On Temporal Variations in Mobile User SNR with Applications to Perceived QoS

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***Abstract*—This paper proposes a stochastic geometry framework to study the temporal performance variations experienced by a mobile user in a cellular network. The focus is on the variations of the Signal to Noise Ratio (SNR) and the downlink Shannon rate experienced when the user moves across Poisson cellular network of the Euclidean plane. The motion is the simplest possible i.e., a user moving at a constant velocity on a straight line. The level crossings of the associated SNR process are shown to form an alternating-renewal process. The two distributions characterizing this process are derived in closed form. The theory of rare events provides simplified expressions for the law of this process for extremes (very large and very small) SNR thresholds. The framework is then leveraged to predict the quality of service experienced by mobile users in two concrete scenarios: that of streaming a video on the downlink and partially buffered on the hand-set to prevent video freezing; and that of downloading a large file, where the main question is the download delay. Finally, discrete event simulation is used to assess practical use of this model on its robustness to perturbations that cannot presently be taken into account in the analysis.**

I. Introduction

The primary goal of this paper is to model and study the temporal capacity variations experienced by wireless users moving through space. These are driven by variation in their spatial and environmental relationships (associations) to the infrastructure, as well as random fluctuations intrinsic to wireless channels, i.e., fast fading, which occur quickly even for stationary users. Our focus on temporal variability due to spatial heterogeneity should be contrasted with the extensive work characterizing spatial variability as seen by randomly located users, e.g., through metrics such as coverage probability, spatial density of throughput, 90% quantile rate, "edge" capacity, spectral efficiency, etc. There are several reasons why devoting some effort to studying the *temporal* variations experienced by a mobile user is of increasing interest.

First, the Quality of Service (QoS) seen by such users is critically dependent on the capacity variations they experience as they move through space. This is particularly true for applications that operate over longer time scales at which the mobile is traveling, e.g., video/audio streaming, navigation/augmented reality, transfers of files for which the capacity variability may determine the user's quality of experience.

Second, an increasing volume of data traffic is being generated by wireless devices [1], and perhaps more interestingly a substantial fraction of this traffic, e.g., 20-30% of cellular data, is being generated during hours most commonly associated with commuting, i.e., by devices most likely on the move [2]. In the future, if driverless cars become prevalent, the amount of traffic associated with moving devices may grow substantially. Hence the importance of understanding the variations alluded to above.

Third, current trends towards heterogeneous network densification leverage opportunistic access to technologies providing different coverage-throughput tradeoffs, e.g., cellular (macro, pico, femto cells), WIFI and perhaps, in the future, mmWave access points. Understanding the effectiveness of offloading or onloading across such networks, in particular for mobile users, depends on the policies and capacity variations the latter will see. Consider an audio/video streaming application to a customer on the move. One might ask what density of WIFI Access Points (APs) will suffice to serve it adequately through periodic opportunistic AP encounters, requiring few resources from the cellular network.

Related work. There is a rich literature on the modeling of the spatial capacity variability that wireless infrastructures can deliver to a typical (stationary) user. Of particular relevance to our work is the line of research based on stochastic geometry, which captures the effect on the typical user of the variability in base station locations, as well as the variability in the environment through shadowing, and in the channel through fading. For a survey on the matter, see e.g. [3]. By contrast work studying the temporal capacity variations for a user on the move is limited.

There has also been a significant related work on Delay-Tolerant Networks (DTN). This literature considers mobile nodes, where the contact duration and the inter-contact time are defined and empirically measured from real traces [4], [5] as well as through mobility models [6]. In addition to mobility induced inter-contact process, [7] considered other factors like user availability. However, this work is in the context of opportunistic ad-hoc communication networks where a set of mobile nodes are moving under different mobility patterns. Our focus is on studying the *continuous-parameter stochastic process* experienced by a tagged mobile user traversing a static pattern of nodes.

To the best of our knowledge the proposed simple level-crossing analysis for the SNR process proposed and analyzed in this paper is new and provides a first order answer to this class of questions.It can be seen as a first and necessary step towards characterizing such temporal variability for more complex random structures and in particular for the study of SINR variations, which in this paper are only explored via simulations for comparison to SNR variations. The fact that comprehensive results are already available for continuous-

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parameter processes extracted from shot noise fields (see [3], Volume 1) indicates that such extensions might be tractable. However, this will require completely different tools which are beyond what will be discussed in the present paper.

There certainly is a lot of interest in studying how to design networks to better address the needs of mobile users or to leverage user mobility for offloading/onloading traffic. For example [8] studies how user mobility patterns and users perceived QoS might drive the selection of macro-cell upgrades. The work in [9] examines the effectiveness of algorithms for optimizing offloading to a set of spatially distributed WIFI APs. The work in [10] evaluates how proactive knowledge of capacity variations could be used in designing new models for video delivery. These works exemplify applications and engineering problems which depend critically on the temporal variability mobile users would experience, but do not directly address the characteristics of such processes.

