents a distributed protocol implementing Local-Coord. We say an AP \(a_m\) is locked, if \(a_m\) is not allowed to switch its channel per other APs’ requests; if \(a_m\) is unlocked, \(a_m\) may switch its channel. First we suppose that each AP has an independent random timer that triggers the AP to initiate the process of switching its channel as described in Fig. 2(a). If \(a_m\) is locked, \(a_m\) will ignore this trigger and wait for the next trigger. The key idea of this protocol is described in Phases 1 and 2 in Fig. 2(a): \(a_m\) needs to lock all the APs indexed by \(\mathcal{V}_m\) (as defined in Section II-B) before \(a_m\) switches to a new channel; then \(a_m\) unlocks those APs after it possibly switches the channels. If any AP indexed by \(\mathcal{V}_m\) cannot be locked, \(a_m\) cannot switch its channel. The procedure to handle locking and unlocking requests are described in Fig. 2(b) and (c), respectively. An AP can be locked multiple times by different APs; Table I describes \(\psi_m\), which denotes the number of times that \(a_m\) was locked. Only when \(\psi_m = 0\) can \(a_m\) initiate the process of channel switching. When \(a_m\) is in the process of switching its channel (denoted by \(\psi_m = -1\)), it cannot be locked.

Deadlock is a problem that needs to be avoided in such distributed algorithm; in this context deadlock means that two or more APs that have initiated the process of switching their channels are waiting for one other, and thus none of these APs can ever finish. In the 6th and 8th steps of Fig. 2(a), \(a_m\) unlocks other APs immediately whether \(a_m\) switches its channel or not; hence, deadlock never arises in the protocol in Fig. 2 (see [17] for detailed description and proof of deadlock prevention).

C. The Global-Coord Algorithm

AP \(a_m\) will switch to a new channel only if the sum interference on the new channel is lower (after \(a_m\) switches there) than the sum interference on its current channel, i.e., the following Global-Coord condition holds:

\[
\sum_{n : f_n = k} W_n^m(\bar{f}) > \sum_{n : f'_n = k'} W_n^n(\bar{f}').
\]

This algorithm requires global coordination among APs using a central network controller that communicates with all APs, and is thus denoted Global-Coord.

D. Implementation Concerns

Note that in the descriptions of the three proposed algorithms, some terms in the weighted interference function are unknown before \(a_m\) switches to the new channel. An implementation may require \(a_m\) to switch to a new channel on a trial basis, and then require one or more cells to measure and compute their weighted interference after \(a_m\) commits to the switch. Only when all the quantities needed for the channel decisions are known can \(a_m\) decide whether switching to the new channel complies with the condition described for each algorithm. If the condition is satisfied, \(a_m\) stays on the new channel; otherwise, \(a_m\) switches back to the old channel or tries another channel. No-Coord requires the weighted interference at cell \(Z_m\), Local-Coord at cells indexed by \(\mathcal{H}_{m,k,k'}(\bar{f})\), and Global-Coord at all cells.

E. Convergence and Characterization of Convergence Points

Theorem III.1. Consider a particular realization of the locations of APs and users and a weighted interference function of the form of (1). Given any set of initial AP channel choices, the channel selection process for Local-Coord and Global-Coord converges in a finite number of steps.

Before characterizing the convergence points for No-Coord, Local-Coord, and Global-Coord, we need some definitions. A vector of frequency allocations denoted by \(\bar{f}\) is a Nash equilibrium (a concept widely used in game theory [19]), if no single cell can lower its weighted interference by unilaterally changing its channel.
Let \( \vec{u} = (u_1, \ldots, u_N) \) and \( \vec{w} = (w_1, \ldots, w_N) \) denote the non-increasing sorted versions of two arbitrary vectors \( \vec{v} = (v_1, v_2, \ldots, v_N) \) and \( \vec{v}' = (v'_1, v'_2, \ldots, v'_N) \), respectively. We say that \( \vec{v} \) lexicographically dominates \( \vec{v}' \) (or \( \vec{v} \succ \vec{v}' \)) if there exists some index \( j \), where \( N \geq j \geq 1 \) for which \( u_j > u'_j \) and \( u_i = u'_i \) for all \( i < j \). Vectors \( \vec{v} \) and \( \vec{v}' \) have the same lexicographic order if \( \vec{u} \) and \( \vec{w} \) are element-wise the same. We say \( \vec{v} \succ \vec{v}' \) if \( \vec{v} \succ \vec{v}' \) or \( \vec{v} \) and \( \vec{v}' \) have the same lexicographic order. We say that a vector of frequency allocations denoted by \( \vec{f} \) is a local lexicographic minimum with respect to a vector function \( \vartheta(\cdot) \), if for any vector of frequency allocations \( \vec{f}' \in \mathbb{K}^M \) that differs from \( \vec{f} \) in only one element, \( \vartheta(\vec{f}') \geq \vartheta(\vec{f}) \) holds true.

