ANALOG COMPRESSED SENSING FOR RF PROPAGATION CHANNEL SOUNDING

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ABSTRACT

Massively broadband® RF channel sounding is severely constrained by the sampling rates required for analog to digital conversion. Analog compressed sensing (CS) techniques based on *Xampling* have demonstrated the ability to lower sampling rates far below the Nyquist rate. Here, we show attributes of the multipath channel sounding problem appear to be well suited to CS approaches for reducing measurement acquisition time while simultaneously estimating time delays, multipath amplitudes, and angles of arrival. This paper presents results of the fusion of CS with modern channel sounding. We show measured propagation data from 60 GHz field trials and note the channel sparsity in time and space. We then propose an architecture for the first massively broadband CS channel sounder based on the Xampling framework (which we call the *Channel Sounding Xampler*) to exploit the sparsity, and we use field measurements to explore tradeoffs between analog and digital signal processing to perform channel impulse response (CIR) parameter estimation in real time. We also offer conceptual approaches for the Channel Sounding Xampler designed to trade off analog and digital components with the goal of improving CIR acquisition at sub-THz frequencies.

Index Terms— 60 GHz, mm-Wave Channel Sounding, Xampling, Sliding Correlation, Analog Compressed Sensing

1. INTRODUCTION

Future *massively broadband*® wireless networks will offer multi-Gbps data rates using millimeter wave devices and highly directional steerable phased array antennas in small form factors [1]. To design reliable broadband communication systems at future sub-THz carrier frequencies, accurate propagation measurements are needed to learn multipath amplitudes, delays, and angles of arrival (AOA) as well as the impact of antenna beamwidth in the channel. Channel sounders use analog front-ends, probes, and detectors to estimate the channel impulse response (CIR).

In this paper, we explore new channel sounding methods based on real propagation data from a 60 GHz outdoor peer-to-peer environment [2] where a *spread spectrum sliding correlation* sounding approach used analog processing methods to estimate the CIR at nanosecond resolution at low sampling rates. Until [2], most millimeter wave propagation measurements were conducted using analog vector network analyzers that required a hardwire connection and thus were limited to indoor applications. The sliding correlator [2] relies on time dilation, which is not truly real time, and thus could limit use at future sub-THz frequencies where Doppler effects will be non-negligible over epochs of a few ms. The sliding correlator averages over an entire *pseudo-noise* (PN) spreading sequence period despite the fact that the multipath CIR is sparse; i.e., multipath is due to a small number of physical objects at discrete delays and angles [1], [2].

Recent results in analog compressed sensing (CS) [3], [4] have led to hardware prototypes that detect sparse signals at sub-Nyquist rates. The Modulated Wideband Converter, which follows the *Xampling* framework [5], was used to sample and recover multiple narrowband transmissions at unknown frequency positions at a rate proportional to the bandwidth occupancy [3]. The Xampling architecture was shown to fit the pulsed radar problem [6] and here is extended to direct pulse channel sounding for CIR estimation.

In this paper, we review the sliding correlator channel sounder and its advantages (low sampling rate and simplicity) and shortcomings (time dilation). We introduce the Xampling framework and explain its application to direct pulse channel sounding. Using real field data, we demonstrate the sparsity of the 60 GHz outdoor propagation environment and show that it is well-suited to sub-Nyquist sampling methods. We compare conventional channel estimation to a low-rate approach and present a new CS channel sounder, the *Channel Sounding Xampler*. This new approach will enable real time channel estimation using commercially available hardware and no hardwire connection.

2. CHANNEL SOUNDING THEORY

Channel sounding measures the impulse response of a multipath channel and allows computation of path loss, number of significant paths, and root-mean-square (RMS) delay spread, etc. For a time-invariant channel with K multipath components, the CIR is described by

$$h(\tau,\theta)=\sum_{k=1}^{K} a_k(\theta)\delta(\tau-\tau_k,\theta)e^{j\phi_k}, \qquad (1)$$

where a_k, τ_k, and ϕ_k denote the real amplitude, excess delay, and RF phase shift for the kth path, respectively, θ represents the pointing angle of the receiver antenna (e.g., AOA), and $\delta(t)$ is the Dirac delta functional. In this work, we neglect θ in the analysis but provide qualitative observations on AOA due to its importance at sub-THz frequencies for determining channel conditions [1], [2].

