Cancellation Error Statistics in a CDMA System Using Successive Interference Cancellation

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Abstract
Successive interference cancellation (SIC) is a promising practical scheme for increasing the capacity of CDMA systems. The interference cancellation is never perfect, and the residual cancellation errors propagate because of the successive nature of the decoding. In fact, these residual errors are the principle capacity-limiting issue in SIC systems. Until now, the statistics of these errors have been assumed to follow standard probability distributions such as lognormal and Gaussian. In this paper, the actual statistics of cancellation error are derived. A mathematical expression for the probability density function and the second order statistics is presented, and shown to depend closely on the channel statistics. These expressions were verified through simulations. A Gaussian distribution allowing accurate modelling of cancellation error is also found. The results will be useful to future research and to industry, allowing accurate modelling of cancellation error when designing CDMA systems with successive interference cancellation.

1 Introduction
In the last two decades an enormous amount of research has been undertaken to increase capacity of a CDMA system using different multiuser detection (MUD) techniques. Details of different type of MUD techniques can be found in [1]. In the past two to three years a growing interest has been shown in SIC and presently its implementation in the industry is being pursued. The main reason for its popularity is its low complexity and in its simplest form, SIC uses decisions produced by single-user matched filters. SIC is a nonlinear type of multiuser detection (MUD) scheme in which users are decoded successively [2],[3], [4]. The approach successively cancels the strongest users by re-encoding the decoded bits, and after making an estimate of the channel, the interfering signal is generated at the receiver. This is then subtracted from the received waveform. In this manner later users do not encounter multiple access interference (MAI) caused by previous users. SIC improves performance for all users: initial users improve because later users are given less power which means less MAI for initial users, and later users improve because early users’ interference has been cancelled out. An optimum power control scheme as shown in [3], [5] can be employed so that all users are decoded with the same Signal to Interference Ratio (SIR).

However, there is a concern in the industry about SIC providing capacity enhancements in a fading channel in the presence of estimation errors. The requirement for unequal power
amongst users in order to provide the same performance to all users is also considered an implementation issue. Capacity enhancements in the presence of cancellation error because of imperfect channel estimates is an area of great interest to the future development of 3G and 4G CDMA systems. Increase in capacity as proposed in [3] is shown to degrade if statistics of cancellation error are not known or incorrectly implemented in the power allocation as proposed in [3] and [5]. Similar degradation in capacity for a multipath fading channels was also highlighted in [6]. This can be seen in Fig. 1 (reproduced below from [3]). The top curve shows that maximum capacity is achieved when variance of the cancellation error is known. The bottom curve is for a system which falsely assumes that there is no cancellation error when designing power control distribution. In summary, capacity degrades if the variance of cancellation error is incorporated incorrectly by the receiver. Better power control can help to overcome this degradation, but in order to incorporate such power allocation schemes, statistics of cancellation error are required.

In order to increase capacity it is therefore important to know the statistics of cancellation error and identify parameters on which these statistics depend upon. In the past, statistics of cancellation error have been assumed to be log normal as in [3]. In this paper we focus on the analysis of cancellation error with a viewpoint to gain knowledge on their statistics so that future work on SIC related to 3G and 4G cellular systems could exploit these results. A simple system model as explained in Section II has been used for determining the analytical expression for cancellation error. Analytical expression for cancellation error is presented in Section III. Optimum power allocation proposed in [3] is also implemented in our model. The paper also provides a Gaussian distribution to model the cancellation error which may be used in future design work for developing better systems. Results found in the paper are verified through simulation which are explained in Section IV. In order to realistically model the received power over a fading channel, it is assumed that users’ amplitudes are Rayleigh distributed with unit mean. In order to keep the model simple, it is assumed that there is only one path from transmitter to receiver, but results could be extended if multipath were to be taken into account using SIC with multipath combining as in [4].
2 System Model

We consider a simple system to accentuate the key results. The transmitter, channel and receiver models are shown in Fig. 2.

