EE 394V ATPs Control. (Part 1)

Note Title 8/29/2007

Review

$$\ddot{z} + a\dot{z} + bz = cu$$

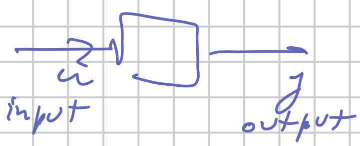
$x_1 = z$, $x_2 = \dot{x}_1 = \dot{z}$ $\longrightarrow$ $\dot{x}_1 = x_2$

$\dot{x}_2 + a\,x_2 + b\,x_1 = cu \longrightarrow \dot{x}_2 = -bx_1 - ax_2 + cu$

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -b & -a \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B} u$$

State space representation of a linear system:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$



$x \to$ vector $\longrightarrow$ $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ $\longrightarrow$ $x_i \Rightarrow$ state variables

A is a matrix

For nonlinear systems: $\dot{x} = f(x, u)$

Equilibrium point(s) $\to$ (only one or many connected in linear systems)

$\Leftrightarrow$ $\boxed{\dot{x}(x_{eq}) = 0}$ $\hookrightarrow$ Recall that $\dot{x}$ is like a velocity so equilibrium means $\dot{x} = 0$ (no velocity)

Other ways of representing linear systems is with a transfer function $\rightarrow$

$$G(s) = \frac{Y(s)}{U(s)}$$ $\rightarrow$ Impedance (Assume by now SISO systems)

$XS = AX + BU \longrightarrow X(IS - A) = BU$

$Y = CX + DU$

$\downarrow$

$X = (IS - A)^{-1} BU$

$$Y = C(IS - A)^{-1}BU + DU$$

$$Y = \left(C(IS - A)^{-1}B + D\right)U$$

"Impedance" $\Rightarrow G(s) = \dfrac{Y}{U} = C(Is - A)^{-1}B + D$ (Y → output, U → input)

For $\dot{x} = Ax + Bu$, $x(t_0) = x_0$, the solution (trajectory)

$$\hookrightarrow x(t) = \phi(t, t_0)x_0 + \int_{t_0}^{t} \phi(t, \sigma)B(\sigma)u(\sigma)\,d\sigma$$

$$x(t) = \underbrace{\phi(t, t_0)x_0}_{\text{related with transient}} + \underbrace{\int_{t_0}^{t} \phi(t, \sigma)B(\sigma)u(\sigma)\,d\sigma}_{\text{related with steady state}}$$

related with transient → driven by internal energy → related with state variables

related with steady state → driven by input

For A non dependent on time

$$\phi(t, t_0) = e^{A(t - t_0)}$$

Matrix exponential

$$\hookrightarrow e^{At} = \sum_{i=0}^{\infty} \frac{A^i t^i}{i!}$$

In linear systems the solution for the state variables take the form:

$$x_i(t) = \underbrace{\sum_{j=1}^{n} C_j e^{\lambda_j t} + \underbrace{X(u, t)}_{\text{steady state}}}_{\text{transient}}$$

$\lambda_j \longrightarrow$ eigenvalues $\longrightarrow$ obtained from solving

$$\det(\lambda I - A) = 0$$

$$(A - \lambda I)x = 0$$

characteristic polynomial: $\varphi(\lambda) \stackrel{\Delta}{=} \det(\lambda I - A) = 0$

Example of eigenvalue use → Stability ⟶ $\mathrm{Re}(\lambda) < 0$ → stable

↳ At least one $\mathrm{Re}(\lambda) > 0$ unstable

So let's go back to power electronics as an application

Consider first the buck converter

Buck converter:

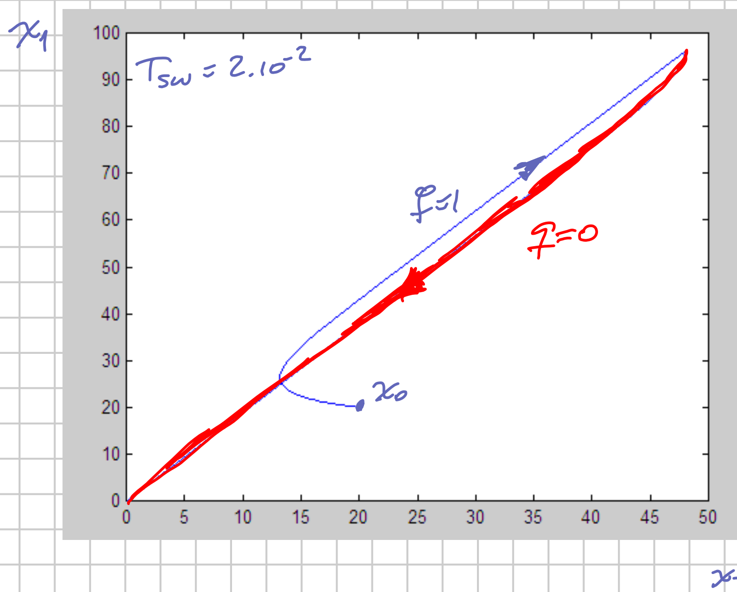
Switched system

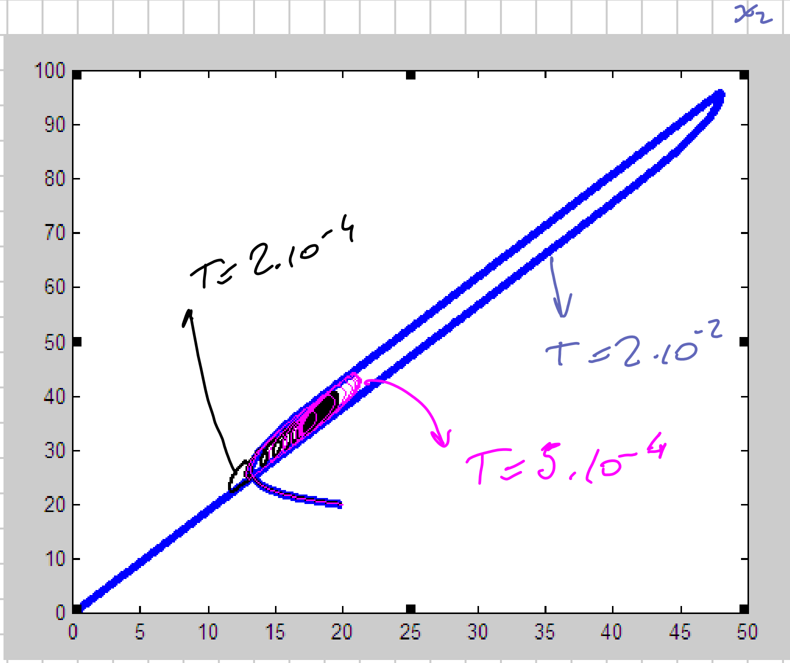
$$\begin{cases} L\dot{x}_1 = f(t)E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases} \qquad x(0) = x_0$$

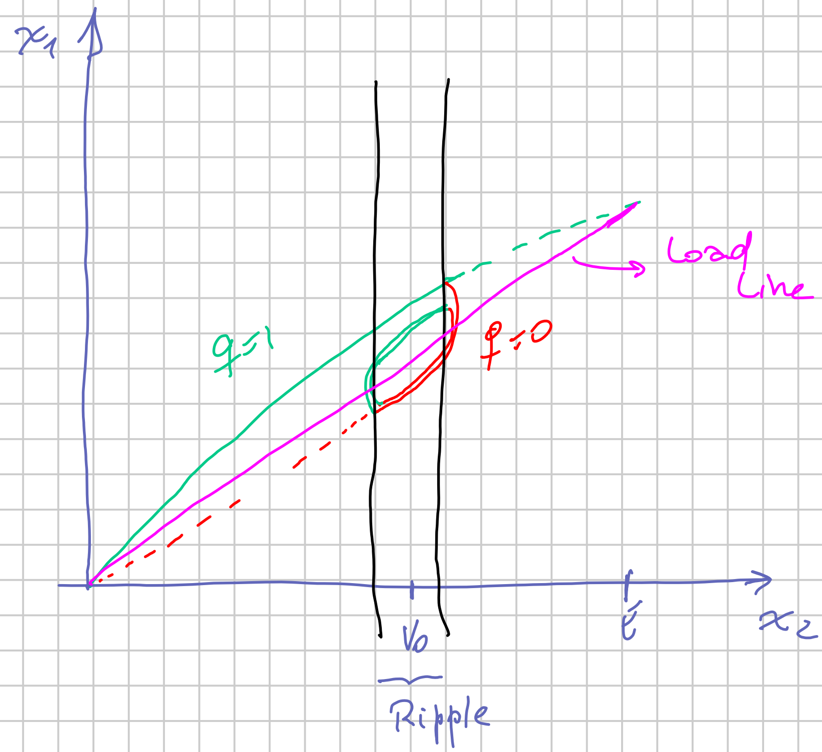
equilibrium

For $f(t) = 0 \longrightarrow x_2 = 0,\ x_1 = 0$

For $f(t) = 1 \longrightarrow x_2 = E,\ x_1 = \frac{E}{R}$







Fast average

$$(1)\quad \begin{cases} L\dot{x}_1 = d\,E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases} \longrightarrow \mathcal{E}\dot{x} = Ax + Bu$$

$$\mathcal{E} = \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix}, \quad A = \begin{pmatrix} 0 & -1 \\ 1 & -1/R \end{pmatrix}$$

