Optical CDMA with Optical Orthogonal Code

SANGWOOK HAN
shan4@ece.utexas.edu
Department of Electrical and Computer Engineering
The University of Texas at Austin
Austin, TX 78712

Abstract - This report examines optical CDMA communication techniques with optical orthogonal codes. Simulations that show the desired properties of these codes and their use in optical CDMA are reported. Based on the simulations, we investigate the properties of optical CDMA. Probability of error is also evaluated.

I. INTRODUCTION

There have been many efforts to take the full advantage of fiber-optic signal processing techniques to establish an all optical CDMA communication systems since CDMA was first applied to the optical domain in the mid-1980s by Prucnal, Salehi, and others [1-3]. Traditional fiber optic communication systems use either TDMA or WDMA schemes to allocate bandwidth among multiple users. Unfortunately, both present significant drawbacks in local area systems requiring large numbers of users.

In a TDMA system, the total system throughput is limited by the product of the number of users and their respective transmission rates since only one user can transmit at a time. For instance, if 100 users wish to transmit at 1 gigabit per second, at a minimum the communication hardware would need to be capable of sustaining a throughput of 100 gigabits per second, a data rate that would strain even the highest performance optical networking equipment. In addition, TDMA systems show significant latency penalties because of the coordination required to coordinate and grant requests for time slots from users by the central node [4].

Unlike TDMA, a WDMA system allows each user to transmit at the peak speed of the network hardware since each channel is transmitted on a single wavelength of light. A WDMA system could easily support a bandwidth of one terabit per second, ideal for the needs of a local area network. Unfortunately, it is difficult to construct a WDMA system for a dynamic set of multiple users because of the significant amount of coordination among the nodes required for successful operation. To build a WDMA network with a dynamic user base, control channels and collision detection schemes would need to be implemented that would waste significant bandwidth.

Fortunately, an alternative to TDMA and WDMA networking schemes, optical CDMA communication systems, require neither the time nor the frequency management systems. Optical CDMA can operate asynchronously, without centralized control, and it does not suffer from packet collisions. As a result, optical CDMA systems have lower latencies than TDMA or WDMA. Furthermore, since time and frequency (or wavelength) slots do not need to be allocated to each individual user, significant performance gains can be achieved through multiplexing. Also, TDMA and WDMA systems are limited by hardware because of the slot allocation requirements. In contrast, CDMA systems are only limited the tolerated bit error rate relationship to the number of users, affording the designer a much more flexible network design [4].

To establish the optical CDMA, we have to overcome the code orthogonality problem. Many researchers have proposed several codes such as prime code, optical orthogonal code, and so on. In this project, we focus on optical orthogonal codes (OOC) among those codes. In section II, we introduce the optical orthogonal codes. Section III discusses three simulations demonstrating the principles of optical CDMA. The first one is for two-user synchronous channels to understand basics of optical CDMA and the second one is for two-user asynchronous channels. The third one is for K-user synchronous channels. Section IV evaluates the probability of error.

II. OPTICAL ORTHOGONAL CODES

An optical orthogonal code is a family of (0, 1) sequences with good auto- and cross-correlation properties. Thumbtack-shaped auto-correlation enables the effective detection of the desired signal (Fig. 2 c), and low-profiled cross-correlation makes it easy to reduce interference due to other users and channel noise (Fig. 2 d). The use of optical orthogonal codes enables a large number of asynchronous users to transmit information efficiently and reliably. The lack of a network synchronization requirement enhances the flexibility of the system. The codes considered here consist of truly (0, 1) sequences (Fig. 1 a) and are intended for “unipolar” environments that have no negative components since you either have light, or you don’t, while most documented correlation sequences are actually (+1, -1) sequences (Fig. 1 b) intended for systems having both positive and negative components.

An \((n, w, 2, \gamma)\) optical orthogonal code \(C\) is a family of \((0, 1)\) sequences of length \(n\) and weight \(w\) which satisfy
the following two properties [5].

1) The Auto-Correlation Property:

\[ \sum_{t=0}^{n-1} x_t x_{t+\tau} \leq \lambda_a \]  

for any \( x \in C \) and any integer \( t \), \( 0 \leq t < n \).

2) The Cross-Correlation Property:

\[ \sum_{t=0}^{n-1} x_t y_{t+\tau} \leq \lambda_c \]  

for any \( x \neq y \in C \) and any integer \( t \).

The numbers \( \lambda_a \) and \( \lambda_c \) are called the auto- and cross-correlation constraints. The \((0, 1)\) sequences of an optical orthogonal code are called its codewords.

