The Capacity of Multicellular Distributed Antenna Networks

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Abstract—Distributed antenna systems (DAS) are being widely considered for state-of-the-art cellular communication systems to cover dead spots. Recent academic studies have shown that in addition to coverage improvements, DAS can also allow a capacity improvement. This paper provides a generalized information theoretic analysis of downlink multicell DAS for two different transmission strategies: selection diversity (where just one of the distributed antennas are used) and blanket transmission (where all antennas in the cell broadcast data). Simple repeaters are a special case of our analysis. The results show that DAS reduces other-cell interference in a multicell environment and hence significantly improves performance and capacity (by about 2-3x), with particularly large improvements for users near cell boundaries.

I. INTRODUCTION

Future wireless systems will need to provide high data rates to a large number of users. Experience tells us that these networks will be interference-limited, since the interference from either in-cell users (as in CDMA) or from other-cell users will determine how heavily the system can be loaded. Multiuser receivers are one potential way to reduce interference in cellular systems (see e.g. [1] and the references therein), but this requires a more complicated receiver. An alternative strategy is to try to reduce the overall transmit power (and hence other-cell interference) using distributed antennas, which has the additional merit of providing better coverage and increasing battery life [2]. Although distributed antennas systems (DAS) were originally introduced to simply cover the dead spots in indoor wireless communications [3], recent studies have identified other potential advantages such as power and system capacity, and expanded its applications [4]–[9].

In DAS, antenna modules are geographically distributed to reduce access distance instead of centralizing at a location. Each distributed antenna module is connected to a home base station (or central unit) via dedicated wires, fiber optics, or an exclusive RF link. Although the connections via the same RF link as that used in a cell are possible, the connections with the same RF link construct information-theoretic relay channels categorized as another research area called cooperative communications [10]–[13]. DAS is rather similar to repeater systems [14]–[16] from the fact that the distributed antenna modules and the home base station are physically connected. However, DAS is distinguished from conventional repeater system by the fact that each distributed antenna module is able to transmit different data in the downlink whereas repeaters just repeat signals from the home base station. Therefore, DAS is a generalization of conventional repeater systems. Since the distributed antenna modules and the home base station together construct a macroscopic multiple input single output (MISO) vector channel, DAS can be also interpreted as a macroscopic multiple-antenna system [17], [18].

From an architectural point-of-view, DAS has manifest advantages over conventional communication systems. DAS can reduce the cost of installing system and simplify maintenance because DAS can reduce the required number of base stations within a target service area. Furthermore, blocking probability can be improved owing to the principle of trunking efficiency because resources for signal processing such as channel cards/elements are centralized and shared at the home base station (or central unit). In addition to these architectural advantages, DAS also has been shown to possess advantages in terms of power, signal-to-interference-plus-noise ratio (SINR), and capacity owing to macro-diversity and the reduced access distance [4]–[8]. Based on these advantages, many cellular service providers or system manufacturers are seriously considering replacing legacy cellular systems with distributed antenna systems or adopting the distributed antenna architecture in the future. However, most of the recent work on DAS has been focused on investigating those advantages and analyzing its performance in the uplink because of its analytical simplicity [4]–[8]. On the other hand, there are few studies on the downlink performance of DAS although the demand for high speed data will be dominant in the downlink. There are also few papers that consider the advantages of DAS in a multicell context.

A recent paper [9] addressed downlink performance of code division multiple access (CDMA) DAS in a multiple cell environment but it relied on computer simulations and only investigated SIR levels perceived at mobile stations. Computer simulations are useful but mathematical analysis
enables us to efficiently identify the key design parameters and to understand their effects. Particularly, information-theoretic analysis provides intuition on achievable system capacity. For this reason, the authors of [17], [18] analyzed downlink capacity in terms of multiple-antenna (MIMO) information theory. However, they did not consider a multicell environment, either. In addition, their analysis was limited to the case with specific assumptions: (1) channel state information is known to the transmitter and to the receiver, and (2) transmit power is shared among distributed antenna modules, i.e., there is no per antenna-module power constraint.

