Multi-code Multicarrier CDMA: Performance Analysis

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Abstract—A novel multi-code multicarrier code division multiple access (MC-MC-CDMA) system is proposed and analyzed in a frequency selective fading channel. By allowing each user to transmit multiple orthogonal codes, the proposed MC-MC-CDMA system can support various data rates, as required by next generation standards, and achieve spreading gain in the time domain. Multicarrier CDMA provides robustness to multipath and spreading in the frequency domain. The bit error rate of the system is analytically derived in frequency selective fading, with Gaussian noise and multiple access interference. The results show that the proposed MC-MC-CDMA system clearly outperforms both single-code multicarrier CDMA (MC-CDMA) and single-carrier multi-code CDMA in a fixed bandwidth allocation. This indicates that MC-MC-CDMA should be seriously considered for next generation cellular systems.

I. INTRODUCTION

Future wireless systems such as fourth generation (4G) cellular will need to flexibly provide subscribers with a variety of services such as voice, data, images, and video. Because these services have widely different data rates and traffic profiles, future generation systems will have to accommodate a wide variety of data rates. Code division multiple access (CDMA) has proven very successful for large scale cellular voice systems, but there is some scepticism about whether CDMA will be well-suited to non-voice traffic [1]. This has motivated research on multi-code CDMA systems which allow variable data rates [2], [3], [4] by allocating multiple codes, and hence varying degrees of capacity to different users. Meanwhile, multicarrier CDMA (MC-CDMA) has emerged as a powerful alternative to conventional direct sequence CDMA (DS-CDMA) in mobile wireless communications [5], [6], [7], [8], and has been shown to have superior performance to single carrier CDMA in multipath fading. This paper proposes and analyzes a new multiple access and modulation technique, a combined multi-code, multicarrier CDMA system for exploiting the best aspects of each of these previous systems.

Multi-rate transmission for single-carrier CDMA systems in AWGN channels has been previously considered, e.g. [9], [10]. Multi-code techniques tradeoff the number of supportable subscribers with the per subscriber data rate, said another way, the number of simultaneous higher data rate users in a multicode CDMA system will be less than the number of equal data rate users in a traditional CDMA system. A variation of the multi-code scheme, which supports variable data rates by varying the set of code sequences assigned to the each user, has been proposed in [4], [11]. The users communicate their data by choosing one sequence from their code set to transmit over the common channel. Also, in [11] the performance of multi-code CDMA was considered only in an AWGN channel.

There have been to our knowledge two previous studies on multi-rate transmission for multicarrier direct sequence CDMA systems [12], [13]. The study of multi-rate transmission for multicarrier direct sequence CDMA systems based on the concepts of multi-code access and variable-spreading gain code access was first presented in [12]. In multi-code CDMA, the data stream of a user with rate $M$ is first multiplexed into $M$ different serial streams with a base data rate, and each serial stream is treated as an individual user. Each of the $M$ serial streams are then converted into $P$ parallel sub-streams and spread by the same spreading code with a constant spreading factor. The system in [12] has $M$ times more interference per user, because each of the $M$ data streams are treated as independent users. Therefore, the system of [12] experiences more interference as the data rate increases, even with a fixed number of users. Also, multi-rate transmission for frequency spread multicarrier CDMA has been studied in [14]. In the multi-rate multicarrier CDMA system in [14], the subcarriers are divided into $M$ groups according to the required data rate. Therefore, when the number of subcarrier is fixed, the spreading gain in frequency domain for each data is decreased with increasing data rate. The single-carrier multi-code CDMA system in [4] addresses the interference scaling problem of multi-code systems by using just one code sequence instead of spreading each of the $M$ multiplexed data streams so that the interference does not increase linearly with the data rate. However, this system [4] does not achieve the frequency diversity benefits of multicarrier modulation.

This paper proposes a multicarrier CDMA system with multi-code. Our proposed multi-code multicarrier CDMA (MC-MC-CDMA) system achieves the advantages of both systems: (i) variable data rates without interference scaling and (ii) enhanced robustness to a multipath fading channel. Moreover, the proposed system has both time and frequency spreading gain.
finally conclusions are presented in Section V. with a performance comparison presented in Section IV. our proposed multi-code multicarrier CDMA system. The bit organized as follows: section II discusses the system model of

Fig. 1. Transmitter and receiver structure of a MC-MC-CDMA system
to exploit the diversity and interference averaging properties of multicarrier modulation and CDMA.