Contributions of this paper. This paper proposes a stochastic geometry framework to model the temporal performance variations experienced by a user moving at constant velocity along a straight line in a Poisson cellular network in the Euclidean plane. This model has the merit of being the simplest parametric setting, which justifies its analysis in a first step. It is clear that it can be generalized to other types of motion and to other types of base station patterns. These generalizations are left for future research. The level crossings of the associated SNR process are shown to form an alternating-renewal process when: (a) the base stations form a realization of a Poisson point process; (b) association is to the closest base station; (c) there is neither shadowing nor fading. The two distributions characterizing this process are derived in closed form using connections with queuing theory. The theory of rare events provides simplified expressions for the law of this process for extreme values of the SNR levels. An interesting observation is the verification of the "rarity hence exponentiality" principle [11], [12] within this context: when properly rescaled, the extreme events in SNR variability converge to Poisson point processes of intensity one on the real line. The framework is then leveraged to predict the QoS experienced by mobile users in two concrete scenarios: (1) a user streaming video on a wireless downlink and opportunistically buffering the video frames on the handset to prevent video freezing during "off" periods; the main question is whether there exists a transmission policy for which such video freezing can be completely avoided in steady state; (2) opportunistic downloading of large file within the WIFI offloading context described above, where the main question is the distribution of the download time resulting from the alternating on and off periods. The paper is completed by discrete event simulations which are used to assess the robustness of this model to perturbations that were not yet taken into account in the analysis. For example the relative impact of variability associated with the changing geometry (proximity of base stations) seen by a mobile vs that associated with channel variability due to channel fades.

Organization. The rest of the paper is organized as follows. We describe the basic model for cellular infrastructure in Section II. We characterize the temporal variations of the SNR seen by a mobile user as an alternating-renewal process and study its asymptotics in Section III. The two applications of our model are discussed in Section IV. The simulation results are presented in Section V and Section VI concludes the paper.

II. Basic Model for Infrastructure

Consider any general network consisting of nodes representing base stations/WIFI hotspots, modeled through their coordinates on the plane $\mathbb{R}^2$. The configuration of the nodes is assumed to be a realization of a Poisson process $\Phi = \{x_1, x_2, ..\}$ of intensity λ. We shall assume that users associate to the closest node. In this case, the region served by station x_i is a convex polygon known as the Voronoi cell with nucleus x_i constructed with respect to the set of stations Φ [3]. The collection of cells forms a tessellation of the plane called the Voronoi tessellation.

Consider a mobile user moving with uniform velocity v on a randomly selected straight line. Without loss of generality, this line can be assumed horizontal and passing through the origin. We shall assume that all nodes transmit at equal and constant power P_{tx}. We shall primarily use the classical power law path loss function, so that in the absence of fading and shadowing, the power received by the mobile user when at a distance d from the closest node is

$$P_{rx} = P_{tx} d^{-\beta} \quad \text{for} \quad \beta > 2, \tag{1}$$

and the Signal-to-Noise Ratio (SNR) is given by

$$\text{SNR} = \frac{P_{tx} d^{-\beta}}{W}, \tag{2}$$

where W denotes the noise power. However many parts of the analysis extend to arbitrary monotonic path loss functions.

III. Characterization of SNR Level Crossing Process and its Asymptotics

Given a certain SNR threshold γ, we analyze the temporal variations of the SNR by characterizing the level crossing process. Note that the SNR seen by the mobile user exceeds γ if its distance to the closest node is less than

$$r_\gamma = \left(\frac{P_{tx}}{W\gamma}\right)^{1/\beta} \tag{3}$$

(see Eq. (2)).Therefore, the characterization of the level crossings of the SNR process is equivalent to that of the process tracking the distance of the mobile user to the closest node.

Thus, let $D_t^{(r)}$, for $r > 0$, denotes the closed disc of radius r on $\mathbb{R}^2$ centered on the mobile user's location at time t. This closed disc follows the mobile user motion along the straight line. Let $C^{(r)}(t) = \Phi \cap D_t^{(r)}$ denote the set of nodes in Φ that are inside the disc at time t and $N^{(r)}(t) = |C^{(r)}(t)|$ denote the number of nodes that are within the disc at time t. The following theorem provides a simple characterization of $(N^{(r)}(t), t \geq 0)$ which helps in characterizing the SNR level crossing process.

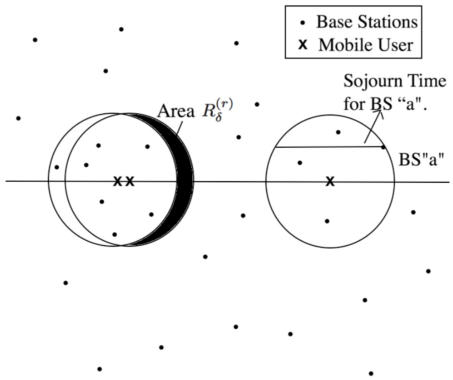

Fig. 1. Figure showing the disc around the mobile user with area $R_\delta^{(r)}$ and the chord along which the node moves

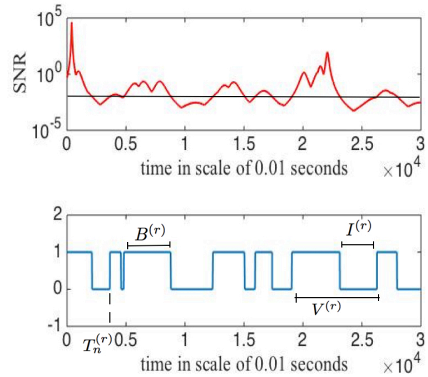

Fig. 2. Level Crossings of SNR process as On-Off Process

Theorem 1. *The process* $(N^{(r)}(t), t \geq 0)$ *can be modeled as an* $M/GI/\infty$ *queue with arrival rate* $\lambda^{(r)} = 2rv\lambda$ *and with independent and identically distributed service times, denoted by* $S^{(r)}$*, with probability density*

$$f_{S^{(r)}}(s) = \begin{cases} \frac{v^2 s}{2r\sqrt{4r^2 - v^2 s^2}}. & \text{for } s \in [0, 2r/v] \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Proof. The entry of a node into the closed disc $D_t^{(r)}$ can be viewed as an arrival to the queue. The amount of time spent by the node in $D_t^{(r)}$ is its service time and the exit from $D_t^{(r)}$ is its departure from the queue.