Based on control input $\longrightarrow$ $B = \begin{pmatrix} E \\ 0 \end{pmatrix}$, $u = d$

or

Based on power input $\longrightarrow$ $B = \begin{pmatrix} d \\ 0 \end{pmatrix}$, $u = E$

or

$$\begin{cases} \dot{x}_1 = \dfrac{1}{L}\left(dE - x_2\right) \\ \dot{x}_2 = \dfrac{1}{C}\left(x_1 - \dfrac{x_2}{R}\right) \end{cases} \longrightarrow \dot{x} = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{pmatrix} x + \begin{pmatrix} E/L \\ 0 \end{pmatrix}.$$

Equilibrium point:

$x_{2_0} = dE$ → Since we want x_{20} constant (dc) then $d(t) = D_0$ ← constant

$$x_{10} = \frac{x_{20}}{R} = \frac{D_0 E}{R}$$

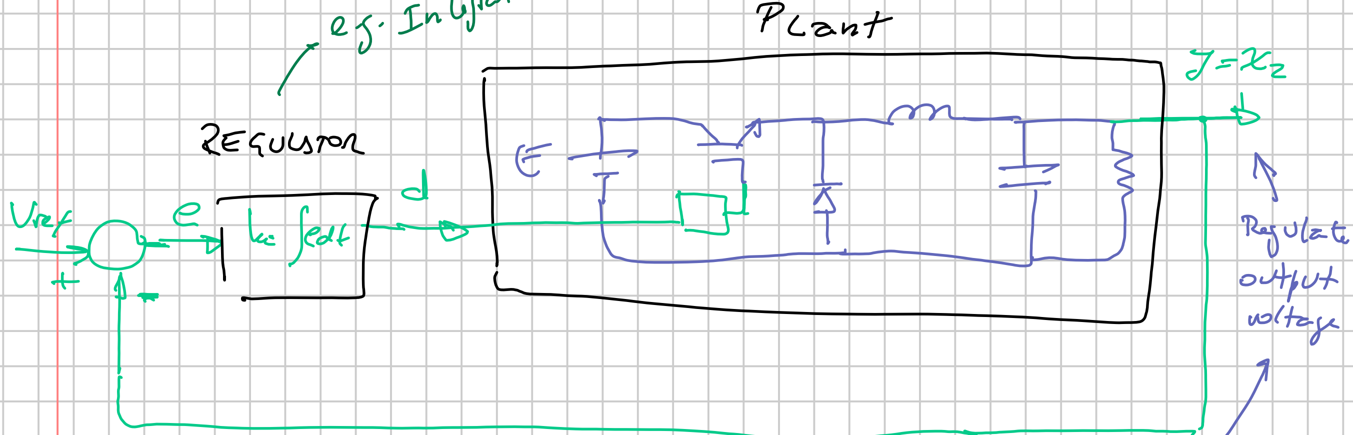
Regulation vs. tracking

with respect to an operation point

Follow some trajectory

Linear controller

e.g. Integral controller



$V_{ref} \rightarrow$ Desired output voltage

$$e = V_{ref} - x_2$$

$$d = k_i \int e\,dt = k_i \int (V_{ref} - x_2)\,dt$$

If $x_2 < V_{ref} \longrightarrow d$ increases, $e > 0$

If $x_2 > V_{ref} \longrightarrow d$ decreases, $e < 0$

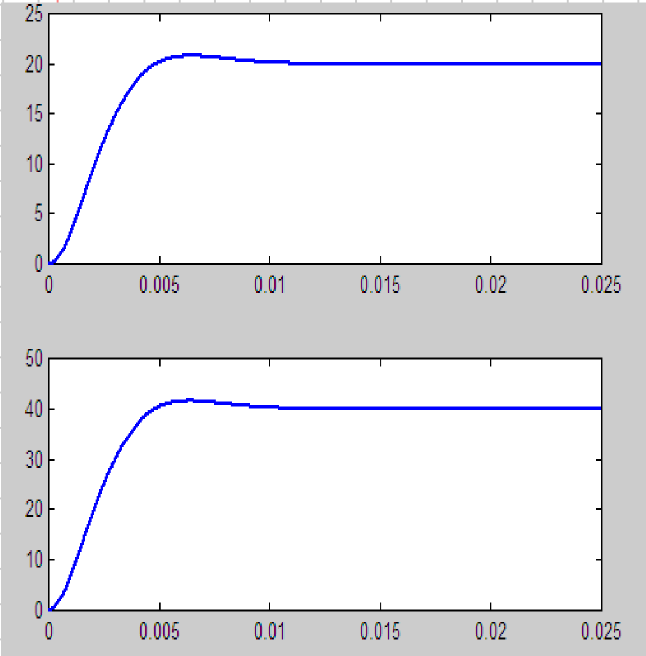
If $x_2 = V_{ref} \longrightarrow d$ constant, $e = 0$