The BER performance of the proposed system is derived analytically and the improvement of the proposed system over an MC-CDMA system is shown through analysis and simulations in a frequency selective fading channel. The rest of the paper is organized as follows: section II discusses the system model of our proposed multi-code multicarrier CDMA system. The bit error probability of the proposed system is derived in section III, with a performance comparison presented in section IV. Finally, conclusions are presented in section V.

II. SYSTEM MODEL

The proposed MC-MC-CDMA system depicted in Fig. 1 uses a set of \( M \) codes called the code sequence set for \( M \)-ary modulation. Each user has the same code sequence set which represents an information data symbol of \( \log_2 M \) bits. The size of the code sequence set depends on the required data rate. In the usual CDMA case, the size of the code sequence set is 2, i.e. there are two sequences in the set, one to represent a ‘0’ and the other to represent a ‘1’. In the proposed system, each user has a set of \( M \) code sequences, where \( \log_2 M \) is the ratio of the required data rate to the base data rate (1 bit/symbol). Therefore, if the data rate is to be made \( \log_2 M \) times the base data rate, the size of the code sequence set is \( M \) and each \( M \)-ary data symbol is mapped to one of the code sequences of length \( N \). This code length \( N \) is fixed over all different values of \( M \). Thus, varying the data rate does not change the code length \( N \), but it does change the size of the code sequence set \( M \). In order to maintain linear independence between the code sets, it is required that \( M \leq N \).

As shown in Fig. 1(a), an \( M \)-ary symbol selects one of \( M \) pre-mapped code sequences for transmission. Each code sequence has a time domain spreading ratio of \( N \). Each bit of the length \( N \) code sequence is copied onto the \( L \) subcarrier branches and multiplied with the user-specific scrambling code of the corresponding branch, \( c_{k,l} \). Note that the \( c_{k,l} \) are static in time so that the spreading at this stage is only in frequency, allowing users to choose specific codes that have low cross-correlations. Each of these branches then modulates one of the \( L \) orthogonal subcarriers and the results are summed. As in the popular orthogonal frequency division multiplexing (OFDM), this process can be implemented using a size \( L \) Inverse Fast Fourier Transform (IFFT) to replace the subcarrier multiplication and summation. Unlike OFDM, which is used to increase the ISI-free data rate, in multicarrier CDMA the same information bit is replicated on all subcarriers to achieve a spreading gain for multiple access. Also, a cyclic prefix is not typically employed in multicarrier CDMA because self-ISI is a minor effect compared to multiple access interference.

To formalize the analysis, consider that each user has the same code sequence set which can be written as

\[ \Omega = \{ \nu(n)|1 \leq m \leq M \} \]  

User \( k \)’s \( i \)-th \( M \)-ary data symbol \( \nu_{k,i} \) is mapped to one of the code sequences in \( \Omega \). Thus the \( i \)-th transmit sequence of user \( k \) before multicarrier modulation can be written as

\[ S_{k,i}(n) = \nu_{k,i}(n) \]  

\[ S_{k,i}(t) = \sum_{n=0}^{N-1} S_{k,i}(n)h(t - nT_c - iT_s), \]  

where \( S_{k,i}(n) \) is the \( n \)-th bit in the \( i \)-th code sequence of user \( k \), \( T_c \) is the bit duration of the code sequence, \( T_s \) is the symbol duration \( (T_s = NT_c) \), and \( h(t) \) is the normalized rectangular waveform defined as

\[ h(t) = \begin{cases} \frac{1}{\sqrt{T_s}}, & 0 \leq t \leq T_c \\ 0, & \text{elsewhere} \end{cases} \]  

According to the block diagram of the MC-MC-CDMA transmitter shown in Fig. 1(a) and (4), the transmitted BPSK signal of user \( k \) can be written as

\[ s_k(t) = \sum_{i=-\infty}^{\infty} \sum_{l=1}^{L} \sum_{n=0}^{N-1} S_{k,i}(n)h(t - nT_c - iT_s) \times c_{k,l}(Ni + n) \cos(\omega_l t + \theta_{k,l}) \]  

where \( c_{k,l}(Ni + n) \) is the \( l \)-th chip of the \( n \)-th bit in the \( i \)-th code sequence of user \( k \), \( \omega_l \) is the \( l \)-th carrier frequency, \( \theta_{k,l} \) is the random phase of the \( l \)-th subcarrier of user \( k \), and uniformly distributed over \([0,2\pi]\), \( L \) is the number of subcarriers, and \( N \) is the length of code sequence.