Now, consider the arrival process $A^{(r)}(t)$ to the queue. The probability that there is an arrival in the next δ seconds is the probability that there is a node in the area $R_\delta^{(r)} = 2rv\delta$ as depicted in Fig. 1. Since the nodes are distributed according to an homogeneous Poisson point process of intensity λ, the number of nodes in any closed set of area A follows the Poisson distribution with parameter λA. Thus, for any $\delta > 0$, the increments in the arrival process have the same distribution. Also, the numbers of nodes in any two disjoint closed sets are independent which implies that the increment over disjoint intervals are independent. Thus, the arrival process has independent stationary Poisson increments. For a small value of δ, the probability that there is a single arrival is given by $\lambda R_\delta^{(r)} + o(\delta)$. Thus, the arrival process is Poisson with rate $2rv\lambda$.

Every node that enters the moving closed ball stays in it for a time that depends on its entry location which is given by the chord shown in Fig. 1.

So, the time taken for a node to travel the chord is its service time in the queue. As the mobile moves with constant velocity, the distribution of the service times can be derived from the distribution of the chord lengths. If the location of the mobile is considered to be the origin, then the y coordinate of the location of a typical node entering the disc $D_t^{(r)}$, $Y^{(r)}$ is uniform on $[-r, r]$. The random variable $S^{(r)}$ representing the service time is given by:

$$S^{(r)} = \frac{2\sqrt{r^2 - (Y^{(r)})^2}}{v}.$$

The density of $S^{(r)}$ is given by (4), as a direct corollary of the change of variables formula, and the mean service time is $E[S^{(r)}] = \frac{\pi r}{2v}$.

Thus, the process $N^{(r)}(t)$ capturing the number of nodes in $D_t^{(r)}$ at time t follows the dynamics of the number of customers in an $M/GI/\infty$ queue with arrival rate $\lambda^{(r)} = 2rv\lambda$ and general independent service times following the distribution of $S^{(r)}$. The stationary distribution for $N^{(r)}(t)$ is thus Poisson with mean $\pi r^2 \lambda$. ∎

A. Characterization of the level crossing processes

The level crossing process can be characterized as an on-off/up-down process which alternates between successive “on” intervals $\{B_n^{(r_\gamma)}, n \geq 1\}$ and “off” intervals $\{I_n^{(r_\gamma)}, n \geq 1\}$ as depicted in Fig. 2. Let $T_n^{(r_\gamma)}, n \geq 1$ and $S_n^{(r_\gamma)}, n \geq 1$ be the sequence of up-crossing and down-crossing times respectively. Let $V^{(r_\gamma)} \sim T_n^{(r_\gamma)} - T_{n-1}^{(r_\gamma)}$ be a random variable whose distribution is that associated with inter arrivals as illustrated in Fig. 2.

Definition 1. *An alternating renewal process switches between two states, called on-off or up-down. The system alternates between successive up intervals and down intervals. If the random vectors* $\{(B_n, I_n), n \geq 1\}$ *are independent and identically distributed, then the sequence* $\{(T_n, S_n), n \geq 1\}$ *is called an alternating renewal process.*

Thus, if the pairs $\{(B_n^{(r_\gamma)}, I_n^{(r_\gamma)}), n \geq 1\}$ are i.i.d random vectors, then the associated up and down level crossings form an alternating-renewal process. Below, we will use the following theorem shown in [13] which characterizes the busy period in an $M/GI/\infty$ queue. In this theorem and below, for a given $\mathbb{R}^+$ valued random variable X, we denote by $\hat{X}$ a random variable distributed like the forward recurrence time of X which is defined by

$$P(\hat{X} > x) = \frac{1}{E[X]} \int_x^{\infty} P(X > z) dz. \quad (5)$$

Theorem 2. *(Makowski [13]) Consider an $M/GI/\infty$ queue with arrival rate λ and generic service time S. Let M denote an $\mathbb{N}$-valued random variable which is geometrically distributed according to*

$$P(M = l) = (1 - \nu)(\nu)^{l-1}, l = 1, 2, \dots \quad (6)$$

with

$$\rho = \lambda E[S] \text{ and } \nu = 1 - e^{-\rho}. \quad (7)$$

Consider the $\mathbb{R}_+$-valued random variable U distributed according to

$$P(U \le u) = \frac{1}{\nu}(1 - e^{-\rho P[\hat{S} \le u]}) \ , u \ge 0, \quad (8)$$

where, $\hat{S}$ is the forward recurrence time associated with the generic service time S. Let $\{U_n, n \ge 1\}$ be an i.i.d sequence independent of the random variable M. Let B denote a typical busy period. Then the forward recurrence time $\hat{B}$ associated with B admits the random sum representation:

$$\hat{B} =_d \sum_{i=1}^{M} U_i, \quad (9)$$

where $=_d$ denotes equality in distribution.

The following theorem characterizes the level crossing process for the SNR for some fixed threshold γ, which is equivalent to level crossing process for the distance to the closest node with threshold r_γ (3) as an alternating-renewal process and gives the distribution of the generic on and off periods.