**Theorem III.2.** Suppose No-Coord converges to a frequency allocation \( \vec{f} \). Then, \( \vec{f} \) is a Nash equilibrium.

Note that No-Coord does not always converge, although simulation results show that No-Coord converges in most cases. Theorem III.2 is true only for the case where No-Coord converges. In order to resolve the non-convergence problem of No-Coord, one may limit the number of iterations or specify a minimum difference of weighted interference (before and after a channel switching) so that No-Coord can stop. Below we state a technical assumption useful in proving Theorem III.3.

**Assumption III.1.** Since the weighted interference in (1) takes a continuum of values, it is reasonable to assume that the weighted interference values at different cells or channels are distinct, i.e., \( \forall k, j \in \mathbb{K}, \forall m, n \in \mathbb{M} \) such that \( k \neq j \) or \( m \neq n \), we have \( W^m_k(\vec{f}) \neq W^n_j(\vec{f}) \) with probability one.

**Theorem III.3.** Suppose Local-Coord or Global-Coord converge to a frequency allocation \( \vec{f} \). Then with probability one, \( \vec{f} \) is a local lexicographic minimum with respect to the vector function \( \tilde{\alpha}(\cdot) \) as defined in (7) for Local-Coord, or with respect to \( \tilde{\beta}(\cdot) \) as defined in (8) for Global-Coord, where

\[
\tilde{\alpha}(\vec{f}) = \left( W^1_1(\vec{f}), W^2_2(\vec{f}), \ldots, W^M_M(\vec{f}) \right) \quad (7)
\]

\[
\tilde{\beta}(\vec{f}) = \left( \sum_{n: f_n = 1} W^n_1(\vec{f}), \sum_{n: f_n = 2} W^n_2(\vec{f}), \ldots, \sum_{n: f_n = K} W^n_K(\vec{f}) \right) \quad (8)
\]

IV. SITE-SPECIFIC KNOWLEDGE-BASED ALGORITHMS

A. The Site-Specific SINR (SS-S) Formulation

We shall consider optimizing a sum of utility functions for all the users’ SINR, assuming all APs are actively transmitting downlink traffic (but not uplink traffic). That is, we optimize the following problem over \( \vec{f} \in \mathbb{K}^M \), which is denoted Site Specific SINR or SS-S in the rest of this paper:

\[
\max_{\vec{f} \in \mathbb{K}^M} \left\{ \sum_{l \in L} U(\gamma_l) \right\} \quad (9)
\]

\[
\gamma_l = \frac{S_{c_l, a_m}}{P_{c_l} + \sum_{n: f_n = m, n \neq m} S_{c_l, a_n}}, \forall l \in L \quad (10)
\]

where \( a_m \) in (10) denotes the AP with which \( c_l \) is associated, \( P_{c_l} \) denotes background interference power that \( c_l \) measures (as described in Section II-C), \( \gamma_l \) denotes the SINR at user \( c_l \) as shown in (10), \( S_{c_l, a_m} \) denotes the average received signal power from \( a_m \) to \( c_l \). Note that the objective in (9) is not optimizing ‘sum SINR’, since such an objective may favor users that are closer to APs and may cause users which are further away to suffer low SINR. A fair SINR distribution can be achieved if we optimize the sum of utility functions in (9), where the utility function \( U(\cdot) \) in (9) can be any function that is concave, continuously differentiable, and strictly increasing. For example, Mo and Walrand have proposed a class of utility functions that capture different degrees of fairness parameterized by \( q \) [20]:

\[
U(\gamma_l) = \left\{ \begin{array}{ll}
(1 - q)^{-1} \gamma_l^{(1-q)}, & \text{if } q \neq 1 \\
\log \gamma_l, & \text{if } q = 1, \gamma_l \in (0, \infty).
\end{array} \right.
\]

This family of utility functions is concave, continuously differentiable, and strictly increasing. Intuitively, as \( q \) increases, the degree of fairness increases, but the sum SINR decreases. A trade-off between sum SINR and fairness of the individual SINR of users that are further away from a serving AP can be observed. By increasing the degree of fairness, we imply that users that are further from APs have higher SINR (which is needed to provide high throughput to distant users). The work in [20] shows that if \( q \to \infty \), the formulation in (9) becomes a special case that achieves max-min fairness. At max-min fairness, the degree of fairness is the highest; however, the sum SINR is the lowest. Simulation results in Section V show that \( q = 2 \) may be a good parameter to capture this trade-off, but this remains a topic for further research. Note that the general form of the weighted interference function in (1) for our measurement-based algorithms could also incorporate utility functions such as the ones in (11).

