

# To spread or not to spread in wireless ad hoc networks?

**Jeffrey G. Andrews**

**Collaborators:**

**Steven Weber<sup>a</sup>, Xiangying Yang, Gustavo de Veciana**

**Wireless Networking and Communications Group**

**The University of Texas at Austin**

**Bell Labs, Lucent Technologies**

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<sup>a</sup>With Drexel Univ.

## A Couple of Open Questions

- How does ad hoc network capacity scale with design parameters like path loss, transmit power, modulation type, spreading factor, target SINR, ...
- Does it make sense to use spread spectrum techniques in an uncentralized wireless network?
  - Can take capacity-centric viewpoint...
  - Or can take holistic viewpoint

## This talk in one slide...

- Traditional capacity (Shannon or otherwise) measures can't answer these questions
- Using an appropriate new capacity framework, it at first appears like CDMA is not the way to go
- Nevertheless, a holistic perspective once again blurs the issue

## Transport Capacity

- The Shannon capacity region of ad hoc networks is  $n(n - 1)$ -D.
- Key Idea of [Gupta and Kumar, 2000]: a fundamental conflict in ad hoc networks exists between capacity and connectivity
  - Maximize instantaneous capacity by having many short range transmissions
  - But to keep the network connected and for information to be moved around, it must travel over some distance
  - Hence the natural measure is bit rate multiplied by the distance the bits travel, or *transport capacity* (bit-meters/sec).
- Key result is that the best-case transport capacity of an ad hoc network is  $O(\sqrt{n})$ , or  $O(\frac{1}{\sqrt{n}})$  per node.
- Various permutations on this such as  $v \uparrow \Rightarrow O(1)$  [GroTse, 02] and  $B \rightarrow \infty \Rightarrow \sim O(n)$  [NegRaj, 04].

## Transmission Capacity

- To explore the non-asymptotic regime, we define the notion of *transmission capacity*.
- *Definition.* Transmission Capacity: the maximum allowable intensity of transmissions per unit area for which  $P[\text{SINR} < \beta] \leq \epsilon$ , where  $\beta$  is the target SINR and  $\epsilon$  is the outage probability.
- How does this differ from transport capacity?

## Transmission Capacity (cont)

- Since transport capacity maximizes the density of allowable transmissions over some radio distance, maximizing transmission capacity will maximize transport capacity, for a given transmit range  $r$ .
- Specifically, let the nodes be randomly spatially distributed according to a Poisson Point Process [Baccelli *et al*] with parameter  $\lambda$ .
  - Higher  $\lambda \Rightarrow$  denser network
  - Lower  $\lambda \Rightarrow$  sparser network
- $\lambda$  has units transmissions/ $m^2$  and  $(1 - \epsilon)\lambda^\epsilon$  is the transmission capacity.
- Unlike Baccelli *et al*, fix the outage probability at  $\epsilon$ , in contrast to their notion of expected forward progress – which advocated  $\epsilon = 1 - e^{-1} \approx 0.63\%$  and wastes a lot of energy.

## Goals of this talk

- Develop upper ( $\lambda_u$ ) and lower ( $\lambda_l$ ) bounds for the transmission capacity of spread spectrum ad hoc networks, specifically:
  - Direct sequence spread spectrum (DS-SS) with spreading factor  $M$ .
  - Frequency hopping spread spectrum (FH-SS) with  $M$  frequency slots.
  - DS-SS with (imperfect) successive interference cancellation (SIC) of  $K$  interfering nodes.
  - Traditional (non-spread) transmission is the special case  $M = 1$ .
- Using these bounds, see how network capacity scales with multiple access type, spreading factor, path loss exponent, outage probability, etc.
- Consider other “softer” metrics and discuss how they might change the conclusions.

## Outage Probability

- Let's first restrict our attention to DS and FH spread spectrum (we'll consider SIC later).
- The outage probability for each can be written as:

$$FH \quad \mathbb{P}^0 \left( \frac{\rho r^{-\alpha}}{\eta + \sum_{i \in \Pi_m} \rho R_i^{-\alpha}} \leq \beta \right) \leq \epsilon,$$

$$DS \quad \mathbb{P}^0 \left( \frac{\rho r^{-\alpha}}{M\eta + \sum_{i \in \Pi} \rho R_i^{-\alpha}} \leq \frac{\beta}{M} \right) \leq \epsilon.$$

- $\eta$  = noise power,  $\rho$  = transmit power (fixed for all nodes),  $\alpha$  = path loss exponent,  $\beta$  = SINR requirement,  $\epsilon$  = outage probability,  $r$  = communication distance,  $R_i$  = interference distance for interferer  $i$ ,  $M$  = spreading factor.
- OK, it's a bit of notation, but pretty standard CDMA expressions.