Theorem 3. *The level crossing process for the SNR for some fixed threshold γ is an alternating-renewal process. If $B^{(r_\gamma)}$ and $I^{(r_\gamma)}$ are random variables representing the typical on and off times respectively, then the distribution of $I^{(r_\gamma)}$ is exponential with parameter $2\lambda vr$ and*

$$P(B^{(r_\gamma)} < x) = 1 - E[B^{(r_\gamma)}] f_{\hat{B}^{(r_\gamma)}}(x). \quad (10)$$

Here, $\hat{B}^{(r_\gamma)}$ is the forward recurrence time associated with the on time; it admits a random sum representation as in (9) for the $M/GI/\infty$ queue given in Theorem 2 with

$$\rho^{(r_\gamma)} = 2rv\lambda E[S^{(r_\gamma)}] = \lambda\pi r^2 \ , \ \ \nu^{(r_\gamma)} = 1 - e^{-\rho^{(r_\gamma)}} \quad (11)$$

and $S = S^{(r_\gamma)}$ defined in (4).

Proof. Given the $M/GI/\infty$ model, the on and off times of the process are characterized by the busy and idle periods of the defined queue. Thus, $V^{(r_\gamma)}$ is the random variable representing the sum of a typical busy period $B^{(r_\gamma)}$ and a typical idle period $I^{(r_\gamma)}$. Since the arrival process is Poisson, the inter arrival times are exponential with parameter $\lambda^{(r_\gamma)} = 2\lambda vr$. Because of the memoryless property, the idle periods all obey the same distribution. Also, the busy period $B^{(r_\gamma)}$ depends upon the arrivals and service times of customers arriving after the customer initiating the busy period which are independent of the past arrivals. Thus the busy period $B^{(r_\gamma)}$ and idle period $I^{(r_\gamma)}$ are independent. The successive busy and idle periods are independent. Hence, the level crossings of the distance form an alternating-renewal process.

The proof for the characterization of the forward recurrence time associated with the busy period of the $M/GI/\infty$ queue follows from Theorem 2. Then, we get (10) from (5). ■

Corollary 1. *In the stationary regime the renewal process is such that the probability that the mobile user is in an "on" period i.e., within a distance r_γ from a node is $\nu^{(r_\gamma)}$ and the probability that it is in an "off" period i.e., within a distance greater than r_γ is $1 - \nu^{(r_\gamma)}$.*

All path loss functions which are monotonic lead to an analogue of Theorem 3.

B. Asymptotics

In this subsection, we use the characterization of the distribution of time for which the SNR is above and below a given threshold γ to study the asymptotic behavior of the level crossing process when the threshold is extreme. More precisely, we analyze the asymptotic behavior of the distribution of the inter arrival time of SNR up-crossings as $\gamma \to \infty$ and as $\gamma \to 0$ which can be seen as "good" and "bad" events respectively.

This is equivalent to the characterization of the asymptotic behavior of the distribution of the random variable $V^{(r_\gamma)}$ corresponding to the up-crossings as $r_\gamma \to 0$ and as $r_\gamma \to \infty$.

Theorem 4. *For all $r_\gamma > 0$, let $T_n^{(r_\gamma)}, n \in \mathbb{Z}$, be the random variables denoting the up-crossings of the distance process for the threshold r_γ i.e., the sequence of times when the mobile user starts being within a distance r_γ from the closest node and the SNR is above γ. Then $\{T_n^{(r_\gamma)}\}$ is a renewal process. Let $V^{(r_\gamma)}$ be the typical interval of this renewal process. Let $f(r_\gamma) = 2\lambda vr_\gamma$. Then*

$$\lim_{r_\gamma \to 0} f(r_\gamma) V^{(r_\gamma)} \xrightarrow{d} \exp(1).$$

Let $g(r_\gamma) = 2\lambda vr_\gamma e^{-\lambda\pi r_\gamma^2}$. Then

$$\lim_{r_\gamma \to \infty} g(r_\gamma) V^{(r_\gamma)} \xrightarrow{d} \exp(1).$$

Proof. The proof is given in the Appendix in the extended version of the paper [14]. As we shall see in section V, these asymptotic results can infact be used for moderate values of γ for parameters typically found in wireless networks. ■

IV. Applications

This section presents direct applications of our model. The generality of the latter allows us to analytically assess some key properties of two very different wireless scenarios: that of video streaming in a both homogeneous and heterogeneous cellular networks, and that of offloading of file downloads in a WIFI network.

A. Mobile User Video Quality of Experience (QoE)

Consider a scenario where a mobile user is viewing a video streamed over a sequence of wireless downlink(s). Within the framework described in the paper, let γ be an SNR threshold, and assume that either the SNR is larger than γ and the serving base station opportunistically sends video-frames to the mobile user at the constant bit rate $\kappa = A\log(1+\gamma)$, or it is not and the serving base station sends nothing. This constant bit rate situation is that when the network does not rely on adaptive coding/decoding. For additional motivation for this scenario, see [10]. Thus, $\nu^{(r_\gamma)}$ is the probability that the alternating renewal process is in the "on" state as given in Corollary 1.