As described in Section II, we assume that the APs and/or the users in the controlled network periodically measure the background interference; hence, \( P_{c_l} \) in (10) is known. We assume that the central network controller has site specific knowledge and the locations of all APs and users, can predict signal power for any pair of AP and user, and can compute \( S_{c_l, a_n} \) for all \( c_l, a_n \) in the denominator of (10). Thus all the quantities in the optimization problem in (9) are known, yet measurement-based algorithms (such as the ones presented in Section III) do not know each individual component of \( S_{c_l, a_n} \) in (10) and thus are not able to solve (9) directly. Because the optimization in (9) is a combinatorial problem, there is no fast algorithm (polynomial-time) that can solve (9). Therefore, we propose an efficient heuristic in Section IV-C that can find the locally optimal solution of (9); simulations show that the algorithm in IV-C outperforms the measurement-based algorithms in Section III as well as all other frequency allocation algorithms in [8]–[12].

B. The Site-Specific Rate (SS-R) Formulation

The formulation in (9) in Section IV-A strives to provision fair SINR across users. From the users’ perspective, however, throughput may be a better metric than SINR for users’ performance. Below we formulate another problem that aims at provisioning fair throughput across users, and this formulation
may be denoted Site Specific Rate or SS-R.

\[
\max_{f \in \mathbb{C}^M} \left\{ \sum_{l \in L} U(\chi_l) \mid \chi_l = \frac{r_l(\gamma_l)}{T_m} \right\}, \tag{12}
\]

\[
\gamma_l = \frac{S_{c_l, a_m}}{P_{c_l} + \sum_{a_f = f, a_r \neq m} S_{c_l, a_r}}, \forall l \in L \right) \tag{13}
\]

where \( L, m \) denote the number of clients that are associated with \( a_m \), \( \chi_l \) denotes the throughput of \( c_l \) from \( a_m \) (\( c_l \) is associated with \( a_m \)), \( r_l(\gamma_l) \) denotes the long-term average data rate that \( c_l \) receives from \( a_m \) if \( c_l \) is the only user associated with \( a_m \); \( r_l \) depends on the SINR seen at user \( c_l \), i.e., \( \gamma_l \), as defined in (10). \( r_l(\gamma_l) \) may also be viewed as the achievable capacity between \( c_l \) and \( a_m \). We assume that the AP \( a_m \) evenly distributes its resource (e.g., time) amongst its \( L_m \) users and therefore has the denominator in (12). There are several ways to model \( r_l(\gamma_l) \); for example, we may use Shannon capacity

\[
r_l(\gamma_l) = \log_2 (1 + \gamma_l) \tag{14}
\]

or an empirical model, e.g., such as introduced in [6], [18] to relate throughput to SINR:

\[
r_l(\gamma_l) = T_{\text{max}} \left( 1 - e^{-A_c(\gamma_l - \gamma_0)} \right), \tag{15}
\]

where the three constants \( T_{\text{max}}, A_c, \) and \( \gamma_0 \) denote peak throughput, slope of throughput variation, and the cutoff SINR, respectively, as described in [6]. Note that the model in (15) captures the downlink throughput of a client \( c_l \) when all other clients associated with the same AP are idle, and the received SINR of this client \( c_l \) is \( \gamma_l \). In our simulation, we use a time division multiplexing (TDM) model for medium access. Hence, at any point of time, an AP is sending data to only one client, and the SINR at this client can be computed by considering interference from all other APs on the same channel. Hence, the model in (15) is valid, as long as we multiply the throughput in (15) by the time fraction that the AP allocates to Client \( c_l \). It is known that IEEE WLAN operates under a carrier sense multiple access / collision avoidance (CSMA/CA) algorithm, and the throughput of a WLAN network is actually more complicated than the simple equation in (15); however, the works in [6], [18] have shown that the empirical model in (15) can serve as a first-order approximation of WLAN throughputs with high accuracy.