![Fig. 1. Examples of sequences (a) sequence for fiber optics (b) sequence for radio frequency](image)

An \((n, w, \tau_a, \tau_c)\) OOC \( C \) can be alternatively considered as a family of \( w \) sets of integers modulo \( n \) in which each \( w \) set corresponds to a codeword and the integers within each \( w \) set specify the nonzero bits. For instance, let’s think of a simple OOC, \( 1101000 \), characterized by \((7, 3, 1, 1)\) in \((0, 1, 3, 1, 1)\) notation. This notation can simply represent codes instead of exhaustively describing long \((0, 1)\) sequences. Table I shows some optimal \((n, 3, 1, 1)\) codes in this notation.

<table>
<thead>
<tr>
<th>( n )</th>
<th>Optimal ((n, 3, 1, 1))-codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>{0,1,3}</td>
</tr>
<tr>
<td>13</td>
<td>{0,1,4}, {0,2,7}</td>
</tr>
<tr>
<td>19</td>
<td>{0,1,5}, {0,2,8}, {0,3,10}</td>
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<tr>
<td>25</td>
<td>{0,1,6}, {0,2,9}, {0,3,11}, {0,4,13}</td>
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<tr>
<td>31</td>
<td>{0,1,7}, {0,2,11}, {0,3,15}, {0,4,14}, {0,5,13}</td>
</tr>
<tr>
<td>37</td>
<td>{0,1,11}, {0,2,9}, {0,3,17}, {0,4,12}, {0,5,18}, {0,6,12}</td>
</tr>
<tr>
<td>43</td>
<td>{0,1,19}, {0,2,22}, {0,3,15}, {0,4,13}, {0,5,16}, {0,6,14}, {0,7,17}</td>
</tr>
</tbody>
</table>

![Table I. Some Optimal \((n, 3, 1, 1)\)-Codes [5]](image)

Let’s think of \( C \) represented by \{\{0, 10, 13, 28\}, \{0, 5, 12, 31\}\} \((mod\ 32)\) with two code words. Two code words are shown in Fig. 2 (a) and (b), respectively. \( C \) also can be represented by \((32, 4, 1, 1)\) in \((n, w, \tau_a, \tau_c)\) notation. Fig. 2 (c) shows auto-correlation of OOC 1. Its maximum value is \( w \) at the auto-correlation time and one or zero at any other time following (1). Fig. 2 (d) shows the cross-correlation between OOC 1 and OOC 2. It always takes one and zero at any time following (2). Here \( 1 \)s are taken as auto-correlation constraints, \( \tau_a \) and cross-correlation constraints, \( \tau_c \), since \( 1 \)s are the lowest value they can be, and correlation is calculated by convolution sum of two sequences.

![Fig. 2. (a) OOC 1 (b) OOC 2 (c) auto-correlation (d) cross-correlation](image)

II. Optical CDMA

II.1. Two-User Synchronous Channel

The two-user synchronous channel simulated here can be characterized by the follows.

\[ y(t) = A_1 x_1(t) + A_2 x_2(t) + \sigma(t) \quad A_1 = A_2 = 1, \ t \in [0,T] \]  

Let an \((32, 4, 1, 1)\) optical code \( C \) with 2 codewords be used. \( C \) can be represented by \{\{0, 10, 13, 28\}, \{0, 5, 12, 31\}\} \((mod\ 32)\) in Fig. 3 (d) and (e). Thus, the system can accommodate 2 transmitters simultaneously. Each transmitter is assigned a \( w(=4) \) set from \( C \), i.e. transmitter1 is assigned a \{0, 10, 13, 28\} set and transmitter 2 is assigned a \{0, 5, 12, 31\} set. At a transmitter, every information bit is encoded into a frame of \( n(=32) \) optical chips in the following way. (A chip is an optical time slot which can assume one of two values: ON or OFF) Let the assigned \( w \) set for a particular transmitter be \( S=\{s_1, s_2, \cdots, s_w\} \). In this case, \( s_1 \) is a \{0, 10, 13, 28\} set and \( s_2 \) is a \{0, 5, 12, 31\} set. Assume the information bit is 1. In the corresponding frame, which consists of \( n \) optical chips, photon pulses (i.e., ON signals) are sent at exactly the \( s_1 \)th, \( s_2 \)th, \cdots and \( s_w \)th chips (Fig. 3 (f) and g). In the other \( n-w \) chips, no photon pulse (i.e., OFF signals).
On the other hand, if the information bit is 0, no photon pulses are sent in the corresponding frame, i.e., all OFF signals are sent. In other words, the codeword set is used as the signature sequence of the transmitter.

