Adaptive Widely Linear Minimum Output Energy Algorithm for DS-CDMA Systems

Jae-Jin Jeon, Jeffrey G. Andrews* and Koeng-Mo Sung

School of Electrical engineering and computer science, Seoul National University
*Dept. of Electrical and Computer Engineering, The University of Texas at Austin

Abstract—A novel blind widely linear (WL) minimum output energy (MOE) algorithm is proposed for the CDMA receiver. Whereas the conventional WL algorithms are only applicable to real-valued modulation, the proposed receiver is applicable to complex-valued modulation. The proposed algorithm is implemented into recursive forms using LMS- and RLS-type methods. Simulation results show that the proposed algorithms outperform the conventional versions in terms of SINR for both LMS- and RLS-type algorithms and convergence speed for only LMS-type algorithm.

I. INTRODUCTION

Multiple access interference (MAI) is the main cause of performance degradation in code division multiple access (CDMA) communication systems. In [1], various types of multiuser detection schemes have been summarized and analyzed. Because optimum multiuser detection algorithm requires much amount of computation, linear multiuser receivers are very attractive due to their low complexity and familiar structure [2] [3] [4]. Among the linear multiuser detection algorithms, a blind technique, minimum output energy (MOE) algorithm, that only requires the spreading code of the desired user and produces an unbiased minimum mean square error (MMSE) filter, is focused in this paper.

Widely linear (WL) filters utilize not only the received signal but also its complex conjugate to reduce estimate error [5] [6]. However, the WL concept does not always guarantee that it outperforms conventional methods. Performance improvement is achievable only when improper signals are considered. An improper signal is defined as having a pseudo-covariance \( E[xx^\dagger] \) that is not close to being a zero matrix. Most complex random signals generally have a pseudo-covariance matrix that is very close or exactly equal to the zero matrix, and are hence referred to as proper signals. On the other hand, real-valued modulation schemes such as ASK, BPSK, GMSK, and QAM result in improper signals and thus have non-zero pseudo-covariance. This observation has led to the recent development of widely linear filtering techniques for the receivers of real-valued modulation systems.

With this motivation, WL-MMSE algorithms have been proposed for suppressing MAI in CDMA systems [7], [8]. The algorithm in [7] is a biased MMSE filter, while the authors of [8] have derived WL-MOE algorithm that produces an unbiased WL-MMSE filter. The algorithm is implemented using LMS-type algorithm. However, the algorithm in [8] cannot handle complex-valued modulation systems because the filter output is always real. In [10], they introduced biased WL-MMSE algorithm that is implemented using data-aided RLS-type algorithm and mentioned that it can be easily reformed into blind algorithm using the approach of [9]. However, it also has the disadvantage that it cannot deal with complex-valued modulation. Since most current commercial systems employ complex-valued modulation, this is a major limitation of prior work. Furthermore, even complex-valued modulation schemes that are proper signals can result in improper received signals due to improper interference or noise [5]. Therefore, a new WL algorithm that can handle complex-valued modulation is highly desirable.

In this paper, we propose a new WL-MOE algorithm that is an unbiased WL-MMSE filter and applicable to both complex-valued and real-valued modulation systems. We develop new constraints that can address both complex-valued and real-valued modulation and propose a new algorithm satisfying the modified constraints. The proposed algorithm is implemented into adaptive filter forms using LMS- and RLS-type algorithms. Derivation of the proposed adaptive algorithms is similar to that of [8] [9] but they have difference filter forms whose dimensionality is doubled. As the conventional LMS- and RLS-type algorithms have computational complexity of \( O(n) \) and \( O(n^2) \), respectively, where \( n \) is the order of filter, the proposed algorithms maintain the same order of complexity though the dimensionality of the filter is doubled.

The rest of this paper is organized as follows. In section II, the CDMA system model is described and the modified WL-MOE algorithm is proposed. The adaptive implementation of the modified WL-MOE algorithm using LMS and RLS algorithm is described in section III. Simulation results and conclusions are presented in section IV and V, respectively.

II. A NEW WIDELY LINEAR MINIMUM OUTPUT ENERGY

A. DS-CDMA

The notation used in this paper is as follows: The notation \( (\cdot)^* \), \( (\cdot)^H \) and \( (\cdot)^T \) denote complex conjugate, complex conjugate transpose and transpose, respectively. Let

\[
\hat{x} \triangleq \begin{bmatrix} x \\ x^* \end{bmatrix},
\]

(1)

\[
\hat{x} \triangleq \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},
\]

(2)
where \( x, x_1 \) and \( x_2 \) are \( L \times 1 \) vectors. It is assumed that the two vectors \( x_1 \) and \( x_2 \) are constructed independently to each other.