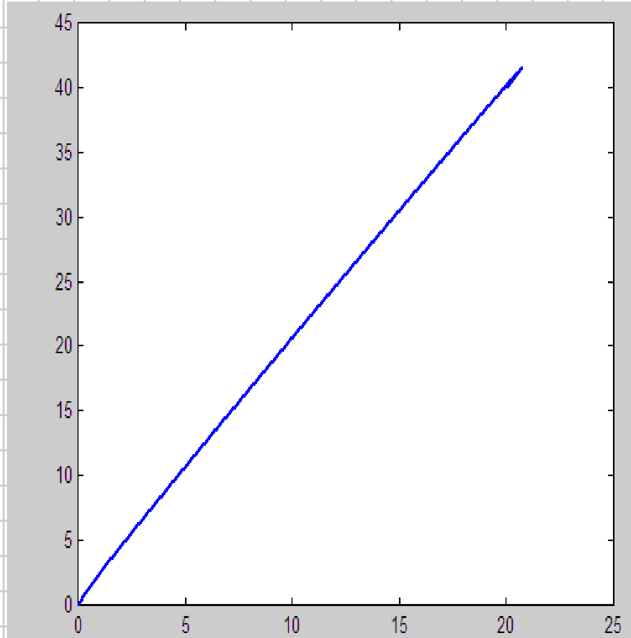
If $e = 0 \Rightarrow \dot{x}_2 = 0$

(because $\dot{e} = \dot{x}_2$)

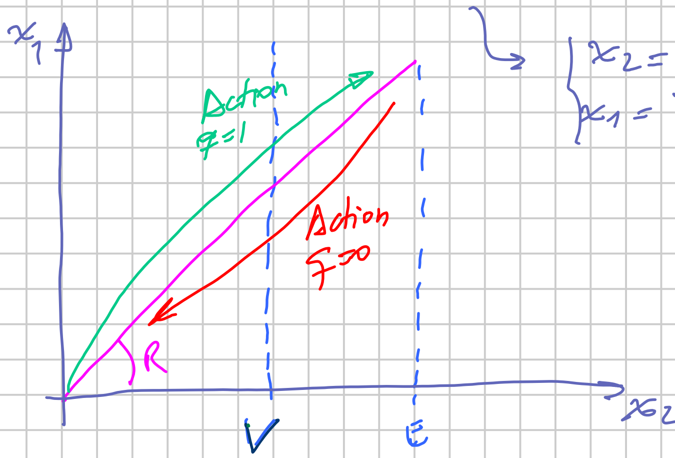
Thus $e=0$ and $d=$ constant at the equilibrium point

Same example, $k_i = 10$





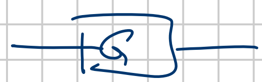
From
$$\begin{cases} L\dot{x}_1 = DE - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases} \Rightarrow \quad x_1 = \frac{x_2}{R} \quad \text{(Line)}$$



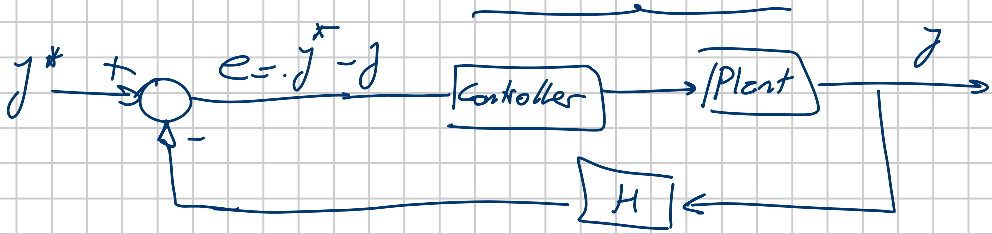
$$\begin{cases} x_2 = DE \\ x_1 = \dfrac{DE}{R} \end{cases}$$

Parametric equation of a line (parameter D)

More on feedback



Consider



↳ Feedback gain

Closed loop gain → $T = \frac{Y}{Y^*} = \frac{G}{1+GH}$

Bandwidth → Max frequency of a disturbance for which there is enough gain to compensate it

↳ It is the frequency at which $T(s)$ dropped 3dB with respect to its low frequency value

Consider that open loop I have → $G = \frac{A_0}{1+ s/\omega_0}$

Closed loop → $T = \frac{G}{1+GH} = \frac{A_0}{1+A_0+s/\omega_0}$

Assume $H=1$

↳ $\omega_c = \omega_0(1+A_0)$

$\omega_c > \omega_0$

when I close the loop I gain bandwidth but I loose gain

Stability definitions (linear approach)