Basically, optimal CDMA scheme is same as radio frequency CDMA scheme except using the special codes. Here, we use the matched filter to convert the received signal in Fig. 3 (h) assuming $n(t)$ in (3) is zero. At the receiving end, correlation-type decoders are used to separate the transmitted signals. The decoder consists of a bank of 2 tapped delay-lines, one for each codeword. The delay taps on a particular line exactly match the signature sequence, i.e., the delays between successive taps are $s_2-s_1, s_3-s_2, \ldots$, optical chips, respectively. Each tapped delay-line effectively calculates the correlation of the received waveform with its signature sequences. In Fig. 4 (b) and (c), there are five different correlation values, 0, 1, 2, 4, and 5. Because of the properties of optical orthogonal codes, the correlation between different signature sequences is low, 0 and 1. Thus the delay-line output is high, 4 and 5, only when the intended transmitter’s information bit is 1. However, a potential problem due to interference can be happened. When correlation value has 2, it is definitely due to the interference. In this case, the value is always below 4 so it
can be discarded by choosing relevant threshold value. But when total number of users goes up, the cross-correlation due to interfering users adds up quickly to severely degrade the system performance. For instance, when \( w = 4 \) like this case, accommodating 4 users make it possible to have 4 as the correlation value even if the intended transmitter’s information bit is 0. To avoid this phenomenon, both high \( w \) which can be considered as the sum of 1s in the sequence, and long \( n \) are required. If we increase only \( w \) fixing \( n \), cross-correlation value due to the interference can be lowered. However, OOC has very sparse marks to keep the cross-correlation low, i.e. a number of zeros is much higher than that of ones in the sequence. It means that cross-correlation increases by itself by increasing only \( w \). Therefore, both \( w \) and \( n \) should be increased simultaneously. But this solution also reveals a drawback, long signal processing time due to long \( n \).

Finally the transmitted information is extracted by thresholding the correlator output in Fig. 4 (d) and (e). In this case, they are successfully recovered as intended. Here, 3 is chosen as the threshold values. Threshold issue will be covered in detail in the section II.2.

II.1.B. Two-User Asynchronous Channel

In optical CDMA, all users are allowed to transmit at any time. There is no network synchronization required. In this section, we simulate a two-user asynchronous channel to investigate above statement. To verify no network synchronization, the same scheme in II.A is used here except the time delay at \( F \) in Fig. 3 (a). Fig. 5 (b) and (c) show the recovered signals from asynchronous and synchronous channel, respectively. They agree well even if synchronous channel does not use any special scheme for synchronization. Therefore, now we can say that optical CDMA does need no network synchronization.

II.2. \( K \)-User Synchronous Channel

In the previous section, we investigated a simple 2-user channel to understand the optical CDMA. In this section, we explore problems faced by increasing \( K \) (Fig. 6 a). Here we choose 7 as \( K \), and \( C \) is \((43, 3, 1, 1)\) having 7 sets, \( \{0, 1, 19\}, \{0, 2, 22\}, \{0, 3, 15\}, \{0, 4, 13\}, \{0, 5, 16\}, \{0, 6, 14\}, \{0, 7, 17\} \).

The first issue is a threshold value. As we saw in the previous section, interfering signal can be effectively discarded by setting a relevant threshold value. The threshold value can be chosen under the following condition.

\[
0 \leq \text{threshold} \leq w \tag{4}
\]

Fig. 6 (d) through (g) show results from several threshold values, 1, 2, 3, and 5. As the threshold value goes up, the recovered signal gets similar to the ideal signal in Fig. 6 (c). However, the threshold value can not be over \( w \). If the intended information bit is 1, the correlation value is \( w \). Therefore, the threshold over \( w \) incorrectly converts it to 0 when the intended information bit is 1. Fig. 6 (g) shows the results by choosing threshold over \( w \). We can clearly see that a dotted line is missed comparing to Fig. 6 (c).

Even if the highest value under (4) is chosen (Fig. 6 f), the recovered signal can not be exactly same as the ideal pattern. In the Fig. 6 (a), if the intended information bit is 1, the correlation value is \( w(=3) \). But as we already discussed in the previous section, the cross-correlation due to interfering users adds up very quickly to severely degrade the system performance. Here, the correlation value due to interference is even higher than \( w(=3) \). This is the reason why the received signal can not be recovered perfectly.