2.1 Transmitter

At the transmitter, each user’s baseband data is split into I and Q branches and spreading gain achieved by directly multiplying the baseband data pulses with independent binary sequences. The resulting I and Q signals are converted to analog signals and are quadrature modulated by the carrier frequency. The resulting transmitted signal for user $k$ is

$$x_k(t) = A_k(t) \sum_{n=-\infty}^{\infty} b_k[n] c_{k}^{I}[n] \cos(w_c t) + c_{k}^{Q}[n] \sin(w_c t)$$

where $A_k$ is the gain factor due to optimum power allocation in presence of cancellation error and $b_k[n]$ is the bit sequence for user $k$ at a bit-rate of $R_b$. The in-phase and quadrature phase spreading sequences $c_{k}^{I}[n]$ and $c_{k}^{Q}[n]$ are at a bit-rate of $R_c$, $w_c$ is the carrier frequency in radians per second. Spreading sequences $c_{k}^{I}[n]$ and $c_{k}^{Q}[n]$ are modelled as pseudorandom Bernoulli $\{-1,+1\}$ sequences. Optimum power control for SIC proposed in [3] is incorporated in our model. This power control ensures that all users are decoded with the same Signal to Interference Ratio (SIR) even in the presence of cancellation error due to imperfect channel estimates. The resulting power control distribution for user $k$ is given as:

$$P_k = P_{k-1} - \frac{(1 - \varepsilon_{k-1})P_{k-1}^2}{V_{k-1} + N}$$

where $\varepsilon_k$ is the fraction of the power not cancelled for user $k$, $N$ is the power of the background additive white Gaussian noise and $V_k$ is the total remaining multiple-access interference (MAI).
for user $k$ is expressed as in [3]:

$$V_k = \sum_{i=1}^{K} P_i - \sum_{i=1}^{k-1} (i - \varepsilon_i) P_i$$ (3)

where $K$ is the total number of users, detailed derivation of (2) and (3) are given in [3].

The discrete-time baseband transmitted signal is:

$$x_k[n] = A_k[n](c_k^I[n] + j c_k^Q[n])b_k[n]$$ (4)

$$= x_k^I[n] + j x_k^Q[n]$$ (5)

We assume that the $I$ and $Q$ signals are uncorrelated.

### 2.2 Channel

The channel is modelled as an asynchronous fading channel with additive white gaussian noise (AWGN). Each user’s signal experiences an independent fading channel. The amplitudes are assumed to be Rayleigh distributed with unit mean value and probability density function (pdf) given by

$$f(x) = \frac{x}{\alpha} e^{-\frac{x^2}{2\alpha^2}}$$ (6)

where $\alpha^2 = 2\pi$. The composite received signal is:

$$y_o[n] = \sum_{k=1}^{K} h_k[n] \cdot x_k[n - \tau_k] + \eta[n]$$ (7)

where $h_k[n]$ is a Rayleigh sequence whose pdf is given in (6). Each user’s signal undergoes a time delay $\tau_k$ which is assumed to be tracked accurately and $\eta[n]$ is the additive white noise with power $N$. We have neglected path loss in the paper, assuming it is mitigated by the open and closed-loop power control.

### 2.3 Receiver

SIC attempts to remove the interference of the $k^{th}$ user (most recently decoded user) from the current composite received signal $y_{k-1}[n]$, by re-encoding the decoded bit sequence for user $k$, modulating it with the appropriate amplitude and phase adjustment, and subtracting it out from $y_{k-1}[n]$ shown in Fig. 2. At the receiver we assume knowledge of the spreading codes of all the users and also the desired decoding order. Once user $k$’s signal is decoded successfully the decoded bits are re-encoded, and the amplitude estimates of user $k$’s in-phase and quadrature branches, $\alpha_k$ and $\beta_k$ respectively are formed as in [3]:

$$\alpha_k = \frac{1}{M} \sum_{n=1}^{M} y_k[n] \cdot \hat{x}_k^I[n]$$ (8)

$$\beta_k = \frac{1}{M} \sum_{n=1}^{M} y_k[n] \cdot \hat{x}_k^Q[n]$$ (9)