## Simplified Outage Probability

- The previous expressions can be massaged into a more convenient format:

$$FH \quad \mathbb{P}^0 \left( \sum_{\Pi_m} R_i^{-\alpha} \geq \kappa \right) \leq \epsilon,$$

$$DS \quad \mathbb{P}^0 \left( \sum_{\Pi} R_i^{-\alpha} \geq M\kappa \right) \leq \epsilon.$$

for  $\kappa = \frac{r^{-\alpha}}{\beta} - \frac{\eta}{\rho}$ .

- Hence, outage probability can be thought of as being entirely dependent on the positions of the interferers.

## Overview of analysis

- Since positions of interferers determine outages, can just integrate Poisson 2-D pdf over the plane to find outage
- Problem: integrating to  $R = \infty$  is not tractable  $\Rightarrow$  exact expressions are not attainable
- Instead, we will attempt to find (reasonably) tight upper and lower bounds.

## Outage Events – Direct Sequence

- The following expressions define the important outage events for a receiver placed at the origin.

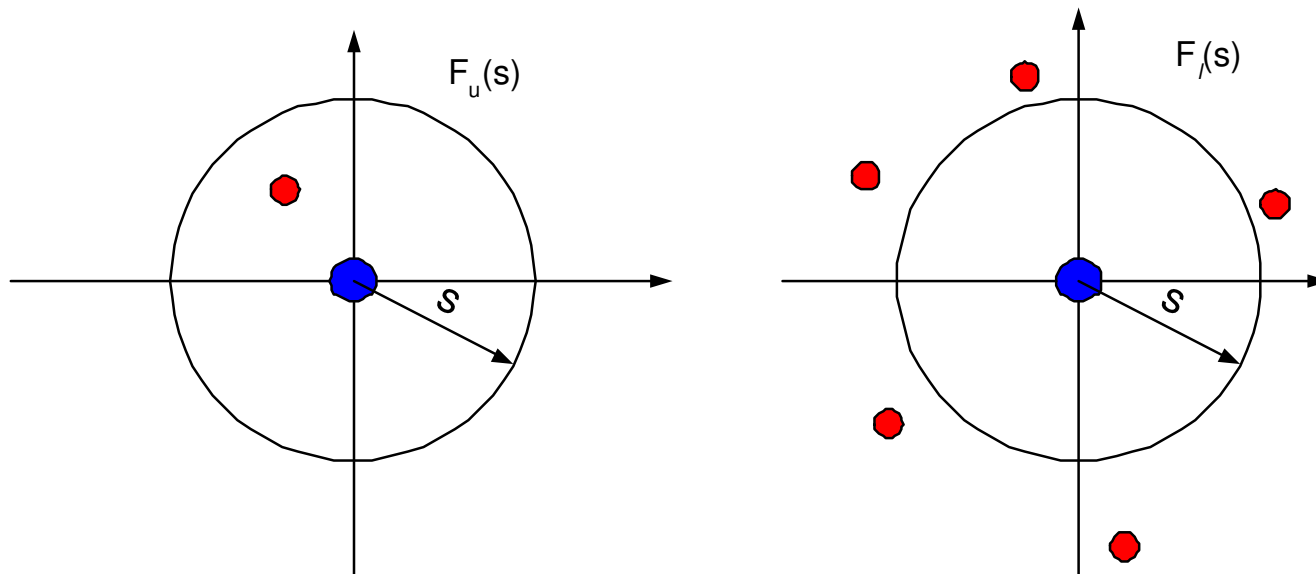
$$F = \left\{ \omega \mid \sum_{i \in \Pi(\omega)} R_i^{-\alpha}(\omega) \geq M\kappa \right\}$$

$$F_u(s) = \left\{ \omega \mid \Pi(\omega) \cap b(0, s) \neq \emptyset \right\}$$

$$F_l(s) = \left\{ \omega \mid \sum_{i \in \Pi(\omega) \cap \bar{b}(0, s)} R_i^{-\alpha}(\omega) \geq M\kappa \right\}$$