To figure it out above problem we introduce optical hard-limiter [3] located before the optical tapped-delay line (Fig. 7 a). An ideal optical hard-limiter is defined as

\[
g(x) = \begin{cases} 1 & \text{if } x \geq 1 \\ 0 & \text{if } 0 \leq x < 1 \end{cases} \tag{5}
\]

Therefore, if an optical light intensity \( (x) \) is bigger than or equal to one, the hard-limiter would clip the intensity back to one, and if the optical light intensity is smaller than one, the response of the optical hard-limiter would be zero. This ideal nonlinear process would enhance the system performance because it would exclude some combinations of interference patterns from causing errors as in the soft-limiter case, i.e., the patterns that caused errors by analog summation of light intensity rather than by exact reproduction of the particular pattern with no analog effect. Fig. 7 shows comparison between the systems with and without the optical hard-limiter.
Fig. 6 (a) schematic diagram of a receiving part for \( K \) users (b) signal at \( H \) (c) ideal signal at \( J \) (d) signal at \( J \) when threshold=1 (e) signal at \( J \) when threshold=2 (f) signal at \( J \) when threshold=3 (g) signal at \( J \) when threshold=5

All optical light intensity which is bigger than or equal to one are clipped back to one in Fig. 7 (d). In Fig 7 (f), the cross-correlation value over \( w \) due to the interference does not exist any more. From these, the thresholding can much effectively recover the information. Fig. 7 (g) and (h) show the recovered signals without and with the optical hard-limiter, respectively. We can clearly see that Fig. 7 (h) is much similar to the intended signal (Fig. 7 i). Consequently, we can say that the optical hard-limiter can effectively lower the interference effect.

Fig. 7. (a) receiving part without optical hard-limiter (b) receiving part with optical hard-limiter (c) signal at \( G \) (d) signal at \( G' \) (e) signal at \( H \) (f) signal at \( H' \) (g) signal at \( J \) (h) signal at \( J' \) (i) ideal signal expected at \( J \)
IV. Probability of Error

The probability of error per bit is defined as

\[ PE = P(LI \leq h \mid b=0)P(b=0) + P(LI < h \mid b=1)P(b=1) \]  

where \( PE \), \( LI \), \( th \), and \( b \) are the probability of error, light intensity, threshold, and intended binary information bit, respectively. Here, an interesting point is that \( P(LI < th \mid b=1) \) is always equal to zero because \( P(LI < th \mid b=1) = p(w+I < th) = p(w+I-th < 0) = 0 \) under (4) where \( I \) is the interference due to other users. According to [4], one can easily calculate the probability of error using the followings.

\[ PE = \frac{1}{2} \sum_{i=0}^{N-1} \left( \sum_{j=0}^{K-1} \frac{w^i}{2n} \left( 1 - \frac{w^i}{2n} \right)^{K-i-1} \right) \]  

Fig. 8 shows several dependency of \( PE \). In Fig. 8 (a), a length of code, \( n \), is tested in different values, 200, 500, 1000, and 2000. As \( n \) goes up, \( PE \) gets lowered resulting in long processing time. So we need to deal with the trade off problem on the low error rate and processing time. Fig. 8 (b) shows \( w \)-dependency of \( PE \). As the sum of 1s in the code, \( w \), goes up, \( PE \) gets lowered. Note that the highest threshold value under (4) would make the lowest \( PE \) on the same \( w \). In the Fig. 8 (c), we can find \( K \)-dependency of \( PE \). As the number of accommodated users, \( K \), goes up, \( PE \) gets higher. This is definitely due to the increasing interference. Observations from Fig. 8 agree well with the results in the section II.1.A.

V. Conclusion

In this report, we introduced the optical CDMA with the optical orthogonal codes. CDMA scheme was successfully applied by using the optical orthogonal codes. However, OOC revealed some drawbacks, requirement of long sequences resulting in long signal processing time and severe degradation due to fast adding of cross-correlation. Optical hard-limiter showed remarkable improvement in reducing interference due to other users. We also presented that the threshold value, code length, and total number of users are important factors for the probability of error.

\[ P_{error} = \frac{1}{2} \sum_{i=0}^{N-1} \left( \sum_{j=0}^{K-1} \frac{w^i}{2n} \left( 1 - \frac{w^i}{2n} \right)^{K-i-1} \right) \]  

Fig. 8. (a) \( n \)-dependency of probability of error (b) \( w \)-dependency of probability of error (c) \( K \)-dependency of probability of error

REFERENCES