Crossover frequency ω_{co} → 1^{st} frequency at which $|G(s)H(s)| = 0dB$

ω_{180} → $\text{Arg}(G(s)H(s)) = 180°$

$|G(j\omega_{180})H(j\omega_{180})| = 1$ → oscilatory

$|G(j\omega_{180})H(j\omega_{180})| > 1$ → unstable

$|G(j\omega_{180})H(j\omega_{180})| < 1$ → stable

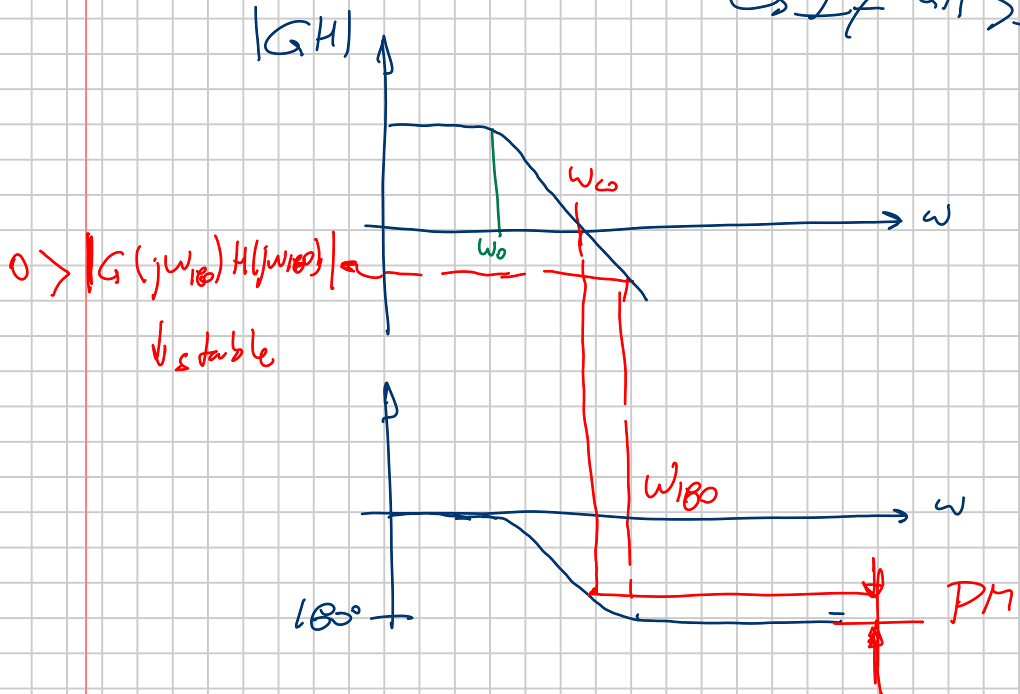
Phase margin $\longrightarrow PM = \operatorname{Arg}\left(G(j\omega_{co})\,H(j\omega_{co})\right) + 180°$

↳ If $PM > 0 \rightarrow$ stable

Gain margin $\longrightarrow$ Represents how much I can increment the system's gain without loosing stability

↳ $GM = -\left|G(j\omega_{180})\,H(j\omega_{180})\right|$

↳ If $GM > 0\,dB \rightarrow$ stable



Linear controllers

- **P control**

$$e \rightarrow \boxed{G} \rightarrow y \quad = \quad e \rightarrow \boxed{k_p} \xrightarrow{d} \boxed{\begin{matrix}\text{Plant}\\ A(s)\end{matrix}} \rightarrow y$$

↳ $Y = A(s)D$

$$d = k_p e = k_p(y^* - y)$$

y^* → desired goal

y → actual output

Now, I close the loop with $H=1$

$$\frac{Y}{Y^*} = \frac{G}{1+GH} = \frac{A(s)}{\frac{1}{k_p} + A(s)}$$

Notice that as $k_p \rightarrow \infty$ $\quad \frac{Y}{Y^*} = 1 \Longrightarrow y = y^*$ → My output equals my desired goal

So a high k_p yields a good tracking (hence regulation)

But → If I have noise then

(little noise) × (big k_p) = lots of noise

→ $d = k_p(y^* - y)$ → Large k_ps yield $d > 1$

since d can only take values between 0 and 1 in reality a large k_p is inneffective

→ large ds are bad for boost, buck-boost and other type of converters

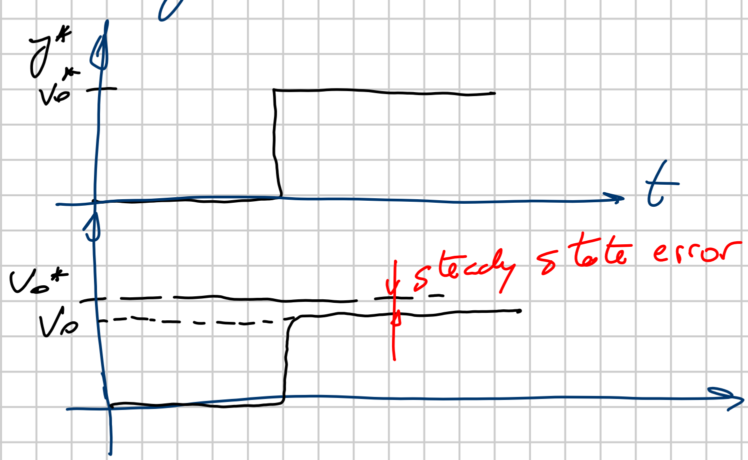
→ If $e = 0$ then $d = 0$ → So when I am meeting my control goal I have no control signal

→ Large k_p improves convergence to goal but worsenes PM.