C. A Local Optimization Algorithm for SS-S and SS-R

The optimization problems in (9) and (12) are combinatorial; solving them exhaustively requires exponential computation time (exponential in the number of APs). Hence, we present an iterative local optimization procedure that yields rapid and nearly-optimal solutions of (9); the same procedure can also solve (12). At the beginning of each iteration, a frequency allocation \( \tilde{f} \) is given, and at the end of the iteration, we find a better frequency allocation \( \tilde{g} \) that improves the objective in (9); \( \tilde{f} \) and \( \tilde{g} \) may differ in several elements, which means that the channels of several APs may change. During each iteration, we do the following steps. First, we select an AP, say \( a_m \). We find \( V - 1 \) other APs that produce the strongest interference on \( a_m \), assuming these \( V - 1 \) other APs and \( a_m \) are on the same channel; for example, \( V = 7 \) implies that we find 6 other APs that are in the vicinity of \( a_m \) so that they will likely interfere with \( a_m \’s \) clients the most. We try all possible \( K^V \) permutations of channels for these \( V \) APs, while fixing the channels at the other \( M - V \) APs. We can find the best out of the \( K^V \) permutations so that (9) is maximized and is strictly larger than the value before this iteration; then we change the corresponding \( V \) elements in \( \tilde{f} \) and thus form \( \tilde{g} \). If these \( V \) APs have operated on the best channel allocation before this iteration, we have \( \tilde{f} = \tilde{g} \); in this case, another AP (instead of \( a_m \)) and its \( V - 1 \) neighboring APs will be selected to restart this iteration. This iterative algorithm runs until every set of \( V \) neighboring APs reaches the best frequency allocation. This iterative algorithm converges in a finite number of steps, since the number of channel permutations is finite, and each iteration strictly increases the objective in (9). In practice, one may limit the number of iterations or specify a minimum difference of weighted interference so that the iterations can be finished in a reasonable amount of time (say 1, 5, or 30 seconds). We expect that the channel allocation found by this local optimization algorithm will be close to the optimum if \( V \) is large enough, since the exhaustive search can explore more possible allocations with a larger \( V \). Nevertheless, the simulations in Section V shows that this local optimization algorithm with \( V = 7 \) outperforms all other algorithms [8]–[12].

The algorithm proposed above solves the SS-S formulation in (9) and the SS-R in (12). When it is applied to solve SS-S, we refer to the algorithm as the SS-S algorithm; similarly, when the algorithm is used to maximize throughput (rate), we refer to it as the SS-R algorithm.

V. SIMULATION SETUP AND RESULTS

We shall begin by describing our simulation setup in Section V-A. Then, in Section V-B we present and discuss our simulation results.

A. Simulation Setup

No-Coord, Local-Coord, and Global-Coord, along with the user-based weighted interference function in (2) and the AP-based in (3), yield six algorithms to be named: No-U, Lo-U, Gl-U, No-A, Lo-A, and Gl-A. The algorithm in [11], denoted as CF, has been shown to outperform [8]–[10]. Hence, we compare our proposed algorithms (the six combinations above as well as SS-S and SS-R) against CF and the algorithm in [12], which is denoted LC. We set the number of orthogonal channels \( K \) to 3 to represent 802.11b/g; other larger values of \( K \) produce very similar trends as to those shown in Figs. 3-5, making our approach applicable to cellular networks and 802.11a. We assume each AP can source a maximum of 54 Mbps per the 802.11g standard. We consider three network sizes, three levels of rogue interference, and two network topologies, and thus have 18 combinations \( (3 \times 3 \times 2) \), as shown in the x-axis of Fig. 4. For each of the 18 combinations, we randomly generate 10 independent cases and compute the average. The three network sizes include a 4-by-4 AP layout.
Fig. 3. *User throughput (in Mbps)* comparison in a setting with APs on a *uniform* 10-by-10 layout, 400 users, and 10 rogue RF interferers. Only the 200 users with lower throughputs are shown.

Fig. 4. Percent of users that have throughputs higher than 512 kbps. The x-axis represents the layout of controlled APs and the percentage of rogue APs compared to the controlled APs. *Nonuniform* and *uniform* AP layouts are denoted 'nu' and 'u', respectively.

Fig. 5. 50 and 25 percentiles of users’ throughputs (50P and 25P) respectively, including both downlink and uplink traffic, for 400 users on a 10-by-10 uniform AP layout with 10 rogues. Vertical arrows and numbers besides the arrows depict the gains of *SS-S* over *LC* in percents (some arrows are omitted when the associated gains are relatively small).