We further consider a fluid queue representing the tagged mobile user's playback buffer state. The fluid queue has frame arrival rate 0 during off periods and rate κ during on periods. Let η denote the playback rate of the video frames by the user. Hence, as long as the buffer is non-empty, the fluid depletion rate of the queue is η. The *load factor* [15] of this queue is thus given by $\rho(\gamma) = \nu^{(r_\gamma)}\kappa/\eta$. In this setting the playback buffer state is still random since the alternating on and off periods associated with arrival process is random in nature. Let us focus on rebuffering as the primary video QoE metric [10]. This is directly linked to the proportion of time the playback buffer is empty.

The first natural question one can ask is whether there is a choice of γ such that the fluid queue is unstable, i.e., ensures no rebuffering in the long term. Denote by $\rho(\gamma)$ the load factor as a function of the SNR threshold γ, i.e.,

$$\rho(\gamma) = \frac{A}{\eta}\log(1+\gamma)\left(1 - e^{\frac{-b}{\gamma^{2/\beta}}}\right), \tag{12}$$

where $b = \lambda\pi\left(\frac{P_{tx}}{W}\right)^{\frac{2}{\beta}}$. In other words, does there exist a $\gamma > 0$ such that $\rho(\gamma) > 1$? It is easy to check that the function $\rho(\gamma)$ has a unique maximum γ^* on $(0,\infty)$ which solves the equation

$$\gamma^{\frac{2}{\beta}} = \left(\frac{\gamma+1}{\gamma}\right)\frac{2b}{\beta}\log(1+\gamma)\frac{e^{-\frac{b}{\gamma^{\frac{2}{\beta}}}}}{1-e^{-\frac{b}{\gamma^{\frac{2}{\beta}}}}}.$$

So either $\rho(\gamma^*) \geq 1$ and the queue can be made unstable through a proper choice of γ, or $\rho(\gamma^*) < 1$ and the queue cannot be made unstable.

More generally, if γ is such that $\rho(\gamma) > 1$, the buffer is never empty, indicating that eventually there is no video rebuffering, whereas if $\rho(\gamma) < 1$, the proportion of time that the buffer is empty and the video is frozen is $1-\rho(\gamma)$. Also,a lower γ limit means a lower transmission rate, but a higher connection probability.

This basic model has two variants with direct engineering implications which we discuss below.

Variant 1 is that where the network supports a homogeneous Poisson process of density ξ of video streaming users. It is assumed that each user has the same behavior as the tagged user, namely moves along a straight line with uniform velocity. In addition, by the Displacement Theorem for Poisson point processes [3] it is guaranteed that users form a Poisson point process of intensity ξ at any time.

At any given time the tagged user is served at a bit rate $A\log(1+\gamma)$, it will share the base station with a random number of users denoted by N. We approximate N Poisson random variable with parameter $\xi E[V^*]$, where V^* denotes the area of the intersection of Voronoi cell of a base station with a closed ball of radius r_γ around it, conditioned on the fact that the tagged user is within a distance r_γ from the base station, which introduces an additional bias. We can compute the expectation with help of integral geometry as shown in Appendix C of [14].

Hence, due to symmetry, the rate available to the tagged user is now approximately

$$\rho(\gamma,\xi) = \frac{A\log(1+\gamma)\left(1-e^{\frac{-b}{\gamma^{2/\beta}}}\right)}{\eta}E\left[\frac{1}{N+1}\right], \tag{13}$$

with b as above and $E[1/N+1]$ calculated by numerical integration. The shapes of (12) and (13) and the optimal value of γ are illustrated in Fig. 3. Notice that for these parameters, the optimal γ increases with ξ.

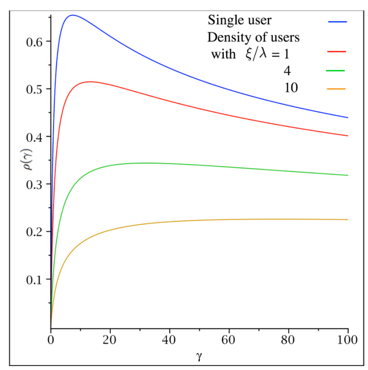

Fig. 3. Load factor of the fluid queue as a function of γ.

Variant 2 is that where the fluid queue model is replaced by its natural discrete time version, which can in particular incorporate fading. Time is slotted with the duration of a slot s equal to the coherence time of the channel, i.e., we assume i.i.d. block fading with an independent fade for each slot. Then in a time slot where the fade power is H and the distance to the closest base station is r, either $HP_{tx}/(Wr^\beta) \geq \gamma$ and the mobile receives at rate $A\log(1+\gamma)$ during s seconds, or $HP_{tx}/(Wr^\beta) < \gamma$ and it receives nothing. Within this framework, depending on the value of the fade, there are slots where the mobile is within distance $r(\gamma)$ of the closest base station and where it nevertheless does not receive frames from its serving base station, and conversely, there are slots where the mobile is not within distance $r(\gamma)$ of the closest base station and where it receives frames from this base station. The load factor of this queue is

$$\tilde{\rho}(\gamma) = \frac{A}{\eta}\log(1+\gamma)2\pi\lambda\int_{r>0} e^{-\lambda\pi r^2}G^c\left(\frac{W\gamma r^\beta}{P_{tx}}\right)dr,$$

where G^c denotes the complementary cumulative distribution function of the fades. The stability condition of this discrete time queue is $\tilde{\rho}(\gamma) < 1$. The same conclusions as above hold.

B. Wifi-Offloading

WiFi offloading helps to improve spectrum efficiency and reduce cellular network congestion. One version of this scheme is to have mobile users opportunistically obtain data through WiFi rather than through the cellular network. Offloading traffic through WiFi has been shown to be an effective way to reduce the traffic on the cellular network. WiFi is faster and typically uses less energy to transmit data when it is available.