There are 2 important issues in proportional controls

$$\begin{cases} y = y^* \text{ only when } k_p \text{ is } \infty \text{, but } \infty \text{ in practice doesn't exist} \\ \text{when } e = 0,\ d = 0 \end{cases}$$

→ In proportional controllers I end up with a steady state error

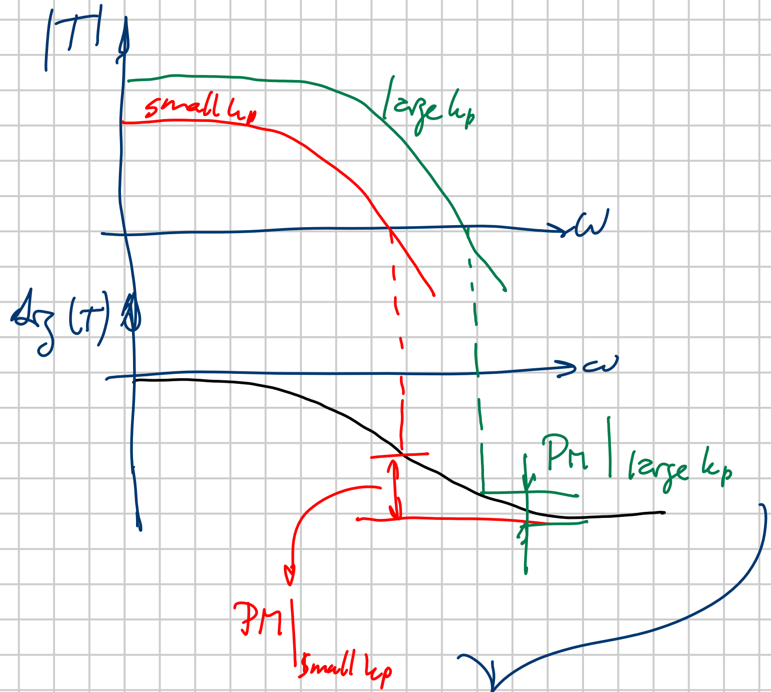


Another problem of P-control is that large k_ps lead to lower PM

↳ e.g. → Assume that $A(s) = \dfrac{1}{(s+a)(s+b)}$

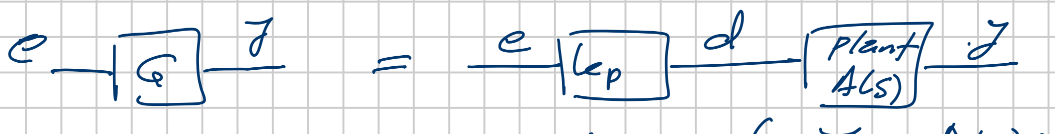
$$\downarrow$$

$$GH = \frac{k_p}{(s+a)(s+b)}$$



$$PM|_{\text{small } k_p} > PM|_{\text{large } k_p}$$

I - control



↳ $Y = A(s)D$

$$d = d_0 + k_i \int e\,dt = k_i \int (y^* - y)\,dt + d_0$$

y^* → desired goal

y → actual output

$\mathcal{L}\left(\int dt\right) = \frac{1}{s}$ ...

$$\frac{Y}{Y^*} = \frac{A(s)\, k_i/s}{1 + A(s)\, k_i/s} = \frac{A(s)\, k_i}{s + A(s)\, k_i}$$

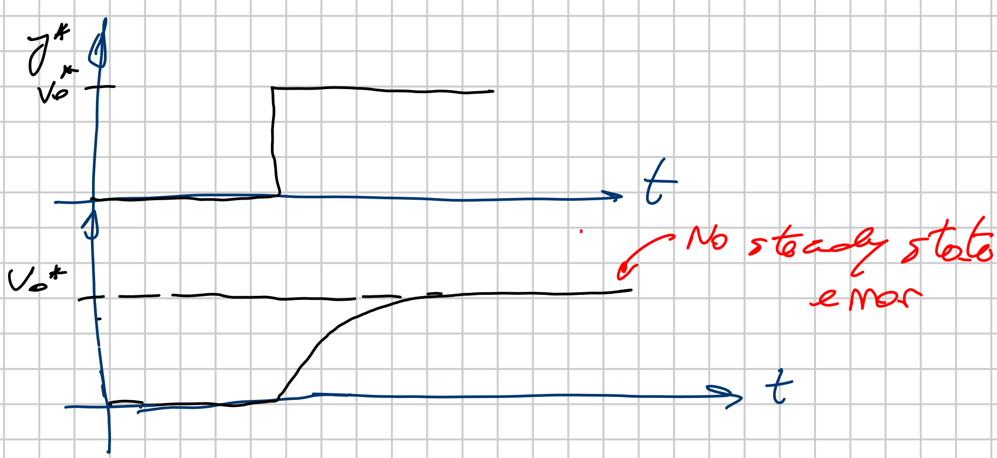
In the same way that $T \to \infty$ implies $\omega \to 0$ then $\underbrace{t \to \infty}_{\text{steady state}}$ implies $\underbrace{s \to 0}_{\text{dc.}}$