To characterize the propagation channel within some frequency band $[f_c-B/2, f_c+B/2]$, we transmit a wideband probe $x(t)=\mathrm{Re}\{p(t)e^{j2\pi f_c t}\}$ and analyze the received signal [7],

$$y(t)=x(t)*h(t)=\mathrm{Re}\{r(t)e^{j2\pi f_c t}\}. \qquad (2)$$

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In general, $p(t)$ is any probing waveform with RF bandwidth of at least B and $r(t)$ is the baseband equivalent of the probe convolved with the channel. Defining $\alpha_k = a_k \exp\big(j(\phi_k - 2\pi f_c \tau_k)\big)$, the baseband equivalent of (2) is

$$r(t) = \frac{1}{2}\sum_{k=1}^{K} \alpha_k p(t - \tau_k). \tag{3}$$

The *power delay profile* (PDP) of a channel is the squared magnitude of $r(t)$, adjusted for channel gain based on system calibration [2]. In practice, PDPs are averaged over time to lower the noise floor, and spatially averaged over a local area to provide a local-average PDP for a specific location. AOA statistics are characterized by using highly directional antennas to measure PDPs for varying combinations of transmitter and receiver antenna orientations. Fig. 1 shows a local-average PDP measured during the 60 GHz field trial.

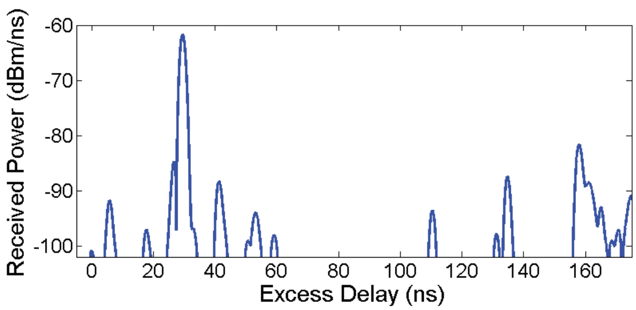

Fig. 1. Local-average PDP for a peer-to-peer 60 GHz channel [2].

3. SLIDING CORRELATOR CHANNEL SOUNDING

The sliding correlator channel sounder uses spread spectrum to boost dynamic range and relies on analog processing to greatly reduce sampling requirements in comparison to direct pulse and direct sequence spread spectrum systems [2], [8]. A PN periodic pulse train with high chip rate R_c is upconverted to f_c Hz and transmitted through a channel $h(\tau)$. The transmitted signal is periodic with period $T = L/R_c$, where L is the PN sequence length, and its spectrum follows the envelope of a sinc centered at f_c with null-to-null bandwidth $B = 2R_c$. At the receiver, the incoming signal is downconverted and mixed with an identical PN sequence with a slightly lower chip rate $R_c' < R_c$. The chip rate difference causes one PN sequence to "slide" past the other and leads to time dilation. The mixed signal is compressed in frequency by the slide factor γ, defined as

$$\gamma = \frac{R_c}{R_c - R_c'}. \tag{4}$$

The null-to-null bandwidth of the time-dilated signal is greatly compressed to $B_{sc} = B/\gamma$ and is then low-pass filtered at bandwidth B_{sc}. The narrowband filter reduces noise power by a factor of γ [8], leading to processing gains on the order of 40 dB. The low-passed output is a time-dilated correlation convolved with the CIR [8],

$$r(t) = \frac{1}{2}\sum_{k=1}^{K} \alpha_k C_{xx'}(t - \gamma\tau_k), \tag{5}$$

where (α_k, τ_k) are the complex gain and excess delay of the kth path, $x(t)$ is the transmitted signal, $x'(t)$ is the slower PN sequence, and $C_{xx'}$ is the correlation between $x(t)$ and $x'(t)$, defined as

$$C_{xx'}(\tau) = \int_{-\infty}^{\infty} {x'}^*(t) x(t - \tau) dt, \tag{6}$$

and $(\cdot)^*$ represents complex conjugation. Intuitively, we see from (5) that the output consists of delayed and attenuated copies of the time-dilated autocorrelation $C_{xx'}(\tau)$. As $x(t)$ propagates through the multipath environment, it arrives at the receiver at a delay corresponding to the path delay and fully slides past $x'(t)$. When the PN sequences line up, they will maximally correlate and despread to a single pulse.