In the receiver of Fig. 1(b), a size \( L \) FFT is applied to the input. The output of the FFT is then despread to generate each chip of the received code sequence. The \( N \) regenerated chips compose one code sequence, and the regenerated code is the input of the matched filter bank to detect the transmitted symbol.
The $N$ despread bits form a degenerated code sequence, which is correlated with each of the possible $M$ code sequences. The sequence that gives maximum correlation is then mapped back into an $M$-ary symbol. The use of this multicarrier scheme provides frequency diversity for multipath mitigation so that no RAKE receiver is required, and a greater percentage of the received energy is actually collected for detection.

III. SYSTEM ANALYSIS

In this section, the output of the matched filter is analyzed and BER expression for the case of $M = 2$ is derived. Because the MC-MC-CDMA transmitted waveform consists of a large number of narrowband subcarriers, the channel model can be reasonably approximated as a frequency selective Rayleigh fading channel where each subcarrier experiences flat Rayleigh fading. The subchannels can be written as

$$h_{k,l}(t) = \beta_{k,l}(t)e^{j\psi_{k,l}(t)} \quad (6)$$

which is a complex Gaussian random variable with zero mean and variance $\sigma^2$, characterized by a Rayleigh distributed amplitude attenuation $\beta_{k,l}(t)$, and a phase shift $\psi_{k,l}(t)$. While in practice there would be some correlation between adjacent subchannels, it is assumed here that $h_{k,m}(t)$ are uncorrelated and identically distributed for different $k$ and $l$.

If there are $K$ active users, the received signal of the synchronous system is

$$r(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} \sum_{l=1}^{L} \sum_{n=0}^{N-1} \beta_{k,l}(t)S_{k,l}(n)h(t-nT_c-iT_s) \times c_{k,l}(Ni+n)\cos(\omega_l t + \phi_{k,l}(t)) + n(t), \quad (7)$$

where $\phi_{k,l}(t) = \theta_{k,l} + \psi_{k,l}(t)$ and $n(t)$ is the additive white Gaussian noise with zero mean and power spectral density $N_0$. The amplitude attenuation and phase shift is considered to be constant over the time interval $[0,T_c]$.

From Fig. 1 and (7), the matched filter output at the receiver can be derived. Assume that the user 1 is a desired user and the $0^{th}$ transmitted $M$-ary symbol of the desired user is $m$ which represents $M$-ary data symbol $b_{1,0}$, then the $0^{th}$ output of the filter matched to the code $m$ of user 1 is

$$U_1 = \int_0^{T_c} x_1(t) \sum_{n=0}^{N-1} \nu_m(n)h(t-nT_c)dt, \quad (8)$$

where the demodulated code sequence of user 1, $x_1(t)$ is

$$x_1(t) = \sum_{j=0}^{N-1} \lambda_{1,j}h(t-jT_c) \quad (9)$$

$$\lambda_{1,j} = \frac{1}{T_c} \int_{jT_c}^{(j+1)T_c} r(t) \sum_{q=1}^{L} c_{1,q}(j)\cos(\omega_q t + \phi_{1,q}(j))\alpha_{1,q}dt. \quad (10)$$

In this paper, we consider Equal Gain Combining (EGC), because of its simplicity as the receiver does not require the estimation of the channel’s transfer function. Thus, $\alpha_{1,q} = 1$ for all $q$. The matched filter output (8) can be written as [6][8]

$$U_1 = D_1 + I_1 + J_1 + \eta, \quad (11)$$

where $D_1$ is desired signal for user 1, $I_1$ is the same carrier interference from other users, $J_1$ is other carrier interference from other users, and $\eta$ is the AWGN term with variance $N_cLN/4T_c$. The desired signal $D_1$ is

$$D_1 = \frac{1}{2} \sum_{n=0}^{N-1} \nu_m(n)\nu_{b_{1,0}}(n) \sum_{q=1}^{L} \beta_{1,q}(0). \quad (12)$$