In this context, we develop a generalized analysis of achievable capacity from an information-theory standpoint for two different transmission strategies: selection diversity (where just one or two of the distributed antennas are used) and blanket transmission (where all antennas in the cell broadcast the data). The result of this paper shows that a distributed antenna architecture effectively addresses other-cell interference (OCI) in a multicell environment, especially at the cell boundaries, and achieves a non-trivial capacity increase over a conventional cellular architecture. An additional contribution of this paper is a theoretical framework for analyzing the achievable capacity and diversity performance of cellular repeater systems, a special case of DAS.

II. SYSTEM MODEL

A. Cellular Architecture

In DAS, the main processing modules such as channel cards are centralized at a location (central unit) and are connected with distributed antenna modules. Each distributed antenna module is physically connected with a home base station via dedicated wires, fiber optics, or an exclusive RF link. A general architecture of DAS in a multicell environment is shown in Figure 1, where a cell is covered by a small base station and six distributed antenna modules. In contrast, the same area is covered by only a single high-power base station in traditional cellular systems. Alternatively, the small base station and 6 distributed antenna modules can be viewed as an alternative to 7 small traditional base stations (pico/micro cells). The actual number of distributed antenna modules would be determined by coverage, user densities, and other environmental factors but we only consider 6 distributed antenna modules as a reasonable example. The total transmit power of the ith distributed antenna module of the jth cell is \( P_i^{(j)} \), where the small base station of each cell and home cell are indexed by \( i = 0 \) and \( j = 0 \), respectively. Throughout this paper, we assume that \( \sum_{i=0}^{6} P_i^{(j)} = P \) where \( P \) is the total transmit power of the conventional base station for a fair comparison. In Figure 1, the radius of a cell (a bold dotted circle) is \( R \) and the radius of a coverage of a distributed antenna module is \( r = R/(1 + \sqrt{3}) \). We also consider the 1-tier cellular structure with universal frequency reuse.

B. Transmission Strategy and Multiple Access Scenario

In cellular DAS, there are several possible transmission strategies using multiple distributed antenna modules. Either all or some of the antenna modules can be used in transmission. Although many methods of using the distributed antenna modules are possible, we consider two likely transmission strategies: the blanket transmission scheme and single transmit selection scheme. The blanket transmission scheme is to transmit signals through all the distributed antenna modules and the home base station. In this scheme, either the same signals or different signals are transmitted through each distributed antenna module. Therefore, the distributed antenna modules construct a macroscopic multiple antenna system. In the single transmit selection scheme, only a single distributed antenna module or the home base station is selected for transmission by the criterion of minimizing propagation pathloss. Although smarter selection algorithms can potentially be used such as maximizing SINR or capacity, we consider this simple one for simplicity of analysis and since it should minimize the required transmit power (and hence the interference caused to other cells). This scheme exploits macroscopic selection diversity and is expected to additionally reduce OCI because the number of OCI sources is reduced.

We also assume a single user scenario for simplicity of analysis, but this assumption holds for most practical multiuser systems as well because time division multiple access (TDMA), frequency division multiple access (FDMA), orthogonal code division multiple access (CDMA), and orthogonal frequency division multiple access (OFDMA) systems basically correspond to this assumption since there is only a single user transmitting in any time/frequency/code dimension.

