

## *Table of Fourier Transform Pairs*

<b>Function, f(t)</b>	<b>Fourier Transform, F(ω)</b>
<i>Definition of Inverse Fourier Transform</i> $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$	<i>Definition of Fourier Transform</i> $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$
$f(t - t_0)$	$F(\omega) e^{-j\omega t_0}$
$f(t) e^{j\omega_0 t}$	$F(\omega - \omega_0)$
$f(\alpha t)$	$\frac{1}{ \alpha } F\left(\frac{\omega}{\alpha}\right)$
$F(t)$	$2\pi f(-\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$\int_{-\infty}^t f(\tau) d\tau$	$\frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$
$\text{sgn}(t)$	$\frac{2}{j\omega}$

$j \frac{1}{\pi t}$	$\text{sgn}(\omega)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$	$2\pi \sum_{n=-\infty}^{\infty} F_n \delta(\omega - n\omega_0)$
$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{Sa}\left(\frac{\omega\tau}{2}\right)$
$\frac{B}{2\pi} \text{Sa}\left(\frac{Bt}{2}\right)$	$\text{rect}\left(\frac{\omega}{B}\right)$
$\text{tri}(t)$	$\text{Sa}^2\left(\frac{\omega}{2}\right)$
$A \cos\left(\frac{\pi t}{2\tau}\right) \text{rect}\left(\frac{t}{2\tau}\right)$	$\frac{A\pi}{\tau} \frac{\cos(\omega\tau)}{(\pi/2\tau)^2 - \omega^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$u(t) \cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
$u(t) \sin(\omega_0 t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega^2}{\omega_0^2 - \omega^2}$
$u(t)e^{-\alpha t} \cos(\omega_0 t)$	$\frac{(\alpha + j\omega)}{\omega_0^2 + (\alpha + j\omega)^2}$

$u(t)e^{-\alpha t} \sin(\omega_0 t)$	$\frac{\omega_0}{\omega_0^2 + (\alpha + j\omega)^2}$
$e^{-\alpha t }$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$
$u(t)e^{-\alpha t}$	$\frac{1}{\alpha + j\omega}$
$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha + j\omega)^2}$

➤ **Trigonometric Fourier Series**

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\omega_0 n t) + b_n \sin(\omega_0 n t))$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt, \quad a_n = \frac{2}{T} \int_0^T f(t) \cos(\omega_0 n t) dt, \quad \text{and}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(\omega_0 n t) dt$$

➤ **Complex Exponential Fourier Series**

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_0 n t}, \quad \text{where} \quad F_n = \frac{1}{T} \int_0^T f(t) e^{-j\omega_0 n t} dt$$

### *Some Useful Mathematical Relationships*

$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$
$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$
$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$2\cos^2(x) = 1 + \cos(2x)$
$2\sin^2(x) = 1 - \cos(2x)$
$\cos^2(x) + \sin^2(x) = 1$
$2\cos(x)\cos(y) = \cos(x - y) + \cos(x + y)$
$2\sin(x)\sin(y) = \cos(x - y) - \cos(x + y)$
$2\sin(x)\cos(y) = \sin(x - y) + \sin(x + y)$

### *Useful Integrals*

$\int \cos(x) dx$	$\sin(x)$
$\int \sin(x) dx$	$-\cos(x)$
$\int x \cos(x) dx$	$\cos(x) + x \sin(x)$
$\int x \sin(x) dx$	$\sin(x) - x \cos(x)$
$\int x^2 \cos(x) dx$	$2x \cos(x) + (x^2 - 2) \sin(x)$
$\int x^2 \sin(x) dx$	$2x \sin(x) - (x^2 - 2) \cos(x)$
$\int e^{\alpha x} dx$	$\frac{e^{\alpha x}}{\alpha}$
$\int x e^{\alpha x} dx$	$e^{\alpha x} \left[ \frac{x}{\alpha} - \frac{1}{\alpha^2} \right]$
$\int x^2 e^{\alpha x} dx$	$e^{\alpha x} \left[ \frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right]$
$\int \frac{dx}{\alpha + \beta x}$	$\frac{1}{\beta} \ln \alpha + \beta x $
$\int \frac{dx}{\alpha^2 + \beta^2 x^2}$	$\frac{1}{\alpha\beta} \tan^{-1}\left(\frac{\beta x}{\alpha}\right)$

# FOURIER TRANSFORM PROPERTIES

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## Abstract

A table of commonly seen transforms, for reference.

## Fourier Transform Properties

### Short Table of Fourier Transform Pairs

$\mathbf{s(t)}$	$\mathbf{S(f)}$
$e^{-(at)}u(t)$	$\frac{1}{i2\pi f+a}$
$e^{(-a) t }$	$\frac{2a}{4\pi^2 f^2+a^2}$
$p(t) = \begin{cases} 1 & \text{if }  t  < \frac{\Delta}{2} \\ 0 & \text{if }  t  > \frac{\Delta}{2} \end{cases}$	$\frac{\sin(\pi f \Delta)}{\pi f}$
$\frac{\sin(2\pi Wt)}{\pi t}$	$S(f) = \begin{cases} 1 & \text{if }  f  < W \\ 0 & \text{if }  f  > W \end{cases}$

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### Fourier Transform Properties

	<b>Time-Domain</b>	<b>Frequency Domain</b>
Linearity	$a_1 s_1(t) + a_2 s_2(t)$	$a_1 S_1(f) + a_2 S_2(f)$
Conjugate Symmetry	$s(t) \in \mathbb{R}$	$S(f) = S(-f)$
Even Symmetry	$s(t) = s(-t)$	$S(f) = S(-f)$
Odd Symmetry	$s(t) = -s(-t)$	$S(f) = -S(-f)$
Scale Change	$s(at)$	$\frac{1}{ a } S\left(\frac{f}{a}\right)$
Time Delay	$s(t - \tau)$	$e^{-i2\pi f\tau} S(f)$
Complex Modulation	$e^{i2\pi f_0 t} s(t)$	$S(f - f_0)$
Amplitude Modulation by Cosine	$s(t) \cos(2\pi f_0 t)$	$\frac{S(f-f_0) + S(f+f_0)}{2}$
Amplitude Modulation by Sine	$s(t) \sin(2\pi f_0 t)$	$\frac{S(f-f_0) - S(f+f_0)}{2i}$
Differentiation	$\frac{d}{dt} s(t)$	$i2\pi f S(f)$
Integration	$\int_{-\infty}^t s(\alpha) d\alpha$	$\frac{1}{i2\pi f} S(f)$ if $S(0) = 0$
Multiplication by $t$	$ts(t)$	$-\frac{1}{(i2\pi)} \frac{d}{df} S(f)$
Area	$\int_{-\infty}^{\infty} s(t) dt$	$S(0)$
Value at Origin	$s(0)$	$\int_{-\infty}^{\infty} S(f) df$
Parseval's Theorem	$\int_{-\infty}^{\infty} ( s(t) )^2 dt$	$\int_{-\infty}^{\infty} ( S(f) )^2 df$