Due to the bandwidth compression, the required sampling rate is reduced by a factor of γ at the expense of increased capture time. The sliding correlator has an inherent delay of $\gamma L/R_c$ – the time it takes for the probing spread spectrum signal to fully slide across and maximally correlate with the slower PN sequence. As a result, the system does not operate in real time. The observed time is related to the real time as $T_{SC} = \gamma T$. Thus, the RF time-bandwidth product of the sliding correlator system is identical to that of the direct pulse system,

$$T_{sc}B_{sc} = TB, \tag{7}$$

and the minimal storage requirements per PDP are identically $N \geq TB$ samples. For a sparse multipath situation, in which the CIR consists of $K \ll N$ nonzero time-instances, most samples contain no information and are effectively discarded. This represents significantly higher storage requirements than necessary and leads to larger processing times.

The WNCG sliding correlator channel sounder was used for 60 GHz field measurements described in [2] and analyzed in Section 5. The system processed a 1.5 GHz null-to-null RF bandwidth probe with minimal sampling rate of 75 KHz (γ =20,000), but the 300 ns propagation measurement interval was stretched over 6 ms. Although this delay does not invalidate measurement results at 60 GHz, it limits the ability to capture time-varying channel statistics and inhibits the extension to sub-THz frequencies and mobile environments where Doppler shift will become important.

4. ANALOG COMPRESSED SENSING

Analog CS extends traditional CS theory to sample sparse continuous-time signals at sub-Nyquist rates [4]. Xampling is a two-stage approach to analog CS [5], in which a broadband signal is compressed in bandwidth and reconstructed from low-rate samples. By incorporating the known structure of a sparse multipath medium, we are able to reduce the time-bandwidth product of direct pulsed systems [9] and to estimate the amplitudes and time-delays by sampling at a rate proportional to the finite rate of innovation, or number of degrees of freedom per unit time [9], [10]. This rate is significantly lower than the corresponding Nyquist rate. For a channel with K multipath components, the minimal sampling rate is $2K/T$, where $T > \tau_{max}$ and τ_{max} is the maximum delay spread of the channel. Thus, a CIR estimate based on M time-averages requires as few as $2KM$ samples. The baseband analog processing and digital recovery stages for pulsed systems are fully described in [9]. The key idea is to obtain a set of Fourier series coefficients and use them to estimate the unknown parameters with standard spectral analysis.

Since $r(t)$ has K unknown delays and amplitudes, we require $N \geq 2K$ samples per period T. However, direct sampling at a rate of N/T will rarely result in nonzero samples because $r(t)$ is sparse in time. Instead, we obtain a sequence of samples $c[n]$ by first prefiltering with a smoothing filter $s^*(-t)$ and then uniformly sampling at times $t = nT/N$. We can relate the sample sequence to our channel parameters through the Fourier series expansion [9]

$$c[n] = \sum_{l\in\mathbb{Z}} R[l] e^{j2\pi ln/N} S^*\left(\frac{2\pi l}{T}\right), \tag{8}$$

where the Fourier coefficients are given by

$$R[l] = \frac{1}{T} P\left(\frac{2\pi l}{T}\right) \sum_{k=1}^{K} \alpha_k e^{-j2\pi l \tau_k / T} \tag{9}$$

and $F(\omega)$ denotes the Fourier transform of a signal $f(t)$.

Given a set $\mathcal{L}$ of $|\mathcal{L}| = L \le N$ consecutive indices such that $P(2\pi\, l/T) \neq 0 \ \ \forall l \in \mathcal{L}$, we can represent (9) in matrix form as

$$\mathbf{r} = \mathbf{P}\mathbf{V}(\boldsymbol{\tau})\boldsymbol{\alpha}, \tag{10}$$

where $\mathbf{r}$ is the L-length vector whose lth element is $R[l]$, $\mathbf{P}$ is the $L \times L$ diagonal matrix with lth diagonal $(1/T)P(2\pi\, l/T)$, $\mathbf{V}(\boldsymbol{\tau})$ is the $L \times K$ matrix with lkth element $\exp(-j2\pi l\, \tau_k/T)$, and $\boldsymbol{\alpha}$ is the K-length vector whose kth element is α_k. We define $\mathbf{g} = \mathbf{V}(\boldsymbol{\tau})\boldsymbol{\alpha}$, where $\mathbf{g}$ is the L-length vector whose lth element is given by

$$\mathbf{g}_l = \sum_{k=1}^{K} \alpha_k e^{-j2\pi l \tau_k/T}. \tag{11}$$

Given $\mathbf{r}$, we can arrive at (11) and use nonlinear methods such as the annihilating filter [9], [10] and matrix-pencil algorithm [6] to solve for the amplitudes and delays. To this end, we construct the filter $s(t)$ such that

$$S(\omega) = \begin{cases} 0 & \omega = 2\pi l/T\,, l \notin \mathcal{L} \\ \text{nonzero} & \omega = 2\pi l/T\,, l \in \mathcal{L} \\ \text{arbitrary} & \text{otherwise} \end{cases} \ . \tag{12}$$