In (11), the same carrier interference term $I_1$ can be written as

$$I_1 = \frac{1}{2} \sum_{k=2}^{K} \sum_{l=1}^{L} \sum_{n=0}^{N-1} \beta_{k,l}(0)\nu_m(n)\nu_{b_{1,0}}(n) \times c_{k,l}(n)\cos(\phi_{k,l}(n) - \phi_{1,l}(n)), \quad (13)$$

and other carrier interference term $J_1$ can be written as

$$J_1 = \frac{1}{2T_c} \sum_{n=0}^{N-1} \sum_{k=2}^{K} \sum_{l=1}^{L} \sum_{q=1}^{L} \beta_{k,l}(0)\nu_m(n)\nu_{b_{k,0}}(n) c_{k,l}(n) c_{1,q}(n)$$

$$\times \int_{0}^{T_c} \cos((\omega_l - \omega_q)t + \phi_{k,l}(t) - \phi_{1,q}(t))dt. \quad (14)$$

As shown in Appendix, $I_1$ and $J_1$ have zero mean and variance

$$\text{var}(I_1) = \frac{1}{4} (K-1)LN\sigma^2 \quad (15)$$

$$\text{var}(J_1) = \frac{\sigma^2 N(K-1)}{8\pi^2} \sum_{l=1}^{L} \sum_{q=1}^{L} \frac{1}{(l-q)^2} \quad (16)$$

respectively. Therefore, assuming that we know the transmitted code sequence, the mean and variance of $U_1$, the statistics of output of the filter matched to the transmitted code sequence at time 0 is as follows:

$$E(U_1) = \frac{1}{2} \sum_{n=0}^{N-1} \sum_{l=1}^{L} \beta_{1,l}(n) \quad (17)$$

and

$$\text{var}(U_1) = \frac{1}{4} (K-1)LN\sigma^2 + \frac{\sigma^2 N(K-1)}{8\pi^2} \sum_{l=1}^{L} \sum_{q=1}^{L} \frac{1}{(l-q)^2} + \frac{N_0LN}{4T_c}. \quad (18)$$

For the case of $M = 2$, we have two orthogonal code sequences for binary signal. Therefore, the probability of bit error conditioned on the collection of subcarrier channels can be obtained by

$$P_{e,M=2} = Q\left(\frac{E(U_1)}{\sqrt{\text{var}(U_1)}}\right) \quad (19)$$

and the bit error rate (BER) can be evaluated by Monte Carlo integration.
IV. PERFORMANCE COMPARISON FOR MULTI-CODE MULTICARRIER CDMA

In this section, the numerical BER performance of MC-MC-CDMA is compared to competing systems for the downlink case in a frequency selective fading channel, and some properties of MC-MC-CDMA are observed. For the MC-MC-CDMA system, \( N = 16 \) for the length of the code sequence, \( L = 16 \) for the number of subcarriers, and \( M = 2, 4, 8, 16 \) for the \( M \)-ary symbols.

Fig. 2 shows the BER performance of the MC-MC-CDMA system with various \( M \), the MC-CDMA system [6], and the multi-code single-carrier CDMA (MC-SC-CDMA) system [4]. In order to fairly compare the performance of these systems which have different subcarrier channel bandwidths, the number of subcarriers in each system is fixed to make the total bandwidth equal for all three systems. For example, when the length of the code sequence \( N = M = 16 \), the MC-MC-CDMA system transmits 16 bits within one symbol time (4 information bits). That means the MC-MC-CDMA system uses 4 times more bandwidth compared to an MC-CDMA system with the same data rate. Therefore, we use 16 subcarriers for the MC-CDMA system and 64 subcarriers for the MC-CDMA system. For the MC-SC-CDMA system, the length of the code sequence is 256. In this way, all three systems use the same total bandwidth in the simulation. As can be seen, even though the MC-CDMA system can get better frequency diversity by using more subcarriers, the proposed MC-MC-CDMA system performs better. By using multicarrier modulation, the MC-MC-CDMA system also easily outperforms the MC-SC-CDMA system in a frequency selective fading channel.

In Fig. 3, the analytical expressions and the simulation results in a Rayleigh fading channel are compared. Here, \( M = 2 \) and \( K = 10 \) or 16 for both. The plot shows that the analytical derivations agree closely with the simulation results.

The BER performance versus the number of users for both systems with an SNR of 10dB is shown in Fig. 4. At the same BER, data rate per user, and consumed bandwidth, the MC-MC-CDMA system can support more users than the MC-CDMA system. For example, at the BER of \( 3 \times 10^{-3} \), the number of users supported by the MC-MC-CDMA is about 13, while it is about 7 for the MC-CDMA system. These are both uncoded systems with a total spreading gain of 64.