Fig. 6. Frequency allocation examples for 49 APs on a 7-by-7 *nonuniform* or *uniform* topology. Three kinds of objects (squares, stars, and circles) signify three orthogonal frequency channels. Filled back objects denote 49 APs; hollow objects denote 196 users; double-layered objects with inner part filled with black denote 20 rogues. The units of X and Y axes are meters.

with 64 users, a 7-by-7 layout with 196 users, and a 10-by-10 layout with 400 users. Each AP may be associated with a different number of users; the average number of users for each AP is four for all three network sizes. We consider low, medium, and high interference from rogue interferers, where the ratio of the number of rogue interferers to the number of APs is 10%, 40%, and 70%, respectively. We consider a *uniform* topology where APs are regularly located on corners of hexagons with less than 5 meters of random perturbation, as illustrated in Fig. 6(a), and a *nonuniform* topology, where APs are perturbed from the uniform layout by a random distance (up to 25% of separation), as shown in Fig. 6(b). The separation between adjacent APs is 240 meters. The path loss exponent is set to be 3. We set a constant transmit power of 10 mW for every AP and client. The noise floor is set
to be 10 dB above the thermal noise to properly represent the RF environment [21]; the thermal noise is modeled as $kT_0B$, where $k$ is Boltzmann’s constant ($k = 1.38 \times 10^{-23}$ Joules/Kelvin), $T_0$ is ambient room temperature (typically taken as 300 K), and $B$ is the equivalent bandwidth of the measuring device ($B = 30$ MHz for the bandwidth of IEEE 802.11b/g systems). We set the fairness parameter $q$ as 2.

B. Simulation Results and Discussions for Two Kinds of Traffic Scenarios

We consider results for two traffic scenarios. The first, in Section V-B1 we present results for saturated downlink networks, i.e., all APs are transmitting downlink traffic. Then, in Section V-B2 we present results for networks with both uplink and downlink traffic.

1) Downlink Only: Fig. 3 shows the users’ throughputs (in ascending order) resulting from different algorithms for 100 controlled APs and 400 users with 10 rogues. Note that the algorithms yield very different throughputs for the first 200 users with lower throughputs but less different for the last 200 users; hence, we emphasize the first 200 users in Fig. 3; more detailed results can be found in [17]. Other scenarios with different numbers of APs, users, and rogue interferers yield similar throughput trends as in Fig. 3, and are therefore omitted for the sake of brevity. Our proposed algorithms outperform the best algorithms in the literature, i.e., LC and CF; particularly, the site-specific knowledge-based algorithms have the best performance. Fig. 3 shows that SS-S and SS-R outperform LC by 16.8% and 13.1% in terms of mean user throughput, 18.5% and 13.6% in terms of median, 97.6% and 87.1% in terms of 25-percentile, 204% and 188% in terms of 20-percentile, and 1180% and 1110% in terms of 15-percentile user throughputs. Although Lo-U achieves lower throughputs than SS-S or SS-R, yet Lo-U is the best measurement-based algorithm, especially good at uplifting throughputs for users with low throughputs. Fig. 3 shows that Lo-U outperforms LC by 12.9%, 14.3%, 81.4%, 168%, and 1010% in terms of mean throughput and 50, 25, 20, and 15 percentiles of user throughputs, respectively.

In Fig. 3, SS-R yields the highest 5 and 3-percentile throughput. Generally, SS-R sacrifices the users with higher throughput to improve the users with lower throughput. Although SS-R is worse than SS-S for users with high throughput, SS-R is still better than Lo-U, the best measurement-based algorithm. Our algorithms yield enormous throughput gains especially for users with low throughputs. Fig. 4 shows the percentage of users with throughputs more than 512 kbps in various scenarios. In Fig. 4, our algorithms enable more users to operate above 512 kbps irrespective of the number of APs and rogues; this trend is true for other throughput thresholds (other than 512 kbps), as well. Fig. 4 shows that SS-R and SS-S accommodate up to 18.8% more users than LC or CF.