We can then write (8) in matrix form as

$$\mathbf{c} = \mathbf{T}\mathbf{S}\mathbf{r}, \tag{13}$$

where $\mathbf{c}$ is the N-length vector whose nth element is $c[n]$, $\mathbf{S}$ is the $L \times L$ diagonal matrix whose lth diagonal is $S^*(2\pi\, l/T)$ for $l \in \mathcal{L}$, and $\mathbf{T}$ is the $N \times L$ matrix with nlth element $\exp(j2\pi n\,(l-1)/N)$ Since $\mathbf{T}$ and $\mathbf{S}$ are invertible,

$$\mathbf{r} = \mathbf{S}^{-1}\mathbf{T}^{\dagger}\mathbf{c}. \tag{14}$$

In addition to sampling at the finite rate of innovation, the above approach allows us to use realizable analog filters, so long as (12) is satisfied. Previous work shows good parameter estimation with noisy measurements when K is known, but new metrics are needed to analyze the performance in a channel sounding framework.

5. MULTIPATH CHANNEL SPARSITY

Propagation measurements at 60 GHz were conducted with a 750 Mcps sliding correlator channel sounder using steerable 7^0 beamwidth antennas at transmitter-receiver distances between 17 and 120 meters in peer-to-peer configurations in an urban setting. Different transmitter and receiver pointing angles offered unique communication links at each location, and local-average PDPs were recorded for each link [2].

The PDPs were analyzed offline to characterize channel statistics. These early results showed that it is possible to establish links in four to six unique transmitter/receiver pointing angles due to scatterers and reflectors in the environment [2]. Objects in the environment affected the transmitted signal's propagation path, leading to multipath.

The 60 GHz multipath measurements clearly show sparsity both in time and in AOA. The peer-to-peer measurements showed that the largest measured excess delay spread never exceeded 200 ns. Given the channel sounder's temporal resolution $T_{res} = 1/R_c = 1.33$ ns, the maximum number of discernable multipath is $\tau_{max}/T_{res} \approx 150$. To measure the CIR's sparsity from actual propagation data, we first estimated and applied a threshold at the noise floor. The threshold was chosen based on manual inspection to correctly distinguish between channel-induced multipath and noise. After accounting for time dilation, we recorded the amplitudes $\{\alpha_i\}_{i=1}^{K}$ and delays $\{\tau_i\}_{i=1}^{K}$ of the local maxima separated by at least T_{res} seconds. Each local peak was assumed to represent a unique multipath component. The CIR was then estimated as the finite impulse train

$$\tilde{h}(\tau) = \sum_{k=1}^{K} \alpha_k \delta(\tau - \tau_k). \tag{15}$$

The *sparse* CIR $h(\tau, P)$ consists of the $K' \le K$ strongest multipath components that comprise P percent of the CIR power:

$$\begin{gathered} \tilde{h}(\tau, P) = \sum_{k=1}^{K'} \alpha_{(k)} \delta\big(\tau - \tau_{(k)}\big) \\ \text{s.t. } K' = \min\left\{ \hat{K} : \frac{\sum_{l=1}^{\hat{K}} \alpha_{(l)}^2}{\sum_{j=1}^{K} \alpha_j^2} \ge \frac{P}{100} \right\}, \end{gathered} \tag{16}$$

where $\{\alpha_{(1)},\ \alpha_{(2)}, \dots,\ \alpha_{(K)}\}$ are the multipath gains in descending order and $\tau_{(k)}$ is the delay associated with the kth strongest path.