Fig. 5 shows the received signal-to-
interference-plus-noise ratio (SINR) before detection versus SNR with the various \( K \) and \( M \). In this system, the mean of all interference power is assumed to be equal. The received SINR of the MC-MC-CDMA system varies according to the variation of \( K \) and SNR, but not \( M \). Since the length of the code sequence \( N \) is fixed over all different value of \( M \), the received SINR is not changed according to \( M \) as shown in Fig. 5. However, because higher \( M \) cause a proportionally reduced spreading gain, the BER after despreading increases with \( M \), as seen in Fig. 2.

V. CONCLUSION

In this paper, the multi-code multicarrier CDMA system was proposed and the BER performance of the system was analyzed. By using the multi-code concept, the MC-CDMA system gets spreading gain as well as frequency diversity. In addition, various data rates can easily be supported by changing the size of the code sequence set. With the same total bandwidth, both analytical and simulation results showed that the proposed MC-MC-CDMA system performs better than both the MC-CDMA system and the MC-SC-CDMA system in terms of BER and capacity in a frequency selective Rayleigh fading channel. This shows that data rate flexibility can be achieved in a multicarrier CDMA system without any sacrifice in performance.

APPENDIX I

DECISION VARIABLE WHICH IS THE OUTPUT OF THE MATCHED FILTER

For the analysis of the BER performance of the proposed system, the matched filter output can be written as (11). From (8) and (11), due to the orthogonality between subcarriers, the desired signal \( D_1 \) can be written as

\[
D_1 = \frac{1}{T_c} \sum_{n=0}^{N-1} \nu_m(n) \nu_{b_{1,q}}(n) \int_{nT_c}^{(n+1)T_c} h(t-nT_c) e^{j2\pi \omega_q t + \phi_{1,q}(n)} dt
\]

\[
= \frac{1}{2} \sum_{n=0}^{N-1} \nu_m(n) \nu_{b_{1,q}}(n) \sum_{q=1}^{L} \beta_{1,q}(n),
\]

The interference term \( I_1 + J_1 \) is given by

\[
I_1 + J_1 = \frac{1}{T_c} \sum_{j=0}^{N-1} \nu_m(j) \int_{jT_c}^{(j+1)T_c} \sum_{k=2}^{K} \sum_{l=1}^{L} \sum_{n=0}^{N-1} \beta_{k,l}(n)
\]

\[
\times \nu_{b_{k,0}}(n) h(t-nT_c) e^{j2\pi \omega_q t + \phi_{k,l}(n)}
\]

\[
\times \sum_{l=1}^{L} c_{1,q}(j) \cos(\omega_q t + \phi_{1,q}(j)) dt.
\]

Both \( I_1 \) and \( J_1 \) can be simplified as (13) and (14). As shown in (13),(14),(20), and (21), the matched filter output is expressed in terms of correlation functions of the code sequences. Now we can derive the variance of the term \( I_1 \) and \( J_1 \) for the EGC case. All cross terms are uncorrelated due to the random phase, and \( I_1 \) and \( J_1 \) are zero mean. Therefore, with the fact that \( E[\beta_{k,l}^2] = 2\sigma^2 \), the variance of \( I_1 \) and \( J_1 \) can be simplified as

\[
\text{var}[I_1] = \frac{1}{4} \sum_{k=2}^{K} \sum_{l=1}^{L} \sum_{n=0}^{N-1} E[\beta_{k,l}^2(n)] E[\nu_m^2(n) \nu_{b_{1,0}}^2(n)]
\]

\[
\times c_{k,l}(n) c_{l,1}(n) E[\cos^2(\phi_{k,l}(n) - \phi_{1,l}(n))]
\]

\[
= \frac{1}{4} (K-1) LN \sigma^2
\]

\[
\text{var}[J_1] = \frac{1}{2T_c^2} \sum_{k=2}^{K} \sum_{n=0}^{N-1} \sum_{l=1}^{L} \sum_{q \neq l} E[\beta_{k,l}^2(n)]
\]

\[
\times E[\nu_m^2(n) \nu_{b_{k,0}}^2(n) c_{k,l}(n) c_{l,1}(n)]
\]

\[
\times E \left[ \left( \int_0^{T_c} \cos((\omega_l - \omega_q t + \phi_{k,l}(n) - \phi_{1,l}(n)) dt \right)^2 \right]
\]

\[
= \sigma^2 N (K-1) \sum_{l=1}^{L} \sum_{q \neq l} \frac{1}{(l-q)^2}
\]

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