- $F$  consists of all outage events
- $F_u(s)$  is when a single transmitter is within a distance  $s$  of the origin. (May or may not cause an outage – that depends on the value of  $s$ ).
- $F_l(s)$  is where the set of transmitters outside of the ball  $b(0, s)$  generate enough aggregate interference to cause an outage

## Outage events



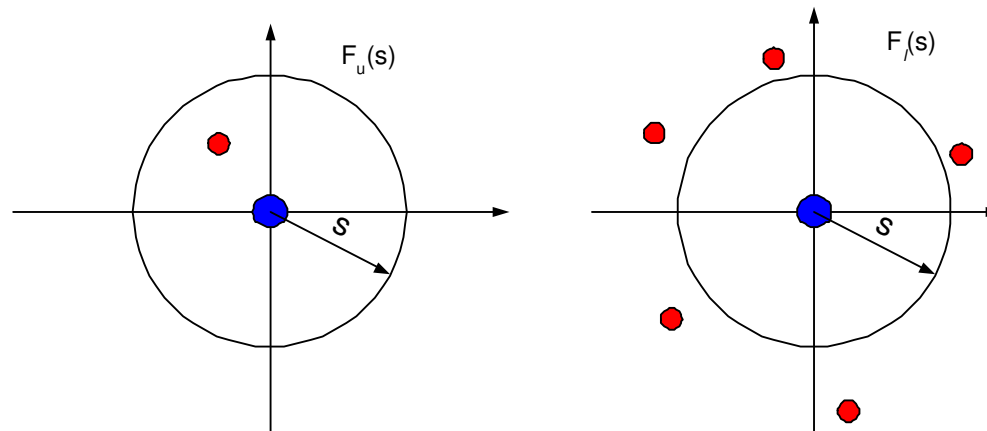
Fact:

- $F_u(s) \subset F$  for all  $s \leq (M\kappa)^{-\frac{1}{\alpha}}$ , where again  $\kappa = \frac{r_{TX}^{-\alpha}}{\beta} - \frac{\eta}{\rho}$ .

That is,  $s \leq (M\kappa)^{-\frac{1}{\alpha}}$  corresponds to a node that is close enough to the receiver to always cause an outage.

## Relationships between outage events

- $F \subset (F_l(s) \cup F_u(s))$  for all  $s$ . Similarly, can show  $F_l(s) \subset F$  and  $F_u(s) \subset F$  for  $s \leq (M\kappa)^{-\frac{1}{\alpha}} \Rightarrow F = (F_l(s) \cup F_u(s)), s \leq (M\kappa)^{-\frac{1}{\alpha}}$ .
  - If there is an outage, but not enough aggregate interference outside of  $s$ , then there must be an interferer inside of  $s$ .
  - If there is no node in  $s$ , then all the interference comes from outside of  $s$ .
  - An outage can also be caused by both, i.e.  $F_l(s) \cap F_u(s)$ .



## Upper bound on Transmission Capacity

- The upper bound  $\lambda_u(s)$  is the maximum transmission density whereby the probability of a node inside the critical radius is below the outage, i.e.  $F_u(s) \subset F = \epsilon$ .

- This is an upper bound since it neglects the interference outside of  $s$ .

$$\mathbb{P}^0(F_u(s)) = 1 - \mathbb{P}^0(\bar{F}_u(s)) = 1 - \mathbb{P}(\Pi \cap b(0, s) = \emptyset) = 1 - e^{-\lambda\pi s^2},$$

- Solving for  $\lambda$  yields (exactly)

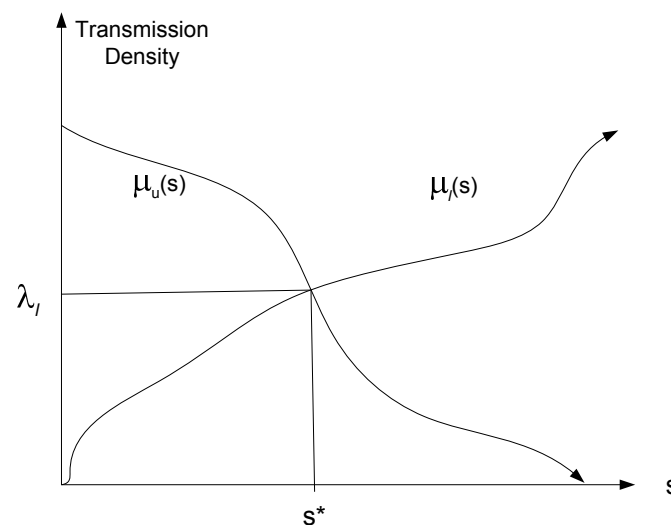
$$\lambda_u(s) = -\frac{1}{\pi} s^{-2} \ln(1 - \epsilon)$$