Here, we consider a situation where a service provider deploys multiple WiFi hotspots as a Poisson point process of intensity λ to offload mobile traffic. The scenario features a mobile user moving on a straight line with uniform velocity v. This user needs to download a file from the service provider, relying on Wifi hotspots rather than on cellular base stations. The question of interest here is the time it takes to complete the download.

Assume that the mobile device connects to Wifi only if it is within a certain distance r from the hotspot. Consider again the case without adaptive coding/decoding. Then the data rate experienced by the mobile user is the constant κ defined above. In addition, the mobile user experiences an alternating on and off process, as characterized in Theorem 3.

Below, for the sake of mathematical simplicity, we assume that the file size F is exponential with parameter δ and that the mobile user starts to download the file at the beginning of an on period. Let T denote the random time taken to download the file. Consider the event $J = \{F > \kappa B^{(r)}\}$ and let

$$\alpha = P(J) = P(F > \kappa B^{(r)}) = \mathcal{L}_{B^{(r)}}(\delta\kappa). \tag{14}$$

Here and below, we let $\mathcal{L}_X(s)$ denote the Laplace transform of the non-negative random variable X at point s, and $B^{(r)}$ is the random variable representing the length of a typical on interval (see Theorem 3).

Now, define the non-negative random variables X and Y by their c.d.f.s

$$P(X < x) = \frac{1}{\alpha}\int_0^x e^{-\delta\kappa z} f_{B^{(r)}}(z)dz \tag{15}$$

$$P(Y < y) = \frac{1}{1-\alpha}\int_0^y (1 - e^{-\delta\kappa z}) f_{B^{(r)}}(z)dz. \tag{16}$$

Notice that

$$\mathcal{L}_X(s) = \frac{1}{\alpha}\mathcal{L}_{B^{(r)}}(s + \delta\kappa) \tag{17}$$

$$\mathcal{L}_Y(s) = \frac{1}{1-\alpha}\left(1 - \mathcal{L}_{B^{(r)}}(s + \delta\kappa)\right). \tag{18}$$

The following representation of the Laplace transform of T is an immediate corollary of the on-off structure:

Theorem 5. *Under the foregoing assumptions,*

$$\mathcal{L}_T(s) = \frac{(1-\alpha)\mathcal{L}_Y(s)}{1 - \alpha\mathcal{L}_X(s)\frac{2\lambda v r}{2\lambda v r + s}}. \tag{19}$$

It is remarkable that the Laplace transform of T admits a quite simple expression in terms of that of $B^{(r)}$. Other and more general file distributions can be handled as well when using classical tools of Laplace transform theory. Note that this setting also leads to interesting optimization questions.

V. Simulation Results

In this section we evaluate when our mathematical model and associated asymptotic results are valid in more realistic settings. We use simulation to study the temporal variations of the SNR process experienced by a mobile user under various scenarios which are are not captured by our analytical framework. The model is challenged in various complementary ways: e.g., by adding fading and accounting for interference from other base stations. In each case the objective is to determine for what parameter values of the additional feature our simplified mathematical model is still approximately valid, providing robust engineering rules of thumb to predict what mobile users will see.

In particular, we will answer the following questions:

- How quickly do the SNR level crossing asymptotics converge as a function of the associated thresholds?
- Under what types of fast fading are our results robust?
- Are there regimes where the temporal characteristics of the SNR process is a good proxy for the SINR process, e.g., high path loss?

We begin by introducing our simulation methodology and the default parameters used throughout this section.

A. Simulation Methodology

We consider a user moving on a straight line (road) at a fixed velocity of 16 m/s. The base stations are randomly placed according to a Poisson point process with intensity λ such that the mean coverage area per base station is that of a disc with radius 200m. Unless otherwise specified, we consider the path loss function given by Eq. (2) with exponent $\beta = 4$ and assume that all base stations transmit with equal power of $P_{tx} = 2$Watts. The signal strength received by the mobile user is recomputed every 10^{-2} seconds.

We calibrate the thermal noise power to the *cell-edge* user. Let D be a random variable denoting the distance from a typical user to the closest base station. Define d_{edge} by the relation $P(D \le d_{edge}) = 0.9$. Since in our simulation setting $P(D \le d) = 1 - \exp(\lambda\pi d^2)$, we have that $d_{edge} = \sqrt{\frac{-\ln(0.1)}{\pi\lambda}}$. If we fix the desired SNR at the cell edge to be SNR_{edge} this then determines the noise power to be $W = \frac{P_{tx} d_{edge}^{-\beta}}{\mathrm{SNR}_{edge}}$.