The top plot of Fig. 2 shows the CDFs of the number of multipath components at 95%, 99%, and 100% of the CIR power for all measured data over 48 locations in an outdoor setting [2]. For nearly all PDPs, fewer than five peaks comprised 95% of the CIR power, suggesting that the signal's sparsity may be exploited for estimating AOA. The bottom of Fig. 2 shows the CDF of the RMS delay spread. Notably, sparsity and RMS delay spread are similar in distribution, suggesting that RMS delay spread is a good indicator of channel sparsity. Using the measured data, we found that the channel multipath increased as the combined transmitter-receiver pointing angle increased. This is attributed to the increase in non-line-of-sight propagation paths (due to scatterers and reflectors) that lead to successful links. However, at 95% CIR power, increase in pointing angle does not imply more multipath since most of the energy is captured in the first few peaks.

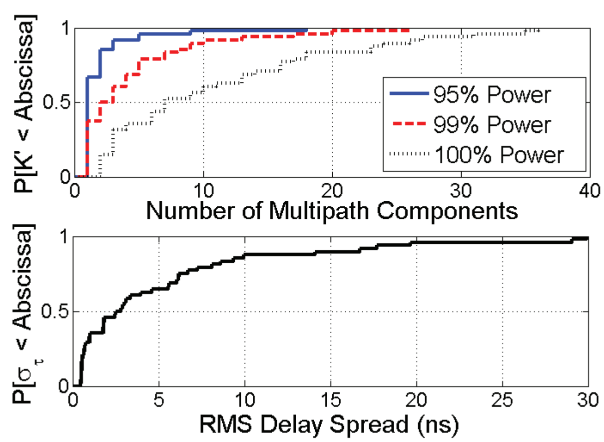

Fig. 2. CDFs of (a) channel sparsity and (b) RMS delay spread for all 60 GHz measurements in [2].

Measured data [2] show that 60 GHz peer to peer outdoor channels are sparse in both time and AOA, and as seen in Figure

2, for more than 95% of the time for 95% of the collected power, we can reasonably approximate the channel with the $K' = 5$ largest multipath components.

6. COMPARING CHANNEL SOUNDER ARCHITECTURES

To simplify the performance comparison of current channel sounders to the analog CS approach, we first consider a direct pulse channel sounder with equivalent parameters to the sliding correlator system discussed in Section 3. Although the direct pulse system does not afford a high processing gain, (7) shows that it has an equivalent time-bandwidth product to the sliding correlator. We simulated a $1/R_c = 1.3$ ns probing pulse sent every $T = \tau_{max} = 200$ ns and estimated the CIR shown in Fig. 3, corresponding to the PDP in Fig. 1 ($K = 16$). The probing pulse was repeated $M = 20$ times (as in [2]) and each transmission was corrupted with AWGN to simulate 30 dB SNR. When operating at the 1.5 Gsps Nyquist rate, the direct pulse system must obtain at least $N = 300$ samples per pulse transmission. In contrast, the time-bandwidth product of the low-rate approach in [9], when searching for $K' = 5$ peaks, is 30 times lower than the direct pulse system, and only 10 samples are required at the 50 Msps minimal sampling rate.

We assessed the performance of the *low-rate estimator* (LRE) discussed in Section 4 by measuring the RMS delay spread of the estimated CIR. The LRE uses the matrix-pencil method to select K' and estimate the multipath from (14). Fig. 3 shows the CIR and the estimates corresponding to the LRE for $N = 299$, 59, and 29. The results show the LRE estimates the CIR exactly when operating at the Nyquist rate. The LRE reasonably predicts RMS delay spread (within 10%) and has close replication of significant multipath components using an approximation limited to 12 multipath peaks, while providing a factor of 5 in sampling/storage savings over the conventional direct pulse method. We found even better approximation using an alternate CS approach detailed in [6]. This early study shows promise for CS methods for channel sounding, but work remains.

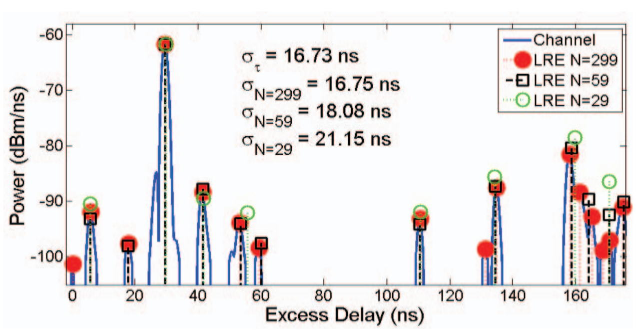

Fig. 3. Estimating a 60 GHz CIR from [2] using LRE at 30 dB SNR.