The received signal in a synchronous DS-CDMA system with \( K \) users and \( L \) chips per symbol can be written as
\[
r[n] = \sum_{k=1}^{K} \sqrt{P_k} e^{j\theta_k} s_k b_k[n] + v[n],
\]

where \( P_k, \theta_k, s_k, \) and \( b_k[n] \) are, respectively, the received power, the channel phase, the \( L \times 1 \) spreading sequence and the transmitted symbol of the \( k \)-th user with power \( P_S = E[|b_k[n]|^2] \). The spreading sequences are normalized to \( s_k^H s_k = 1 \). The additive white Gaussian noise \( v[n] \) has the distribution \( N(0, \sigma^2 I) \). It is assumed that user 1 is the desired user in this paper.

### B. Modified WL-MOE

As previously noted, WL-MOE technique requires real-valued modulation, i.e., \( b_k[n] \) should be a real-valued sequence such as ASK or BPSK sequence [8]. The received signal \( \tilde{r} \) is processed by a filter \( \tilde{w} \) that is augmented in the same manner as \( r \). Therefore, the filter output is always real, i.e. \( \tilde{w}^H \tilde{r} = 2\Re\{\tilde{w}^H \tilde{r}\} \) where \( \Re\{\cdot\} \) is the real part of a complex value. Therefore, the algorithm proposed in [8] is not applicable to complex-valued modulation types such as M-PSK and M-QAM.

In order to consider complex-valued modulation the constraint \( \tilde{w}^H \tilde{S}_1 = 1 \), that is, \( \tilde{r} \) should be modified because the filtered output for the desired user \( \tilde{w}^H \tilde{r} \) becomes 
\[
2\Re\{\tilde{w}^H \tilde{r}\} \text{ and } 2\Re\{\tilde{w}^H \tilde{r}\},
\]
the information of \( \tilde{r}_1 \) and the information of \( \tilde{r}_2 \). With (16) and (17), the stochastic gradient updates for \( \tilde{c}_1 \) and \( \tilde{c}_2 \) can be obtained as
\[
\frac{\partial J}{\partial \tilde{c}_1} - (s_1^H \frac{\partial J}{\partial \tilde{c}_2}) s_1 = (\tilde{w}^H \tilde{r})^* (r^* - (s_1^H r) s_1) .
\]
In the same manner, that of the gradient \( \frac{\partial J}{\partial \tilde{c}_2} \) can be expressed as
\[
\frac{\partial J}{\partial \tilde{c}_2} - (s_1^H \frac{\partial J}{\partial \tilde{c}_2}) s_1 = (\tilde{w}^H \tilde{r})^* (r^* - (s_1^H r) s_1) ,
\]
with (16) and (17), the stochastic gradient updates for \( c_1 \) and \( c_2 \) are respectively given by
\[
c_1[n] = c_1[n-1] - \mu((\tilde{w}[n-1]^H \tilde{r}[n])^* (r[n] - (s_1^H r[n]) s_1),
\]
where the covariance matrix of the augmented received signal is given as

\[
\tilde{R} \equiv E[\tilde{r}\tilde{r}^H] = E[r\tilde{r}^H] E[r\tilde{r}^T] E[r^T] E[\tilde{r}^H] \rightleftharpoons \tilde{R} = \begin{bmatrix} \tilde{R} & \tilde{R}^* \tilde{R}^* & \tilde{R} \end{bmatrix},
\]

III. ADAPTIVE IMPLEMENTATION OF WL-MOE

Fig. 1 shows an adaptive WL filter that satisfies the modified constraints. The error, \( e = \tilde{w}^H \tilde{r} \) is utilized to update the filter coefficient. LMS- and RLS-type implementations are possible during the recursive process. In this section, two kinds of adaptive algorithms will be derived.

A. LMS type implementation

Firstly, we propose the LMS-type implementation of the modified WL-MOE algorithm. Generally, the conventional MOE filter can be rewritten in the canonical form as
\[
w = s_1 + c,
\]
where \( c \) is orthogonal to \( s_1 \), i.e., \( e^H s_1 = 0 \). This representation makes the constraint \( \tilde{w}^H s_1 = 1 \) valid.

Because the proposed WL-MOE algorithm has the constraint of (4), the canonical representation should be modified as
\[
\tilde{w} = \begin{bmatrix} s_1 \\ 0 \end{bmatrix} + \tilde{c} = \tilde{s}_1 + \tilde{c},
\]
where \( e^H \tilde{s}_1 = [e_1^H s_1 e_1^H s_1^*] = [0 0] \).