C. Channel Model and Received Signal

When the blanket transmission scheme is used, the multiple distributed antenna modules and the home base station together construct a macroscopic MISO vector channel given by

\[
h = \sqrt{L_i^{(0)}} h_i^{(0)} \sqrt{L_1^{(0)}} h_1^{(0)} \cdots \sqrt{L_6^{(0)}} h_6^{(0)}
\]

where \( h_i^{(j)} \) denotes short term fading from the ith distributed antenna module in the jth cell and is an i.i.d. complex Gaussian random variable \( \sim \mathcal{CN}(0, 1) \). The fading channel is assumed to be static during a symbol duration. \( L_i^{(j)} \) denotes propagation pathloss and shadow fading from the ith distributed antenna module in the jth cell. Let the transmitted signal vector be \( x = [x_0^{(0)} x_1^{(0)} \cdots x_6^{(0)}]^T \), where \( x_i^{(j)} \) is the transmitted signal from the ith distributed antenna module in the jth cell and is i.i.d. Gaussian random variable \( \sim \mathcal{CN}(0, 1) \). Then, the received signal at a mobile station at a given symbol duration is given by

\[
y = hx + \sum_{j=1}^{6} \sum_{i=0}^{6} h_i^{(j)} \sqrt{L_i^{(j)}} x_i^{(j)} + n\]

where \( n \) is the additive Gaussian noise with variance \( \mathbb{E}[nn^H] = \sigma_n^2 \). Since the number of interfering source is sufficiently large and interfering sources are independent of each other, the interference plus noise is assumed to be a
complex Gaussian random variable $z$ with variance $\sigma_z^2$ by the Central Limit Theorem (CLT).

When the single transmit selection scheme is used, the received signal is given by

$$y = \sqrt{L_m} h_m x_m + \sum_{j=1}^{6} h_j x_j + n$$  \hspace{1cm} (4)

$$= \sqrt{L_m} h_m x_m + z_1$$  \hspace{1cm} (5)

where $m = \arg \max_{i \in \{0, 1, \ldots, 6\}} \{ h_0^0, h_1^0, \ldots, h_6^0 \}$ and $z_1$ is a complex Gaussian random variable with variance $\sigma_z^2$, representing interference plus noise.

III. ACHIEVABLE CAPACITY

In this section, the Shannon capacity of the cellular DAS is derived in order to upper bound the theoretically achievable system capacity.

A. Ergodic Capacity only with CSIR

If we assume the channel state information is known only at the receiver (CSIR) and the channel is ergodic, the ergodic Shannon capacity at a given location of the target mobile station can be achieved by

$$C_e = \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{1}{\sigma_z^2} \| h \| \| h \|^H \right) \right]$$  \hspace{1cm} (6)

where $\mathbb{E}_h$ is the statistical expectation with respect to the random variable $h$ given by (5).

If ergodicity of the channel is assumed, the ergodic capacity can be obtained as

$$C_e = \mathbb{E}_h \left[ \log_2 \left( 1 + \sum_{i=0}^{6} \frac{1}{\sigma_z^2} | h_i^0 |^2 \| L_i^0 \| \| P_i^0 \| \right) \right]$$

$$= \int_0^\infty \log_2 (1 + \gamma) f_\gamma(d\gamma)$$  \hspace{1cm} (7)

where $\gamma = \sigma_z^2 \sum_{i=0}^{6} | h_i^0 |^2 \| L_i^0 \| \| P_i^0 \|$ is a weighted chi-squared distributed random variable with pdf given by [19], [20]

$$f_\gamma(\gamma) = \sum_{i=0}^{6} \frac{\sigma_z^2 \pi_i}{\| L_i^0 \| \| P_i^0 \|} \exp \left( -\frac{\gamma}{\| L_i^0 \| \| P_i^0 \|} \right)$$  \hspace{1cm} (8)

where $\pi_i = \prod_{k=0, k \neq i}^{6} \frac{L_k^0 P_k^0}{L_i^0 P_i^0}$. Then, the ergodic capacity can be obtained in a simple form by [21]

$$C_e = -\frac{1}{\ln(2)} \sum_{i=0}^{6} \pi_i \exp \left( -\frac{\sigma_z^2}{\| L_i^0 \| \| P_i^0 \|} \right) Ei \left( -\frac{\sigma_z^2}{\| L_i^0 \| \| P_i^0 \|} \right)$$  \hspace{1cm} (9)

where $Ei(x)$ is the exponential integral function ($Ei(x) = -\int_{-x}^{\infty} e^{-t} / t \, dt$) and can be easily calculated with popular numerical tools such as MATLAB, MATHEMATICA, and MAPLE.