7. CONCLUSION: A CHANNEL SOUNDING XAMPLER

New propagation measurement campaigns at sub-THz frequencies will require channel sounders capable of measuring multi-GHz bandwidths in real time and with minimal sampling and storage. This paper outlines a new channel sounder approach we are developing with National Instruments. The architecture presented in Fig. 4 is a preliminary concept for the Channel Sounding Xampler and uses fundamental ideas of both conventional channel sounding (e.g. direct pulse) and sub-Nyquist sampling. We are investigating direct pulse and spread spectrum signaling, and considering the utility of new low rate CS channel sounding techniques to reduce sampling time and storage requirements. Using 60 GHz field data, we showed that the outdoor propagation environment is well-approximated by its five strongest multipath components (when using antennas with less than 60 degree beamwidths). Further, we note sparsity is not greatly affected by antenna positioning [2]. Here, we demonstrated the minimal sampling rate to detect five multipath components is 50 Msps – a 30-fold reduction from today's systems. Oversampling reduces the impact of noise and is still within the limits of commercial analog to digital converters [9]. Because estimation is hindered when the channel is not sparse, we recognize the need to adjust the system to estimate varying numbers of multipath components. We plan to use FPGAs for digital recovery so that the sampling rate and digital processing can be adjusted according to the number of paths.

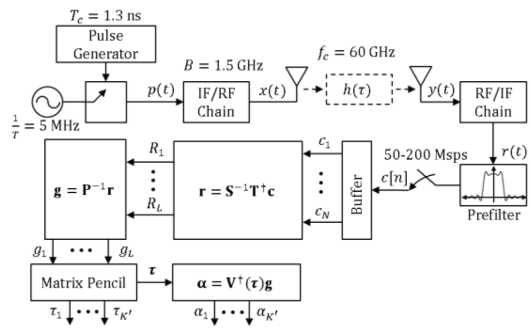

Fig. 4. Channel Sounding Xampler System Diagram.

8. REFERENCES

[1] T.S. Rappaport, J.N. Murdock, and F. Gutierrez, "State of the Art in 60 GHz Integrated Circuits and Systems for Wireless Communications," *Proc. IEEE*, vol. 99, no. 8, pp. 1390-1444, Aug. 2011.

[2] E. Ben-Dor, T.S. Rappaport, et al., "Millimeter-wave 60 GHz Outdoor and Vehicle AOA Propagation Measurements using Broadband Channel Sounder," *IEEE Globecom*, Houston, TX, Dec. 5-9, 2011.

[3] M. Mishali and Y.C. Eldar, "From Theory to Practice: Sub-Nyquist Sampling of Sparse Wideband Analog Signal," *IEEE J. Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 375-291, Apr. 2010.

[4] Y.C. Eldar, "Compressed Sensing of Analog Singals in Shift-Invariant Spaces," *IEEE Transactions on Signal Processing*, vol. 57, no. 8, pp. 2986-2997, Aug. 2009.

[5] M. Mishali, Y.C. Eldar, and A.J. Elron, "Xampling: Signal Acquisition and Processing in Union of Subspaces," *IEEE Trans. Signal Processing*, vol. 59, no. 10, pp. 4719-4734, Oct. 2011.

[6] W.U. Bajwa, K. Gedalyahu, and Y.C. Eldar, "Identification of Parametric Underspread Linear Systems and Super-Resolution Radar," *IEEE Trans. Sig.Processing*, vol. 59, no. 6, pp. 2548-2561, June 2011.

[7] T.S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed.: Prentice Hall, 2002.

[8] R.J. Pirkl and G.D. Durgin, "Optimal Sliding Correlator Channel Sounder Design," *IEEE Trans. on Wireless Comm.*, vol. 7, no. 9, pp. 3488-3497, Sept. 2008.

[9] R Tur, Y.C Eldar, and Z. Friedman, "Innovation Rate Sampling of Pulse Streams With Application to Ultrasound Imaging," *IEEE Trans. Signal Processing*, vol. 59, no. 4, pp. 1827-1841, Apr. 2011.

[10] M. Vetterli, P. Marziliano, and T. Blu, "Sampling Signals With Finite Rate of Innovation," *IEEE Trans. Signal Processing*, vol. 50, no. 6, pp. 1417-1428, June 2002.