- Evaluating this at the critical radius  $s = (M\kappa)^{-\frac{1}{\alpha}}$  gives

$$\lambda_u^{DS} = -\frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \ln(1 - \epsilon) \approx \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

## Lower Bound Derivation

- Lower bound is more challenging to find, but since  $F \subset (F_l(s) \cup F_u(s))$ , and since  $F_l(s) \perp F_u(s)$ , can investigate individually:
  - $\mu_u(s)$  as the largest density possible such that the probability of an interferer  $\in b(0, s) \leq \epsilon$ .
  - $\mu_l(s)$  as the largest density possible such that  $\mathbb{P}^0(F_l(s)) \leq \epsilon$ . (This is hard to compute).



- The maximum of  $\min[\mu_l(s), \mu_u(s)]$  occurs at their intersection.

## Lower bound

- After application of Campbell's theorem, a Chebychev approximation, and some simplifications, we find (for small  $\epsilon$ ):

$$\mu_u(s) = -\frac{1}{\pi} s^{-2} \ln(2 - \sqrt{1 + \epsilon})$$

$$\mu_l(s) = \frac{(\alpha - 1)(M\kappa)^2}{\pi} s^{2(\alpha-1)} (\sqrt{1 + \epsilon} - 1).$$

- Setting these equal and finding  $s^*$  leads to:

$$\lambda_l^{DS} = h(\alpha) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

where  $h(\alpha) = \frac{1}{2}(\alpha - 1)^{\frac{1}{\alpha}} \approx 0.6$  for most reasonable values of  $\alpha$ .

- This is a lower bound (rather than an equality) due to the use of the Chebychev inequality to bound  $\mu_l(s)$ .

## Frequency Hopping vs. Direct Sequence

- Using similar analysis, we find the following bounds for FH spread spectrum ad hoc networks, which leads to the following summary:

$$h(\alpha) \frac{1}{\pi} M \kappa^{\frac{2}{\alpha}} \epsilon \leq \lambda^{FH} \leq \frac{1}{\pi} M \kappa^{\frac{2}{\alpha}} \epsilon$$

$$h(\alpha) \frac{1}{\pi} (M \kappa)^{\frac{2}{\alpha}} \epsilon \leq \lambda^{DS} \leq \frac{1}{\pi} (M \kappa)^{\frac{2}{\alpha}} \epsilon$$

- Hence the capacity of ratio (for both bounds) is:

$$\frac{\lambda^{FH}}{\lambda^{DS}} = M^{1-\frac{2}{\alpha}}$$

## Comparison of Transmission and Transport Capacity

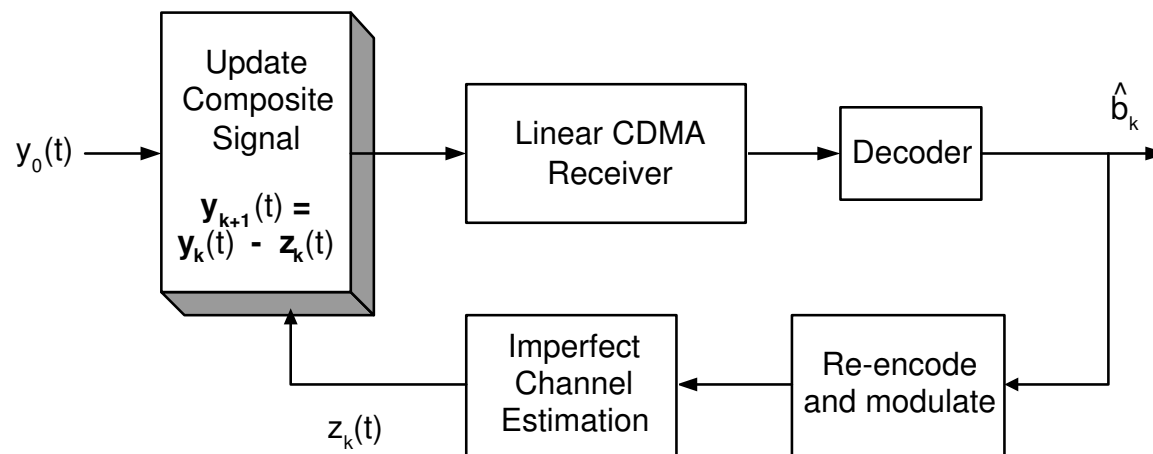
- As we have seen,  $\lambda \propto \kappa^{\frac{2}{\alpha}}$ , where  $\kappa = \frac{r_{TX}^{-\alpha}}{\beta} - \frac{\eta}{\rho}$ .
- Hence,  $\lambda \propto r_{TX}^{-2} \Rightarrow r_{TX} \propto 1/\sqrt{\lambda}$ , i.e. as the transmission distance increases, the transmission density must go down according to an inverse square law.
- The transport capacity is roughly  $\lambda \cdot r_{Tx}$ , i.e. the density of transmissions times the distance of those transmissions.
- Hence, transport capacity is  $O(\frac{\lambda}{\sqrt{\lambda}}) = O(\sqrt{\lambda})$ , which recovers the scaling trend of Gupta and Kumar.
- The advantage of transmission capacity is that it provides a statistical measure of outage for randomly distributed nodes, and closed-form results in the non-asymptotic regime.