The gradient of \( J' = \|\tilde{w}^H \tilde{r}\|^2 \) with respect to \( c_1 \) and \( c_2 \) can be obtained as \( \frac{\partial J'}{\partial c_1} = (\tilde{w}^H \tilde{r})^* \text{r and } \frac{\partial J'}{\partial c_2} = (\tilde{w}^H \tilde{r})^* r^* \), respectively. The relevant component of the gradient \( \frac{\partial J'}{\partial c_1} \) orthogonal to \( s_1 \) is obtained by subtracting the projected component of the gradient onto \( s_1 \) from the gradient as
\[
\frac{\partial J'}{\partial \tilde{c}_2} - (s_1^H \frac{\partial J'}{\partial \tilde{c}_2}) s_1 = (\tilde{w}^H \tilde{r})^* (r^* - (s_1^H \text{r}) s_1) .
\]

In the same manner, that of the gradient \( \frac{\partial J'}{\partial \tilde{c}_2} \) can be expressed as
\[
\frac{\partial J'}{\partial \tilde{c}_2} - (s_1^H \frac{\partial J'}{\partial \tilde{c}_2}) s_1 = (\tilde{w}^H \tilde{r})^* (r^* - (s_1^H \text{r}) s_1) .
\]
\[ c_2[n] = c_2[n-1] - \mu(\bar{w}[n-1]^H \hat{r}[n])^* (r[n] - (s_1^T r[n]) s_1^*). \]  

Finally, the update equation of \( \bar{e} = [c_1^T \ c_2^T]^T \) can be expressed as

\[ \bar{e}[n] = \bar{e}[n-1] - \mu(\bar{w}[n-1]^H \hat{r}[n])^* (I - \bar{S}_1 \bar{S}_1^H) \hat{r}[n]. \]

The major difference of the proposed algorithm from the conventional algorithms is the form of the projection matrix, \( I - S_1 S_1^H \), while the conventional WL-MOE [8] and the conventional linear MOE [4] have \( I - \bar{s}_1 \bar{s}_1^H \) and \( I - s_1 s_1^H \), respectively.

### B. RLS type implementation

RLS-type algorithm should construct the filter that minimizes the exponentially windowed cost function given by

\[ \bar{w}[n] = \arg \min_w \sum_{m=1}^{n} \lambda^{n-m} \bar{w}^H[m], \quad \text{s.t.} \quad \bar{w}^H S_1 = e_1^T. \]

The filter can be obtained by introducing Lagrange multiplier like (7), yielding

\[ \bar{w}[n] = \hat{R}^{-1}[n] S_1 (\hat{S}_1^H \hat{R}^{-1}[n] S_1)^{-1} e_1, \]

where the correlation matrix can be adaptively estimated as

\[ \hat{R}[n] = \sum_{m=1}^{n} \lambda^{n-m} \bar{r}[m] \bar{r}^H[m], \]

and \( 0 < \lambda < 1 \) is a forgetting factor that is determined according to statistical variations of the received signal.

To reduce computational complexity, the inverse of the correlation estimate is obtained by matrix inversion lemma as

\[ \hat{P}[n] \triangleq \hat{R}^{-1}[n] = \frac{1}{\lambda} (\hat{P}[n-1] - k[n] \bar{r}^H[n] \hat{P}[n-1]), \]

where the gain vector is given by

\[ k[n] \triangleq \frac{\hat{P}[n-1] \bar{r}[n]}{\lambda + \bar{r}^H[n] \hat{P}[n-1] \bar{r}[n]}. \]

Let

\[ A[n] \triangleq (S_1^H \hat{P}[n] S_1)^{-1}. \]

With (24) and (26), the inverse matrix of \( A[n] \) can be expressed as

\[ A[n]^{-1} = \frac{1}{\lambda} (A[n-1]^{-1} - \bar{S}_1^H k[n] \bar{r}^H[n] \hat{P}[n-1] S_1). \]

After applying matrix inversion lemma to (27), recursive update of (26) can be written as

\[ A[n] = \lambda (A[n-1] - A[n-1] \bar{S}_1^H k[n] \bar{r}^H[n] \hat{P}[n-1] S_1 A[n-1] \) \[ \quad + A[n-1] \bar{S}_1^H k[n] \bar{r}^H[n] \hat{P}[n-1] S_1 A[n-1] \] \[ \quad - (1 - \bar{r}^H[n] \hat{P}[n-1] S_1 A[n-1]) \bar{S}_1^H k[n] \]

\[ = \lambda (A[n-1] + b[n] \bar{r}^H[n] \hat{P}[n-1] S_1 A[n-1]) \]

where

\[ b[n] \triangleq A[n-1] \bar{S}_1^H k[n] (1 - \bar{r}^H[n] \hat{P}[n-1] S_1 A[n-1])^{-1}. \]