In the sequel we evaluate how quickly the convergence to exponential studied in Theorem 4 arises. To that end we compare the renormalized distributions obtained via simulation to the reference exponential distribution with parameter 1, using the Kolmogorov-Smirnov (K-S) test. The K-S test finds the greatest discrepancy between the observed and expected cumulative frequencies– called the "D-statistic". This is compared against the critical D-statistic for that sample size with 5% significance level. If the calculated D-statistic is less than the critical one, we conclude that the distribution is of the

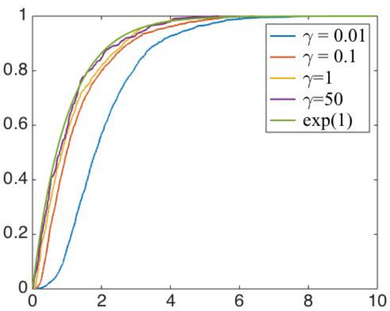

Fig. 4. CDF of interarrival of up-crossings for various thresholds

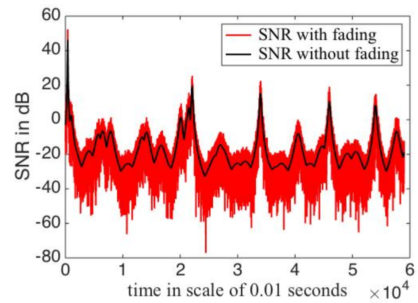

Fig. 5. SNR process in presence of fading with mean 1

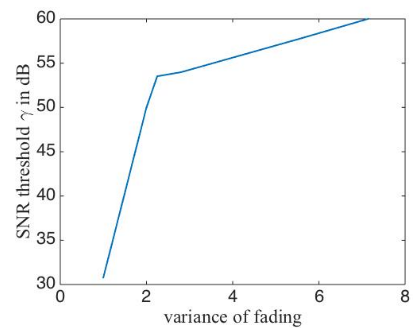

Fig. 6. Threshold above which the inter-arrival of up-crossing converge to exponential with parameter 1 for different variance of fading

expected form, see e.g. [16]. The same methodology is used for the other core studies.

B. Convergence of Level-crossing Asymptotics

Theorem 4 indicates that as the SNR threshold γ increases the rescaled distribution for up-crossings of the SNR process becomes exponential. The question is how large γ needs to be for this result to hold. To that end we simulated the level crossing process for various γ and computed the D-statistic mentioned above. The empirical CDF for up-crossing inter-arrivals rescaled by $f(r_\gamma)$ as introduced in the theorem can be seen in e.g., Fig. 4. As expected we found that as the threshold threshold increases the distribution becomes exponential, and for a threshold value of $\gamma = 50$ or more, it is exponential with unit mean.

In reality an SNR of 50 is not realistic for wireless users. However, as seen from figure 4, for moderate values of γ such as $0.1, 1$, the up-crossing inter-arrivals can be approximated by an exponential with parameter $1/f(r_\gamma)$. For $\gamma = 1$, the empirical mean for the inter-arrival time for up-crossings is $72.4649s$ and the asymptotic approximated mean i.e., $f(r_\gamma)^{-1}$ is $63.0780s$.

C. Robustness of Level-crossing Asymptotics to Channel Fading

Next we study the effect that channel fading might have on the level-crossing asymptotics. We consider channels with Rayleigh fading with unit mean, so that the SNR experienced by the tagged mobile user at a distance d from the base station is given by $HP_{tx}d^{-\beta}/W$, where H is fading random variable which is exponential with unit mean. The coherence time is set to $t_c = 0.423/f_d$, where f_d is the Doppler shift given by $f_d = \frac{v}{c} f_o$ where v is the vehicle velocity, c is the speed of lights and $f_o = 900$MHz is the operating frequency. This gives a coherence time $t_c = 0.007s$. Thus, fading (power) changes every 0.007 seconds. The SNR process with fading is illustrated in Fig. 5.

Clearly when we incorporate channel fading in the SNR process, even when one fixes a high SNR threshold, the level-crossing process will fluctuate up and down before it goes down again for some time, see Fig. 5. Thus to recover the on-off structure and asymptotics we consider a modified process defined as follows. After the first up-crossing, we suppress all subsequent up crossings (if any) for an appropriate time scale, and then look for the next up-crossing taking place after this time. We take a time scale for the suppression of up-crossings equal to twice the expected on time $2E[B^{(r)}]$ [13].

In order to vary the variance keeping the mean of the fading one, we now consider fading which is a mixture of exponentials. For this process, we would expect that for fading with mean one, if variance is small, the appropriately rescaled inter-arrival distribution for up-crossings which are not suppressed would once again asymptotically become exponential with parameter 1. In other words we expect the geometric variations associated with base station locations to dominate channel variations. Whereas, if fading variance is big, one might expect the SNR threshold required to obtain convergence to an exponential to increase. Fig. 6 exhibits such thresholds as a function of the fading variance. We also see that for fading variances exceeding 8, the channel variations dominate the geometric variations, leading to up-crossing asymptotics which no longer match Theorem 4.

D. Robustness of Level-crossing Asymptotics to the Inclusion of Interference

So far we have focused on the SNR process. One might ask to what degree the Signal-to-Interference-plus-Noise Ratio (SINR) process, shares similar characteristics.

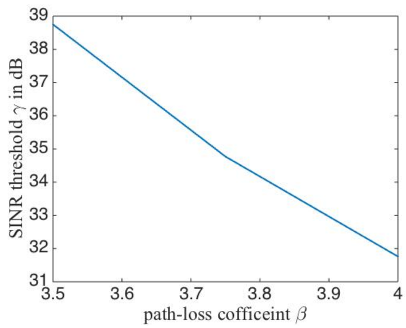

Fig. 7. Threshold above which the inter-arrival of up-crossing converge to exponential with parameter 1 for different path loss exponent

We first simulated the SINR process for a setting with a high path loss exponent of $\beta = 4$ and found once again that the rescaled distribution for the up-crossing inter-arrivals converges to an exponential with parameter 1. The test requires a threshold $\gamma = 31.76,$. However, as seen above this asymptotic is already useful for moderate values of γ. We then evaluated, for different path loss β, what threshold values were needed to obtain a similar convergence. As shown by Fig. 7, the threshold in question increases as β decreases. Further we found that for $\beta < 3.5$, we no longer have the desired convergence property. In summary, for high path-loss exponents $\beta = 3.5 - 4$, the up-crossing asymptotics for the SNR and SINR processes are similar.