2) Both Downlink and Uplink: Above we assumed all traffic was downlink and optimized the frequency allocation for the most active case where all APs are transmitting downlink traffic. It is reasonable to optimize frequency allocation for this most active case, since in this case, frequency allocation is most crucial for interference mitigation at each user. In this subsection, we examine the performance of the optimized frequency allocations in the presence of both downlink and uplink traffic. It has been shown in [22] that uplink and downlink capacities in multiple cells are mutually coupled due to inter-cell interference, and no system-level analytic model has been found to model activities of multiple APs. In our next simulation, we consider that time is slotted, and propose an approximate probabilistic model where APs independently choose one of the three possible activity states at each time slot. An AP can be transmitting downlink traffic, receiving uplink traffic, or idle, with probabilities $p_d$, $p_u$, and $p_s = 1 - p_d - p_u$, respectively. For any AP that is transmitting downlink traffic or receiving uplink traffic at a certain time slot, a user is randomly chosen (with uniform probability distribution) out of all the users associated with this AP to be the recipient or the sender of the traffic. We fix the ratio of $p_d$ to $p_u$ as 5:1 [6], and simulate 5 cases where $p_d + p_u$ (the probability that an AP is active) is 0.2, 0.4, …, 1.0, respectively. We intend to see the effect of $p_d + p_u$ on the performance of the proposed algorithms. The assumption that the activity of each AP is independent from the other APs simplifies the simulations and provides a rule-of-thumb for the performance comparison. Fig. 5 shows that our algorithms consistently yield throughput gains (including both downlink and uplink) irrespective of the probability of AP activity; particularly the gains are high (up to 71% for 25-percentile throughput and 19% for median throughput) when APs are highly active (i.e., when the network traffic load is heavy). In Fig. 5 we still see the same trend as in Fig. 3 that SS-S and SS-R have the best performance in providing high throughputs for users who suffer the lowest throughputs.

VI. Conclusions

A central network controller with site specific knowledge can predict the path loss between any AP and client, and therefore predict the impact of SINR and throughput on every AP and user when the channel of any AP is changed. This site specific knowledge leads to vast network improvements which we have demonstrated by using two site-specific algorithms that can incorporate the importance of fairness across users. Our proposed algorithms are particularly useful when the traffic load of the network is high and APs are highly active. The two algorithms, SS-S and SS-R, are better in uplifting the throughputs of users that suffer low throughputs when particular utility functions are chosen. Judicious selection of utility function is a topic of future research. We believe that site specific knowledge is also useful for other wireless communication problems in both cellular networks and WLANs, which will be validated by ongoing and future work; for example, work in [23] use site specific knowledge to perform load balancing in wireless networks.

When site specific knowledge is not available, the proposed measurement-based algorithms are good alternatives. Among the three measurement-based algorithms, Local-Coord is the best in uplifting the throughputs of users that suffer low throughputs. For Local-Coord, a scalable distributed proto-
col is given, and the convergence is proven; hence, Local-Coord is our best measurement-based algorithm for frequency allocation in wireless networks. If coordination among APs cannot be realized as required in Local-Coord, No-Coord is also a good option, since it does not require coordination among APs. Although No-Coord is not guaranteed to converge, simulations show that it converges in most cases and has comparable throughput gain as Local-Coord. We present practical approaches to implement these measurement-based algorithms.

**APPENDIX**

**Proofs of Theorems III.1-III.3**

Lemma A.1. Suppose two vectors \( \vec{v} = (v_1, v_2, \ldots, v_N) \) and \( \vec{v}' = (v'_1, v'_2, \ldots, v'_N) \) differ in at least one element. Assume all elements in \( \vec{v} \) are distinct, and so are those in \( \vec{v}' \). Let \( \Omega \) denote indices where \( \vec{v} \) and \( \vec{v}' \) differ, i.e., \( \Omega = \{ i : v_i \neq v'_i \} \). Then we have \( \vec{v} \succ \vec{v}' \) if \( \max_{i \in \Omega} v_i > \max_{i \in \Omega} v_i' \).

**Proof Sketch:** We sort the elements of \( \vec{v} \) and \( \vec{v}' \) respectively in descending order, and compare their elements one by one from the largest to the smallest. Then the first different pair of elements between the two sorted vectors is \( \max_{i \in \Omega} v_i > \max_{i \in \Omega} v_i' \). Since \( \max_{i \in \Omega} v_i > \max_{i \in \Omega} v_i' \), we have \( \vec{v} \succ \vec{v}' \) according to the definition of lexicographic order in Section III-E. A detailed proof is given in [17].

Lemma A.2. Suppose \( a_m \) is a representative AP switching its channel from \( k \) to \( k' \) according to the Local-Coord Condition in (5) or Global-Coord Condition in (6), and the channels of all the other APs remain unchanged. Then we have \( \vec{a}(\vec{f}) \succ \vec{a}(\vec{f}') \) for Local-Coord, or \( \vec{b}(\vec{f}) \succ \vec{b}(\vec{f}') \) for Global-Coord \( (\vec{a}(\vec{f}) \) defined in (7) and \( \vec{b}(\vec{f}) \) defined in (8)).