So with PI control at dc $\left.\frac{Y}{Y^*}\right|_{dc} = 1$

And as $t \to \infty$ (regardless of dc or ac signal)

$$y(t) \xrightarrow{t \to \infty} y^*(t) \quad \text{so} \lim_{t \to \infty} e = 0$$

Since $d = d_0 + k_i \int e\, dt \longrightarrow d \neq 0$ when $e = 0$ even if $d_0 = 0$ because the integral is a "sum" so the result at $t \to \infty$ when $e = 0$ is not necessarily 0 because of all the contributions to the sum from $t = 0$ to $t = \infty$



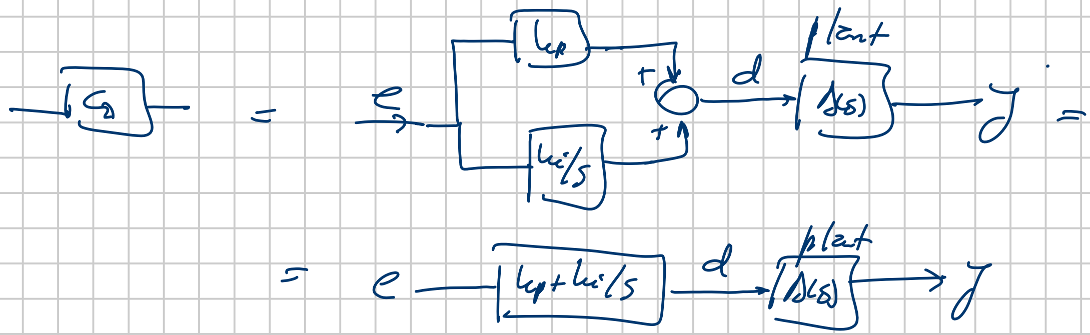
If $A(s) = \frac{A_0}{s + p} \longrightarrow T = \frac{k_i A_0}{s^2 + sp + k_i A_0}$

$$\lambda_{1,2} = \frac{-p \pm \sqrt{p^2 - 4 k_i A_0}}{2}$$

Hence, → 1) If k_i is very large we can get complex solutions (in the frequency domain) which yield oscillatory behavior and overshoots in time domain

→ 2) Provided that $k_i > 0$, if $p > 0$ (o.l. stable) then the closed loop equilibrium point is also stable.

PI control



$$\frac{y}{y^*} = \frac{s(k_p\Delta(s)) + k_i\Delta(s)}{s(1 + k_p\Delta(s)) + k_i\Delta(s)}$$

So as $t \to \infty$ and $s \to 0$ then $\frac{y}{y^*} = 1 \longrightarrow e = 0$ as $t \to \infty$

PI controllers can achieve faster convergence without having large k_i's that may create overshoots or oscillations, but without ending up with steady state error

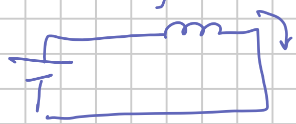
Boost converter

$$\begin{cases} L\dot{x}_1 = E - g'(t)\,x_2 \\ C\dot{x}_2 = g'(t)\,x_1 - \frac{x_2}{R} \end{cases}$$

For $g'(t) = 0$

$$\begin{cases} L\dot{x}_1 = E \\ C\dot{x}_2 = -\frac{x_2}{R} \end{cases} \xrightarrow{\text{eq. point}} \quad 0 = E\,? \quad x_1 = ?$$

If $g'(t)=0 \Rightarrow g(t)=1$

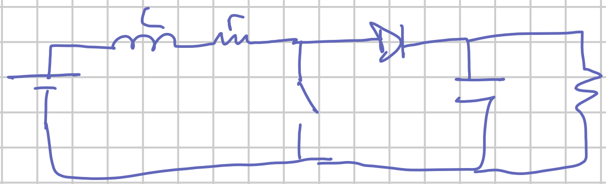


$L\dot{x}_1 = E \Rightarrow$ No equilibrium

$$x_1(t) = \frac{E}{L}t + \frac{E}{L}x_1(0)$$

Linear increase tending to infinity

Now assume that the inductor has some resistance r



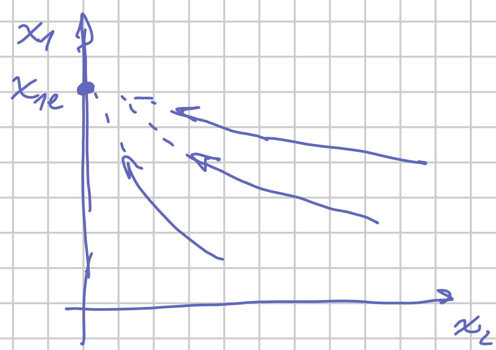
$$\begin{cases} L\dot{x}_1 = r x_1 - g'(t)\, x_2 + E \\ C\dot{x}_2 = g'(t)\, x_1 - \frac{x_2}{R} \end{cases}$$