$M$ is the number of symbols in a frame and $\hat{x}_k^I[n]$ and $\hat{x}_k^Q[n]$ are $c_k^I[n]\hat{b}_k[n]$ and $c_k^Q[n]\hat{b}_k[n]$ respectively.
Using the amplitude estimates in (8) and (9), the estimate of the signal for user $k$ is:

$$\hat{z}_k[n] = \alpha_k \hat{x}_k^I[n] + j \beta_k \hat{x}_k^Q[n]$$ (10)

The stored composite signal may then be updated as:

$$y_k[n] = y_{k-1}[n] - \hat{z}_{k-1}[n]$$

$$= y_o - \sum_{i=1}^{k-1} \hat{z}_i[n]$$ (12)

3 Cancellation Error

Cancellation error for user $k$ is defined as the residual signal of user $k$ in the remaining composite signal after the subtraction of the recreated signal. Cancellation error is primarily due to the limitation that the amplitude and phase estimation are never perfect, therefore the received signal cannot be perfectly re-created. The other source of cancellation error is incorrect bit decisions. In the presence of bit decision errors SIC actually enhances the MAI instead of removing it. Because the bit-error rate (BER) is assumed to be low on the order of $10^{-4}$, virtually all of cancellation error comes from amplitude and phase estimation error.

Cancellation error for user $k$ is defined as:

$$e_k[n] = h_k[n]x_k[n] - \hat{z}_k[n]$$

$$= e'_k[n] + je'_Q[n]$$ (14)

where $x_k[n]$ and $\hat{z}_k[n]$ are given in (4), and (10) respectively.

Looking at the error $e'_k[n]$, for only the I branch and assuming low bit errors, i.e. $b_k[n] = \hat{b}_k[n]$, the I branch error can be expressed as:

$$e'_I_k[n] = h_k[n]x'_I_k[n] - \alpha_k \hat{x}_k^I[n]$$ (15)

Substituting $x'_I_k[n]$ and $\hat{x}_k^I[n]$ into (15) and dropping the $I$ notation, error for user $k$ for the $I$ branch is:

$$e_k[n] = (A_k h_k[n] - \alpha_k)c'_I_k[n]b_k[n]$$ (16)

When re-creating the signal, $A_k[n]$ is set to $\alpha_k$ for the duration of the frame, however each symbol in the frame undergoes Rayleigh fading given by $h_k[n]$. We can also express $y_k[n]$ in (11) as:

$$y_k[n] = \sum_{k=1}^{K} x_k[n]h_k[n] - \sum_{i=1}^{k-1} \hat{z}_i[n] + \eta[n]$$

$$= \underbrace{x_k[n]h_k[n]}_{\text{Desired term}} + \underbrace{\sum_{j=1}^{k-1} [x_j[n]h_j[n] - \hat{z}_j[n]]}_{\text{Residual interference}}$$

$$+ \underbrace{\sum_{i=k+1}^{K} x_i[n]h_i[n] + \eta[n]}_{\text{Uncancelled users}}$$ (18)
The first term in (18) is the received signal for user $k$, second term is the residual interference not cancelled out for $(k-1)$ users, third term is the composite signal for $(k+1)$ to $K$ user's which have not yet been decoded, and the fourth term is the AWGN.

Substituting (13) in (18), $y_k[n]$ can be expressed as:

\[
y_k[n] = \sum_{j=1}^{k-1} e_j[n] + \sum_{i=k+1}^{K} x_i[n] h_i[n] + x_k[n] h_k[n] + \eta[n]
\]  

(19)

In order to calculate $\alpha_k$ we substitute, (19) in (8),

\[
\alpha_k = \frac{1}{M} \sum_{n=1}^{M} \left[ x_k[n] h_k[n] + \sum_{j=1}^{k-1} e_j[n] + \sum_{i=k+1}^{K} x_i[n] h_i[n] + \eta[n] + \hat{x}_k'[n] \right]
\]

(20)

where $\hat{x}_k'[n] = c_k'[n] b_k[n]$. Expansion of (20) will result in four terms. All four terms are averaged over $M$ symbols. The second term in (20) can be expressed as:

\[
\frac{1}{M} \sum_{n=1}^{M} \sum_{j=1}^{k-1} e_j[n] \cdot \hat{x}_k'[n] = \frac{1}{M} \sum_{n=1}^{M} \sum_{j=1}^{k-1} \left[ A_j h_j[n] - \alpha_j b_j[n] c_j'[n] c_k'[n] b_k[n] \right]
\]