Figure 2 shows the ergodic capacity of cellular DAS versus the normalized distance from the home base station in the direction of the worst case position $W$ when the pathloss exponent is 4.0. Again, the transmit power of each distributed antenna module is $0.1P$ and the transmit power of the home base station is $0.4P$ in DAS whereas the transmit power of the base station in the conventional cellular system is $P$. As expected from the SINR-based results, the single transmit selection scheme achieves the highest throughput owing to the OCI reduction and macroscopic selection diversity. Although the achieved throughput of the blanket transmission scheme in the cellular DAS is slightly lower than that of conventional cellular system near the home base station due to reduced transmit power, the achieved throughput of the blanket transmission scheme in DAS has substantially higher throughput beyond the normalized distance 0.5. Note that 80% of the users will be in $(0.5R, R)$ assuming a uniform distribution, while the other 20% have very high SINR since they are close to the home base station and farther from the interfering cells.

Assuming that the target mobile stations are uniformly distributed, we can obtain the average Shannon capacity which represents achievable average cell throughput. In Figure 3, the achievable average cell throughput is provided according to pathloss exponents. When the pathloss exponent is 3.5, the throughput of the blanket transmission scheme and the single transmit selection scheme in cellular DAS are about 120% and 190% of throughput of conventional cellular system, respectively. This large improvement is due to the fact that three times more users are outside of the radius $1/2R$ within a cell assuming uniform distribution.

B. Ergodic Capacity with CSIT

When channel is known to the transmitter side, the ergodic Shannon capacity at a given location of the target mobile station can be given by [22]

$$C_e = \mathbb{E}_h \left[ \max_{S_i \subseteq \{P_i^0\}_{i=0}^6} \log_2 \left( 1 + \frac{1}{\sigma_z^2} \| h \| \| h \|^H \right) \right]$$  \hspace{1cm} (10)

where $S_i$ is the $i$th diagonal element of $\mathbf{S}$. The maximum capacity is achieved by a proper design of the transmit power spectrum matrix $\mathbf{S}$ under individual power constraints of the distributed antenna modules and the home base station. If power cooperation among the distributed antenna modules and the home base station is allowable, this problem reduces to a standard water-filling problem in a macroscopic MISO vector channel [17], [22] and the maximum capacity is simply given by

$$C_e = \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{P \| h \|^2}{\sigma_z^2} \right) \right]$$  \hspace{1cm} (11)

where $P = \sum_{i=0}^{6} P_i^0$. However, the per-antenna power constraints make it meaningless to apply the simple water-filling algorithm. Therefore, we should find another solution instead of the conventional water-filling algorithm.

Let the eigenvalue decomposition of the symmetric positive definite matrix $\mathbf{S}$ be $\mathbf{S} = \mathbf{U} \Lambda \mathbf{U}^H$ where $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and $\Lambda$ is the concentration matrix of the received signal.
diag(λ_0, λ_1,⋯, λ_6). Then, the capacity formula in (10) can be given by

\[ C_c = \mathbb{E}_h \left[ \max_{\lambda_i \leq P_i^{(0)} / \nu_i} \log_2 \left( 1 + \frac{1}{\sigma_i^2} f(\lambda \rho H) \right) \right] \]

(12)

\[ = \mathbb{E}_h \left[ \max_{\lambda_i \leq P_i^{(0)} / \nu_i} \log_2 \left( 1 + \frac{1}{\sigma_i^2} \sum_{i=0}^{6} |f_i|^2 \lambda_i \right) \right] \]

(13)

where \( f = hU \) and \( f_i \) is the \( i \)th element of a \( 1 \times 7 \) vector \( f \).