**End Part I of Talk**

- Questions?

## Successive Interference Cancellation (SIC)

- Adding SIC to ad hoc networks seems interesting. What if we could just cancel out the strongest few interferers?
- Some recent work [Toumpis and Goldsmith, 2003] found that ad hoc network rate regions were maximized using CDMA + SIC.
- SIC has been shown to be information-theoretic optimal for a variety of multiuser channel scenarios [Cover, Rimoldi and Urbanke '96, Tse and Viswanath].



## Some challenges with SIC

- Imperfect interference cancellation is to be expected
- Latency increases linearly with  $K$  (the number of cancelled users)
- But...
  - Hardware complexity is not very dependent on  $K$
  - SIC is naturally suited to unequal received powers
  - Imperfect interference cancellation still should be better than no interference cancellation
- How well does SIC fare in ad hoc networks?

## The SIC Model for DS-CDMA

$(K, \zeta)$  SIC: Cancellation of a fraction  $(1 - \zeta)$  of the strongest  $K$  users' signals.

- $\zeta \rightarrow 1 \Rightarrow$  Traditional Matched Filter CDMA
- $\zeta \rightarrow 0 \Rightarrow$  Perfect cancellation
- On average  $K$  can be related to a “cancellation radius”, which should be less than  $r$ .

## The Results

Perfect Interference Cancellation, ( $K > 0, \zeta = 0$ ):

$$\lambda_l = \left(\frac{2K}{\epsilon}\right)^{1-\frac{1}{\alpha}} h(\alpha) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

$$\lambda_u = \left(1 + \frac{K}{\epsilon}\right) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon.$$

Partial Cancellation of All Nodes, ( $K = \infty, \zeta$ ):

$$\lambda_l = \zeta^{-\frac{2}{\alpha}} h(\alpha) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

$$\lambda_u = \zeta^{-\frac{2}{\alpha}} \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