From (22), (24) and (28), the filter can be obtained adaptively as

\[ \bar{w}[n] = \hat{R}[n]^{-1} S_1 A[n] e_1 \]

\[ = (\hat{P}[n-1] - k[n] \bar{r}^H[n] \hat{P}[n-1]) S_1 \]

\[ \cdot (A[n-1] + b[n] \bar{r}^H[n] \hat{P}[n-1] S_1 A[n-1]) e_1 \]

\[ = \hat{w}[n-1] + (\lambda \hat{P}[n] S_1 b[n] - k[n]) e^*[n], \]

where \( e[n] \triangleq \bar{w}^H[n-1] \bar{r}[n] \).

In Table I and II, the proposed adaptive WL-MOE algorithms are summarized.

### IV. Simulation results

Experiment conditions for all the numerical simulations are as follows: (1) The number of users and chips per symbol are both 16 and the sequences for users are chosen randomly, (2) The power of the transmitted signal is \( P_k = 1 \) for BPSK, or \( P_k = 2 \) for QPSK (3) the SNR of the received signal before despeading is 5dB, (4) the channel phases are assumed to be uniformly distributed from 0 to 2\( \pi \), and (5) the step-size and the forgetting factor are \( \mu = 10^{-3} \) and \( \lambda = 0.999 \) for LMS- and RLS-type algorithm, respectively.

In Fig. 2, BPSK modulation is used for all the users and LMS-type algorithms are applied. It shows the performance of conventional MOE, conventional WL-MOE and the proposed WL-MOE. The proposed WL-MOE algorithm clearly outperforms the other two algorithms in terms of received SINR after a reasonable number of iterations. Although the orthogonal components of the filters are set to zero, the conventional WL-MOE algorithm has higher SINR than the others after the first
iteration because the cost function of the conventional WL-MOE has the form $\Re\{s^H r[1]\}$ instead of $s^H r[1]$. However, as the iterations continue, the proposed algorithm increases SINR faster, i.e. the convergence speed of the proposed algorithm is also better than the other algorithms.

In Fig. 3, interference is composed of BPSK and QPSK signals, i.e. interference is improper while the desired signal is proper. Including the desired user, 5 users use QPSK modulation while the other users are still based on BPSK modulation. LMS-type algorithms are applied as well. In this case, the conventional WL-MOE algorithm is not feasible. In Fig. 3 it can be confirmed that the proposed algorithm is superior to the conventional MOE algorithm since the improper aspects of the aggregate interference can be exploited.

In Fig. 4, BPSK modulation is used for all users and RLS-type algorithms are applied. It is well-known that RLS-type algorithms require more computational power than LMS-type algorithms and the former has faster convergence speed. Though we cannot tell that the proposed algorithm has faster convergence speed over the conventional MOE algorithm, it has better steady-state SINR performance.

In Fig. 5, both BPSK and QPSK modulation are used to construct improper interference while the desired user uses QPSK modulation. The number of users based on QPSK modulation is 5. Here, RLS-type algorithms are applied. The proposed algorithm also has improved performance in terms of steady-state SINR.

Direct comparison between LMS-type and RLS-type algorithm is difficult because transient and steady-state performance is dependent on the parameters such as step-size $\mu$ and forgetting factor $\lambda$. However, it is obvious that RLS-type algorithms have faster convergence speed over LMS-type algorithms from the simulation results.

**V. CONCLUSION**

In this paper, a new widely linear minimum output energy (WL-MOE) algorithm has been proposed to mitigate multiple access interference in multiuser communication systems such as CDMA cellular systems. The proposed algorithm is applicable to the complex-valued modulation types needed for high-speed wireless data systems, whereas prior work was applicable only to real-valued (one dimensional) modulation. It is implemented into two recursive forms such as LMS-type and RLS-type algorithm. Numerical results have further confirmed that the proposed algorithm outperforms the conventional MOE and WL-MOE algorithms in terms of SINR when either the desired signal or a non-negligible portion of the interference is improper. In case of LMS-type algorithm, the proposed method also has faster convergence speed than the conventional methods. The resulting improved SINR will allow improved BER or capacity, while the increased tracking ability will improve operation in fading channels.

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**REFERENCES**

Fig. 3. SINR with BPSK+QPSK modulation when LMS-type algorithms are applied

Fig. 4. SINR with BPSK modulation when RLS-type algorithms are applied

Fig. 5. SINR with BPSK+QPSK modulation when RLS-type algorithms are applied