**Proof:** If \( a_m \) switches from channel \( k \) to \( k' \), only the cells indexed by \( \mathbb{H}_{m,k,k'}(\vec{f}) \) see changes in their weighted interference. Note that the \( n \)-th element of \( \vec{a}(\vec{f}) \) signifies the weighted interference of \( Z_n \). Hence, the different elements between \( \vec{a}(\vec{f}) \) and \( \vec{a}(\vec{f}') \) are those indexed by \( \mathbb{H}_{m,k,k'}(\vec{f}) \). According to Lemma A.1, it suffices to show that the maximum of these different elements in \( \vec{a}(\vec{f}) \) is greater than the maximum of those in \( \vec{a}(\vec{f}') \), i.e., \( \max_{i \in \mathbb{H}_{m,k,k'}(\vec{f})} W_i(\vec{f}) > \max_{i \in \mathbb{H}_{m,k,k'}(\vec{f})} W_i(\vec{f}') \), which is equal to the Local-Coord condition in (5). Hence, the proof for Local-Coord is done. The proof for Global-Coord is similar and is omitted for the sake of brevity (see [17] for a proof).

**Proof of Theorem III.1:** We will first prove the convergence of Local-Coord. We form a directed graph \( G \) with all possible channel vectors \( \vec{f} \) as nodes (hence the number of nodes is finite), and all channel adjustments that satisfy Local-Coord Condition in (5) as edges, assuming only one AP switches its channel at any point of time. We will show that this graph is acyclic; then since \( G \) is acyclic and finite, any initial node will converge to a sink in a finite number of steps of channel adjustments. Note that lexicographic order possesses the transitive property, that is, if \( \vec{v} \succ \vec{v}' \) and \( \vec{v}' \succ \vec{v}'' \), then \( \vec{v} \succ \vec{v}'' \) [24]. Suppose there exists a cycle on \( G \), and \( \vec{f}_1, \vec{f}_2, \ldots \) are nodes on this cycle. As we travel through this cycle once, we will see that \( \vec{a}(\vec{f}_1) \succ \vec{a}(\vec{f}_2) \succ \ldots \succ \vec{a}(\vec{f}_0) \) according to Lemma A.2. This implies \( \vec{a}(\vec{f}_0) \succ \vec{a}(\vec{f}_1) \) according to the transitive property, which is a contradiction since \( \vec{a}(\vec{f}_0) \) does not lexicographically dominate itself. Therefore \( G \) is acyclic, and the proof is done. The proof of Global-Coord is the same as the above proof, except that the edges of \( G \) are the channel adjustments satisfying the Global-Coord Condition in (6), and \( \vec{a}(\vec{f}) \) is replaced with \( \vec{b}(\vec{f}) \).

**Proof of Theorem III.2:** Suppose No-Coord converges at a frequency allocation \( \vec{f} \), then \( \vec{f} \) is not a Nash equilibrium. Then there exists at least one AP, say \( a_m \), and one channel \( f'_m \) (\( f'_m \neq f_m \)) so that \( a_m \) can switch from its current channel \( f_m \) to \( f'_m \) to strictly decrease the weighted interference of \( Z_m \). Then, the frequency allocation has not converged, since \( a_m \) can switch to channel \( f'_m \) according to No-Coord condition in (4). This proof is done by contradiction.

**Proof of Theorem III.3:** Recall from the proof of Lemma A.2 that \( \vec{a}(\vec{f}) \) differs from \( \vec{a}(\vec{f}') \) only in the elements indexed by \( \mathbb{H}_{m,k,k'}(\vec{f}) \). In order to prove that \( \vec{a}(\vec{f}) \succ \vec{a}(\vec{f}') \) holds with probability one, it suffices to show that

\[
\max_{i \in \mathbb{H}_{m,k,k'}(\vec{f})} W_i(\vec{f}) = \max_{i \in \mathbb{H}_{m,k,k'}(\vec{f})} W_i(\vec{f}')
\]

holds with probability one, according to Lemma A.1. Since Local-Coord converges at \( \vec{f} \), no AP can move to a new channel so that Local-Coord condition in (5) is satisfied. Hence, for every AP \( a_m \) (say it is currently on channel \( k \)) and every new channel \( k' \) (\( k' \neq k \)), the converse of (5) holds. The inequality in the converse of (5) holds with probability one according to Assumption III.1, and is the same as (16); thus, the proof is done. The proof for Global-Coord is similar and is omitted for the sake of brevity (see [17] for the proof).