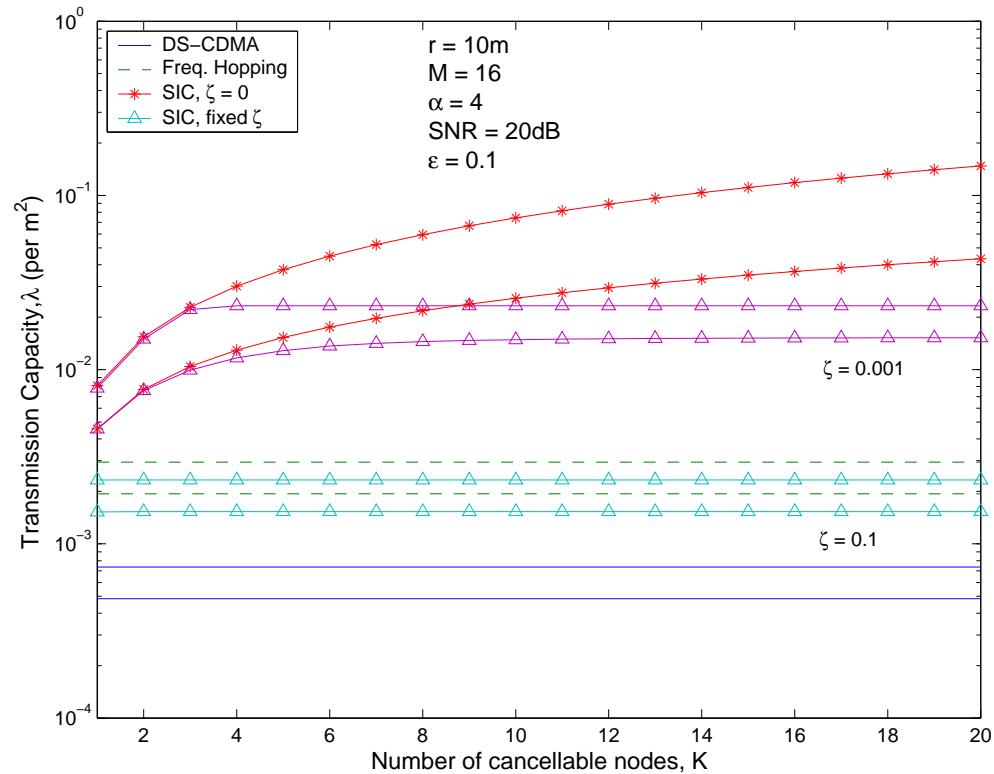
Partial Cancellation of Nearby Nodes ( $K > 0, \zeta$ ):

$$\lambda_l = \left( (1 - \zeta^2) \left(\frac{\epsilon}{2K}\right)^{\alpha-1} + \zeta^2 \right)^{-\frac{1}{\alpha}} h(\alpha) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

$$\lambda_u = \min\left( \frac{1 + \frac{K}{\epsilon}}{1 + \zeta^{\frac{2}{\alpha}}}, \zeta^{-\frac{2}{\alpha}} \right) \frac{1}{\pi} (M\kappa)^{\frac{2}{\alpha}} \epsilon$$