(21)

c_j'[n], c_k'[n] and $c_k'[n]$ are PN sequences with an arbitrarily long period and are modelled as i.i.d random variables with values $\pm 1$ with probability 0.5. Therefore, expected value of the summation over $M$ symbols of $c_j'[n] c_k'[n]$ would be 0 and second moment of the crosscorrelation would go down as $1/M$. The third term would also not contribute to $\alpha_k$ because of the summation of $I_i[n] c_k'[n]$. By using the average correlations over $M$ symbols, the variance of the noise in the estimate of the amplitude decreases by a factor of $1/M$. Therefore, only the first term in (20) will determine $\alpha_k$, which can be expressed as:

\[
\frac{1}{M} \sum_{n=1}^{M} x_k[n] h_k[n] \hat{x}_k'[n] = \frac{1}{M} \sum_{n=1}^{M} A_k h_k[n] b_k[n] c_k'[n] \hat{x}_k'[n]
\]

(22)

\[
= \frac{1}{M} \sum_{n=1}^{M} A_k h_k[n] b_k[n] c_k'[n] c_k'[n] b_k[n]
\]

(23)

\[
= \frac{1}{M} \sum_{n=1}^{M} A_k h_k[n]
\]

(24)

Therefore, $\alpha_k$, can be expressed by ignoring II,III and IV terms. This has been verified through simulation in Section IV.

\[
\alpha_k = \frac{A_k}{M} \sum_{n=1}^{M} h_k[n]
\]

(25)

Applying the central limit theorem to (25), $\alpha_k$ can be written as:

\[
\alpha_k = A_k \left[ \Theta_k \cdot \sigma_h + \mu_h \right]
\]

(26)

where $\Theta_k$ is $\mathcal{N}(0, 1)$ and $\sigma_h$ and $\mu_h$ are the standard deviation and mean of $h_k$. Therefore, $\alpha_k$ is also a Gaussian random variable with unbiased mean $A_k$ and its variance decreases by a factor
of $1/M$. Estimate of the amplitude $\alpha_k$ for user $k$ is approximately equal to $A_k$ with standard deviation approaching zero. This was also verified through simulation and explained in Section IV. Using the results of (26) and substituting that in to (16), the cancellation error for user $k$ is expressed as:

$$e_k[n] = A_k b_k[n] c_k^I[n] \left[ h_k[n] - \Theta_k \cdot \frac{\sigma_h}{\sqrt{M}} - \mu_h \right]$$

(27)

evaluating the mean and variance of (27) shows that cancellation error $e_k[n]$ has a zero mean and variance equal to $\sigma_h^2$. Therefore, statistics of the cancellation error due to imperfect estimation of the channel parameters, is directly related to the channel statistics. If effective power control is incorporated so as to neutralize Rayleigh fading, it can be seen that the standard deviation of the cancellation error would be reduced. However, for a realistic SIC system, the results of (27) can be used to determine the variance of the cancellation error and incorporated for the optimum power allocation as in [3].

For a sufficiently large size $M$, $\alpha_k$ approaches $A_k$, which was also verified through simulations shown in the next section. The probability distribution of the normalized cancellation error, can be determined with the following substitution in (16):

$$\hat{e}_k[n] = c_k^I[n] b_k[n] [h_k[n] - 1]$$

(28)