Since the \( \log \) function is a monotonic increasing function with respect to \( \lambda_i \) \( \forall i \), the maximum capacity is achieved if \( \lambda_i = P_i^{(0)} / \nu_i \) as

\[ C_c = \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{1}{\sigma_i^2} \sum_{i=0}^{6} |f_i|^2 P_i^{(0)} \right) \right] \]

(14)

We should emphasize here that the best transmission strategy of DAS with per antenna module power constraints is for each antenna module and home base station to transmit allowable full power. This result indicates that the channel state information at the transmitter (CSIT) in DAS does not provide any advantage over the case that the channel state information is provided only to the receiver if there are per-antenna module power constraints. However, if each antenna module and home base station has multiple antennas, CSIT will certainly help increase capacity.

For simplicity of analysis, we assume \( P_i^{(0)} = P/7 \) \( \forall i \). Then, since \( U \) is the unitary matrix, it holds that \( \sum_{i=0}^{6} |f_i|^2 = hU^H H = \sum_{j=0}^{6} |h_j^{(0)}|^2 L_j^{(0)} \) and the capacity formula is given by

\[ C_c = \mathbb{E}_h \left[ \log_2 \left( 1 + \frac{P}{7\sigma_i^2} \sum_{i=0}^{6} |h_i^{(0)}|^2 L_i^{(0)} \right) \right] \]

(15)

\[ = \frac{1}{\ln 2} \sum_{i=0}^{6} \nu_i \exp \left( \frac{-7\sigma_i^2}{L_i^{(0)}} E_i \right) \left( 1 + \frac{7\sigma_i^2}{L_i^{(0)}} \right) \]

(16)

where \( \nu_i = \prod_{k=0, k \neq i} \frac{L_i^{(0)}}{L_k^{(0)} - P_k^{(0)}} \). Equation (16) is derived from the fact that \( \sum_{i=0}^{6} |h_i^{(0)}|^2 L_i^{(0)} \) is a weighted chi-squared distributed random variable.

Figure 4 shows the ergodic capacity when channel state information is known to the transmitter according to the normalized distance from the home base station when the pathloss exponent is 4.0. In this figure, we assume that the transmit power of each distributed antenna module and the home base station in DAS is \( P/7 \) whereas the transmit power of the base station in the traditional cellular system is \( P \). Similarly to the CSIR case, the single transmit selection scheme achieves the highest throughput owing to the OCI reduction and macroscopic selection diversity. Although the achieved throughput of DAS is slightly lower than that of conventional cellular system near the home base station due to reduced transmit power, the achieved throughput of DAS has substantially higher throughput beyond the normalized distance 0.5. When the target mobile stations are uniformly distributed in space, the average Shannon capacity is plotted versus pathloss exponent in Figure 5. This figure also shows the similar result to the CSIR case.

IV. CONCLUSIONS

In this paper, we have analyzed achievable capacity of various transmission schemes in cellular DAS from an information theoretic standpoint. The results of this paper suggest that distributed antenna systems effectively reduce OCI and improve SINR compared to conventional cellular systems in an interference-limited multicell environment. As a result, distributed antenna systems achieve higher capacity than conventional cellular systems. Based on the results of this paper, the distributed antenna architecture might be an effective solution to reduce OCI in an interference-limited cellular environment. An additional contribution of this paper is a theoretical framework for analyzing the achievable capacity and diversity performance of conventional repeater systems, a special case of DAS.

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Fig. 1. Structure of a distributed antenna system

Fig. 2. Ergodic capacity with CSIR versus the normalized distance from the home base station

Fig. 3. Average Ergodic capacity with CSIR versus the pathloss exponent
Fig. 4. Ergodic capacity with CSIT versus the normalized distance from the home base station

Fig. 5. Average Ergodic capacity with CSIT versus the pathloss exponent