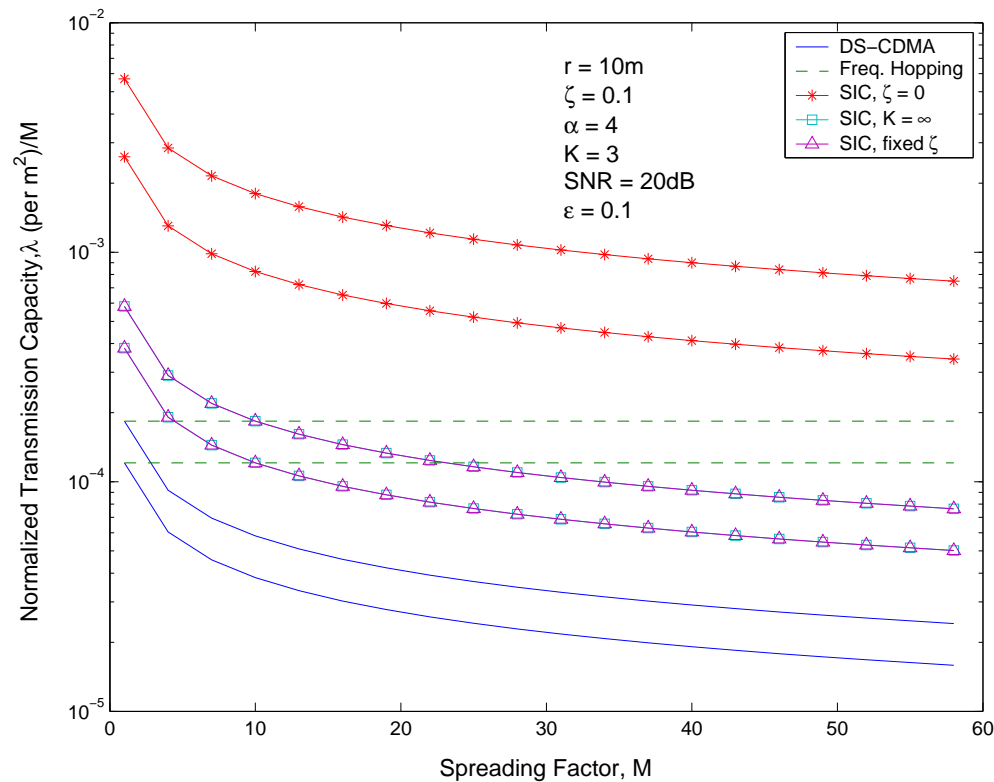
## Numerical Results

## How many interferers should be cancelled?



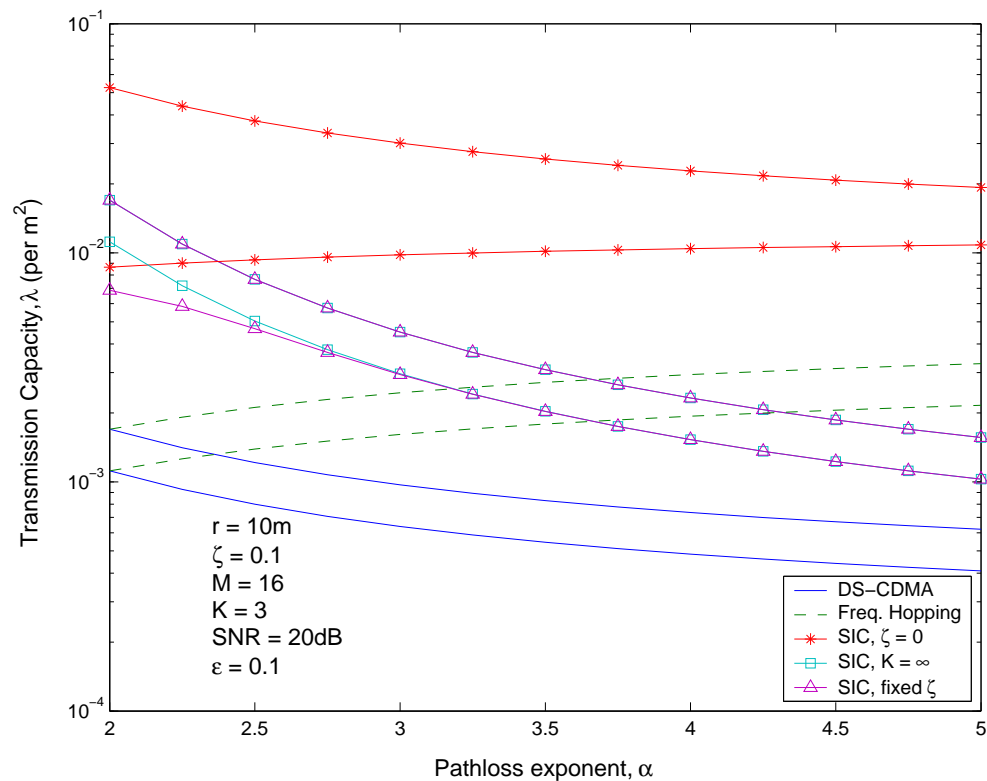
- Just one! Good for complexity – but not a huge capacity gain.
- Pathloss is severe, so closest node dominates interference.

## Does spreading help or hurt?



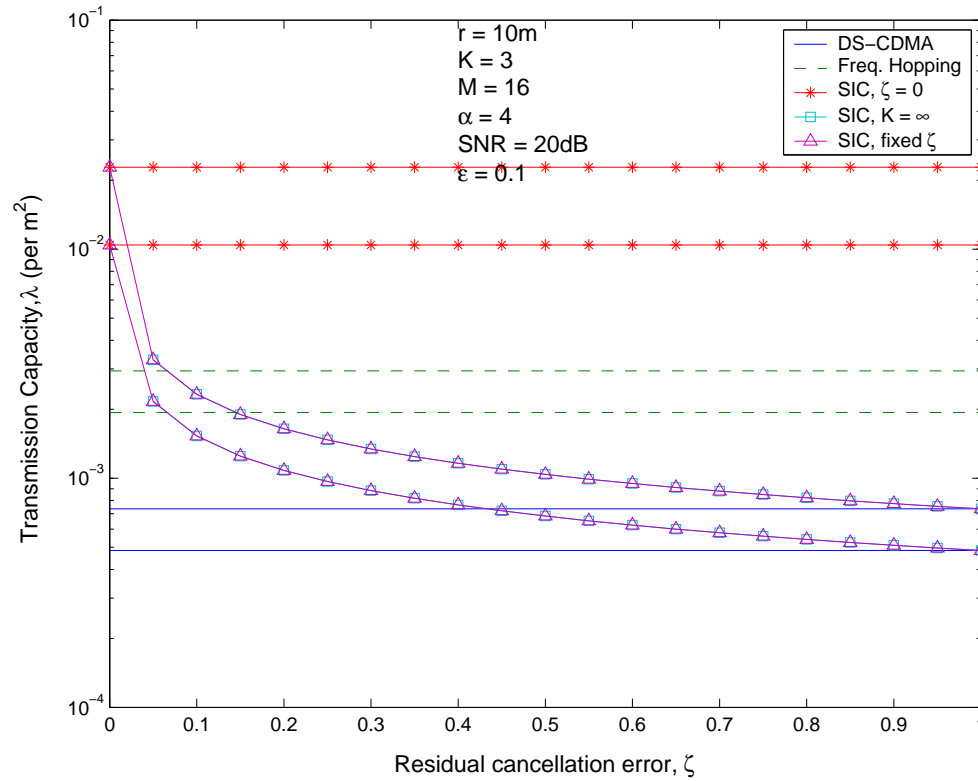
- Frequency hopping is indifferent to  $M$ .
- DS capacity does not increase quickly enough with  $M$  to compensate for data rate  $R \rightarrow \frac{R}{M}$ .