where $e_k$ is normalized by $A_k$. Since $h_k[n]$ is modelled as a Rayleigh sequence with unit mean, $[h_k[n] - 1]$ results in a shifted Rayleigh sequence with zero mean. The sequence is then multiplied with a Bernoulli sequence of $\{+1, -1\}$ with probability of 0.5 each. The pdf of $h_k$ and $h_k - 1$ are shown in Fig. 3 (a) and (b). The pdf’s of $c_k^I[n] b_k[n] [h_k[n] - 1]$ with $\{c_k^I[n] b_k[n] = +1, -1\}$ each with probability 0.5 are also shown in Fig. 3 (c) and (d). Finally if we add the two pdf’s of Fig. 3 (c) and (d) we get the pdf for $\hat{e}_k[n]$ shown in Fig. 3 (e). In Fig. 4 (a) $e_k$ is modelled as a Gaussian distribution with $\sigma^2 = \sigma_h^2$. It can be seen that $\mathcal{N}(0, \sigma_h^2)$ is a close fit for $e_k$. This result is useful as it can be used to accurately model the cancellation error with SIC.
Table 1: Simulation Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>J</td>
<td>Spreading factor</td>
<td>32</td>
</tr>
<tr>
<td>M</td>
<td>Number of symbols/frame (100 bits)</td>
<td>3200</td>
</tr>
<tr>
<td>K</td>
<td>Number of full-rate users</td>
<td>10</td>
</tr>
<tr>
<td>P_b</td>
<td>Target bit error-rate (BER)</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>-</td>
<td>Number of bits used in simulation</td>
<td>200,000</td>
</tr>
<tr>
<td>σ_e</td>
<td>Estimation error, std. deviation</td>
<td>variable</td>
</tr>
<tr>
<td>ε_k</td>
<td>Fractional cancellation error for user k</td>
<td>variable</td>
</tr>
<tr>
<td>Ŕ_k</td>
<td>Receiver estimate of ε_k</td>
<td>variable</td>
</tr>
<tr>
<td>N</td>
<td>Noise Power, with $E_b/N_0 \geq 10dB \forall k$</td>
<td>$P_k J/20$</td>
</tr>
</tbody>
</table>

4 Simulation Results

To verify the analytic results, simulation for normalized $\alpha_k$ by $A_k (\hat{\alpha}_k)$ and $\hat{\epsilon}_k$ were generated using the parameters shown in Table 1 and system model shown in Fig. 2. Optimum power allocation of [3] was also incorporated with $\hat{\epsilon}_k = 0.3$: $\hat{\epsilon}_k$ is the estimate of the fractional cancellation error used in the power allocation so that all users are decoded with the same signal-to-interference ratio even in the presence of cancellation error. In order to realistically model the channel a Rayleigh fading model was used with a high Doppler [7]. Pdf for $\hat{\alpha}_k$ with 20,000 samples is shown in Fig. 5 (a) and Fig. 5 (b) is for $\hat{\epsilon}_k$ found in (26). It can be seen that for a frame size of 100 bits the results found in (26) conforms to the simulation results. More than 300,000 samples of normalized cancellation error were collected and its pdf is shown in Fig. 4 (b). A chi-square fitness test [8] for the sampled data was done with the model found for the cancellation error in (28). The fitness test was found to be within 5% tolerance for all 10 users and within 1% if the frame size was increased to 200 bits. The chi-square fitness was carried out for more than 300,000 samples using the interval size of 20. It was also seen that chi-square fitness test performed poorly if $\hat{\epsilon}_k < 0.25$, since power allocation for $\hat{\epsilon}_k < 0.25$ results in higher bit error-rate as later users are allocated much less power and the assumption, $b_k[n] = \hat{b}_k[n]$ is no more valid. If $\hat{\epsilon}_k$ is guessed incorrectly, SIC performs far worse off as compared to no interference cancellation at all.
5 Conclusion

Cancellation error in any realistic SIC system results primarily from imperfect channel estimation and is the main source of degradation in a SIC system. Yet the statistics of the cancellation error have previously been unknown. In this paper the channel was modelled as a fading channel with a unit mean Rayleigh envelope. It has been shown mathematically and verified through simulation that the mean of the cancellation error is zero and standard deviation is equal to the standard deviation of the channel. The pdf of the cancellation error based on the model used in our paper is also presented (27) and verified through simulation. These pdf’s can be seen in Fig. 3 and Fig. 4 respectively. It can also be seen that the variance in the cancellation error can be reduced if the variance of the amplitude envelope is reduced, this amounts to better power control techniques. The optimum power allocation scheme required for SIC does not add additional complexity relative to a conventional CDMA system as shown in [9],[10]. Therefore in a realistic CDMA system in which power control is implemented to overcome a fading channel, second order statistics of the cancellation error will be the same as that of power control error. With perfect power control it will be an AWGN channel where estimation/cancellation error would be negligible.

References