**References**


Jeremy K. Chen (S’03-M’07) received his B.S.E.E. from National Taiwan University in 2001 and his M.S. and Ph.D. in electrical and computer engineering from The University of Texas at Austin, in 2004 and 2007 respectively. He is currently with Qualcomm. He was with Qualcomm Flarion Technologies from 2007 to 2008. He was a research assistant in the Wireless Networking and Communications Group (WNCG) at UT Austin during 2003 - 2007. He has extensive experiences in networks, wireless communications, algorithms, and software engineering. He held summer internships with Wireless Valley Communications (now Motorola) in 2004 and Intumit Technology in 2000. During 1998 and 2001, he was an undergraduate research assistant at Academia Sinica, Taipei, Taiwan, where he co-developed a cross-platform Chinese-input software package called Chewin, which won the Taiwan Free Software Community Awards in 2003 and is now widely used in the Chinese community. He has won the Asian Championship and world final’s 10th place (out of 1,400) in the International Collegiate Programming Contest held by the ACM in 1999. He received the Silver Medal in the International Olympics in Informatics in 1997. He ranked the first place in the Annual Joint College Entrance Exam in Republic of China (Taiwan) in 1997.

Gustavo de Veciana (S’88-M’94-SM’01-F’09) received his B.S., M.S., and Ph.D. in electrical engineering from the University of California at Berkeley in 1987, 1990, and 1993 respectively. He is currently a Professor at the Department of Electrical and Computer Engineering at the University of Texas at Austin. He served as the Associate Director and then Director of the Wireless Networking and Communications Group (WNCG) 2004-2008. His research focuses on the design, analysis and control of telecommunication networks. Current interests include: measurement, modeling and performance evaluation; wireless and sensor networks; architectures and algorithms to design reliable computing and network systems. Dr. de Veciana has served as editor for the IEEE/ACM Transactions on Networking, and as co-chair of ACM CoNEXT 2008. He is the recipient of General Motors Foundation Centennial Fellowship in Electrical Engineering, an NSF Foundation CAREER Award 1996, co-recipient of the IEEE William McCalla Best ICCAD Paper Award 2000, and co-recipient of the Best Paper in ACM Transactions on Design Automation of Electronic Systems, 2002-2004.

Theodore S. Rappaport (S’83-M’84-SM’91-F’98) is the William and Bettye Nowlin Chair in Engineering at the University of Texas at Austin and is the founding director of the Wireless Networking and Communications Group (WNCG) (http://www.wnscg.org/) at the university’s Austin campus, a center he founded in 2002. Prior to joining UT Austin, he was on the electrical and computer engineering faculty of Virginia Tech where he founded one of the world’s first university research and teaching centers dedicated to the wireless communications field. Prof. Rappaport has been a pioneer in the fields of radio wave propagation and wireless communication system design, and his work has influenced many international wireless standard bodies. He is one of the world’s most highly cited authors in the wireless field, having authored or co-authored over 200 technical papers, 100 US and international patents, and several books. In 2006, Rappaport was elected to serve on the Board of Governors of the IEEE Communications Society (ComSoc), and was elected to the Board of Governors of the IEEE Vehicular Technology Society (VTS) in 2008. In 1999, his pioneering work on site-specific RF propagation and system design received the IEEE Communications Society Stephen O. Rice Prize Paper Award. In 1989, he founded a cellular radio/PCS software radio manufacturer that he sold in 1993 to what is now CommScope, Inc (NYSE: CTN). In 1995, he founded Wireless Valley Communications Inc., a site-specific wireless network design and management firm that he sold in 2005 to Motorola, Inc. (NYSE: MOT). Rappaport has testified before the US Congress, has served as an international consultant for the ITU, has consulted for over 30 major telecommunications firms, and works on many national committees pertaining to communications research and technology policy. He is a highly sought-after consultant and technical expert. When he is not working or teaching, he enjoys singing, marathon training, amateur radio (N9NB), and traveling. He received B.S., M.S., and Ph.D. degrees in electrical engineering from Purdue University in 1982, 1984, and 1987 respectively.

Rappaport and his wife have taught classes at the University of Texas through the Osher Lifelong Learning Institute, and are frequently sought to talk on various aspects of wireless communications to a variety of audiences.