VI. Conclusion

As explained in the introduction to this paper, the analysis of the time variations of the SNR or the SINR experienced by a mobile user requires the characterization of the functional distribution of a continuous parameter stochastic process constructed on a random spatial structure (e.g. the Poisson Voronoi tessellation) and is hence a challenging mathematical question. This paper addressed the simplest question of this type by focusing on the SNR process in the absence of fading. This allowed us to derive an exact representation of the level crossings of the stochastic process of interest as an alternating renewal process with a full characterization of the involved distributions and of their asymptotic extreme behavior. The simplicity and the closed form nature of this mathematical picture are probably the most important messages of the paper. We also showed by simulation that this very special case actually provides a quite good representation of what happens for much more concrete scenarios, like e.g. those with fading when the fading variance is small enough, or those based on SINR rather than SNR when the path loss exponent is large enough. This model is hence of potential practical use as is, in addition to being a first glimpse at a large field of new research questions. The most challenging questions on the mathematical side are probably (1) the understanding of the tension between the randomness coming from geometry (studied in the present paper) and that coming from propagation (only studied by simulation here): it would be nice to analytically quantify when one dominates the other. (2) the extension of the analysis to SINR processes, which are the long term aim of the present paper and which will require significantly more sophisticated mathematical tools, e.g. based on functional distributions of shot noise fields, than those used so far. On the practical side, the main future challenges are linked to the initial motivations of this work, namely in the prediction and optimization of the user quality of experience. Many scenarios refining those studied here should be considered. For instance, the stationary analysis of the fluid queue representing video streaming should be completed by a transient analysis and by a discrete time analysis. This alone opens an interesting and apparently unexplored connection between stochastic geometry and queuing theory with direct implications to wireless quality of experience.

VII. Acknowledgment

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References

[1] Cisco, "Cisco visual networking index: Global mobile data traffic forecast update, 2013–2018," *San Jose, CA*, 2014.

[2] Wandera, "Enterprise mobile data report - q3," Oct 2013. [Online]. Available: http://viroam.iroam.com/sites/all/themes/elegantica/documents/Enterprise%20Mobile%20Data%20Report%20-%20Q3.pdf

[3] F. Baccelli and B. Blaszczyszyn, *Spatial Modeling of Wireless Communications – A Stochastic Geometry Approach*. NOW Publishers, 2009.

[4] T. Karagiannis, J.-Y. Le Boudec, and M. Vojnović, "Power law and exponential decay of intercontact times between mobile devices," *Mobile Computing, IEEE Transactions on*, vol. 9, no. 10, pp. 1377–1390, 2010.

[5] V. Conan, J. Leguay, and T. Friedman, "Characterizing pairwise intercontact patterns in delay tolerant networks," in *Proceedings of the 1st international conference on Autonomic computing and communication systems*. ICST (Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering), 2007, p. 19.

[6] H. Cai and D. Y. Eun, "Toward stochastic anatomy of inter-meeting time distribution under general mobility models," in *Proceedings of the 9th ACM international symposium on Mobile ad hoc networking and computing*. ACM, 2008, pp. 273–282.

[7] C.-H. Lee, J. Kwak *et al.*, "Characterizing link connectivity for opportunistic mobile networking: Does mobility suffice?" in *INFOCOM, 2013 Proceedings IEEE*. IEEE, 2013, pp. 2076–2084.

[8] S. Mitra, S. Ranu, V. Kolar, A. Telang, A. Bhattacharya, R. Kokku, and S. Raghavan, "Trajectory aware macro-cell planning for mobile users," *arXiv preprint arXiv:1501.02918*, 2015.

[9] H. Deng and I.-H. Hou, "Online scheduling for delayed mobile offloading," in *INFOCOM, 2015 Proceedings IEEE*, April 2015.

[10] Z. Lu and G. de Veciana, "Optimizing stored video delivery for mobile networks: The value of knowing the future," in *INFOCOM, 2013 Proceedings IEEE*, April 2013, pp. 2706–2714.

[11] J. Keilson, *Markov chain model-rarity and exponentiality*. Springer Science & Business Media, 2012, vol. 28.

[12] D. Aldous", *"Probability Approximations via the Poisson Clumping Heuristic"*. Springer-Verlag, 1989.

[13] A. M. Makowski, "On a random sum formula for the busy period of the M/G/Infinity queue with applications," DTIC Document, Tech. Rep., 2001. [Online]. Available: http://www.researchgate.net/publication/235086381_On_a_Random_Sum_Formula_for_the_Busy_Period_of_the_MGInfinity_Queue_With_Applications

[14] P. Madadi, F. Baccelli, and G. de Veciana, "On temporal variations in mobile user SNR with applications to perceived QoS (to appear on arXiv)." [Online]. Available: https://www.dropbox.com/s/2ik7vnc0w3kelr8/ext_ver.pdf?dl=0

[15] F. Baccelli and P. Bremaud, *Palm probabilities and Stationary Queues*. Springer Verlag, March 1987.

[16] E. W. Weisstein, "Kolmogorov-Smirnov test," *MathWorld–A Wolfram Web Resource*. [Online]. Available: http://mathworld.wolfram.com/Kolmogorov-SmirnovTest.html