## What role does the channel play?



- DS-CDMA: Pathloss  $\uparrow \Rightarrow$  transmit power  $\uparrow \Rightarrow$  worse near-far problem
- FH: nearby interference no worse than before (still the same  $1/M$  chance of collision), plus aggregated interference decreases.

## How accurate should the interference cancellation be?



- Perfect interference cancellation increases capacity by nearly 100X.
- Capacity drops off quickly with  $\zeta$ .

## Interpretations

- Results appear pessimistic for DS-CDMA. But basic cellular analysis produced similar conclusions.
- Frequency hopping looks promising for ad hoc networks (perhaps Bluetooth had it right from the start!)
- SIC has a large gain for DS-CDMA, but is very sensitive to inaccurate cancellation  $\Rightarrow$  claims that multiuser receivers “solve” the near-far problem in ad hoc networks should be met with extreme scrutiny.
- Scheduling/CSMA is promising for DS-CDMA since nearest node dominates [Hasan and Andrews 2004]. (But this won't help frequency hopping).

## Let's step back...

Capacity is important but there are other important metrics:

- End-to-end delay
- Energy-efficiency
- Security
- Scalability
- Complexity
- Reliability...

These aren't as easy to quantify as capacity (which is already hard!), but to designers, often equally or more important.

## Capacity tradeoffs

- We may be willing to sacrifice some capacity to improve these other metrics
- CDMA provides flexibility in this regard due to allowance of low received SINRs
- Examples:
  - Longer relay distances  $\Rightarrow$  reduced delay
  - Enables more nodes to go into sleep mode and maintain network connectivity
  - Guard zone (carrier sense region) can be smaller by factor of  $M^{1/\alpha} \Rightarrow$  no hidden node problem [Hasan and Andrews, 2004].
  - Allows clustering and efficient MAC design [Yang and de Veciana, Infocom 2005]
  - Well-known security advantages

## Final Comments

- Introduced Transmission Capacity framework for quantitative ad hoc network analysis
- Capacity is good for FH-CDMA  $\Rightarrow$  Interference Avoidance is good
- Capacity is bad for DS-CDMA  $\Rightarrow$  Interference averaging is not effective
- Current work is extending this analysis to other scenarios.

## More information

- Pre-prints and technical reports with the complete analysis are available at my web page: <http://www.ece.utexas.edu/~jandrews>
  - S. Weber, X. Yang, J. G. Andrews, and G. de Veciana, “Transmission capacity of wireless ad hoc networks with outage constraints”, under revision, *IEEE Trans. on Information Theory*, Nov. 2004.
  - S. Weber, J. G. Andrews, X. Yang, and G. de Veciana, “Transmission Capacity of Wireless Ad Hoc Networks with Successive Interference Cancellation”, to be submitted to *IEEE Trans. on Information Theory*.
  - Conference versions appeared at *ISSSTA 2004* and *Globecom 2004*.
  - J. G. Andrews, X. Yang, A. Hasan, and G. de Veciana, “The flexibility of CDMA mesh networks”, submitted to *IEEE Wireless Communications Magazine*, Apr. 2005.
- Francois Baccelli maintains a web page on stochastic geometry at:  
<http://www.di.ens.fr/~mistral/sg/>