For $\underline{g'(t) = 0}$ $\longrightarrow g(t)=1$

$$\begin{cases} L\dot{x}_1 = r x_1 + E & \longrightarrow x_{1e_1} = E/r \\ C\dot{x}_2 = -\frac{x_2}{R} & \longrightarrow x_{2e_1} = 0 \end{cases}$$

$\hookrightarrow \dot{x}_2 < 0$

$\dot{x}_1 > 0$



For $\underline{g'(t)=1}$ $\longrightarrow g(t)=0$

$$\begin{cases} L\dot{x}_1 = -r x_1 - x_2 + E \\ C\dot{x}_2 = x_1 - \frac{x_2}{R} \end{cases} \longrightarrow x_{1e_0} = \frac{x_{2e_0}}{R}$$

$$-\frac{r}{R}x_{2e_0} - x_{2e_0} + E = 0$$

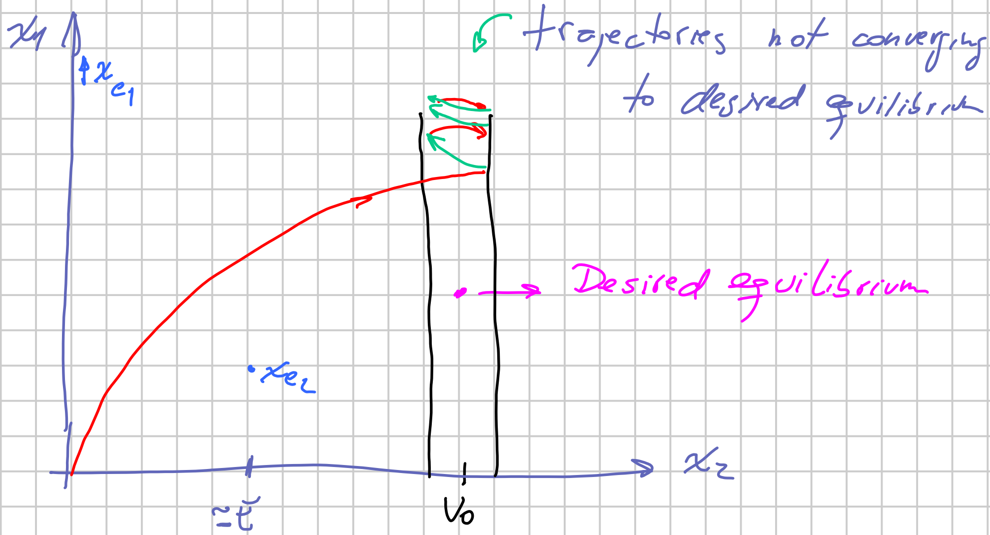
$$\hookrightarrow x_{2e_0} = \frac{E}{1 + r/R}$$

$\hookrightarrow x_{2e_0} \simeq E$ because $r \ll R$

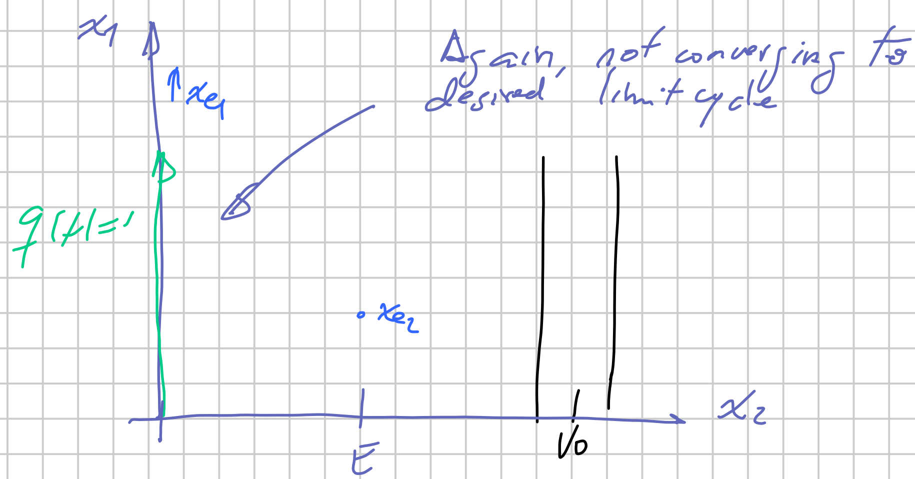
lets attempt a geometric control to regulate output voltage

Consider $x(0)=0$

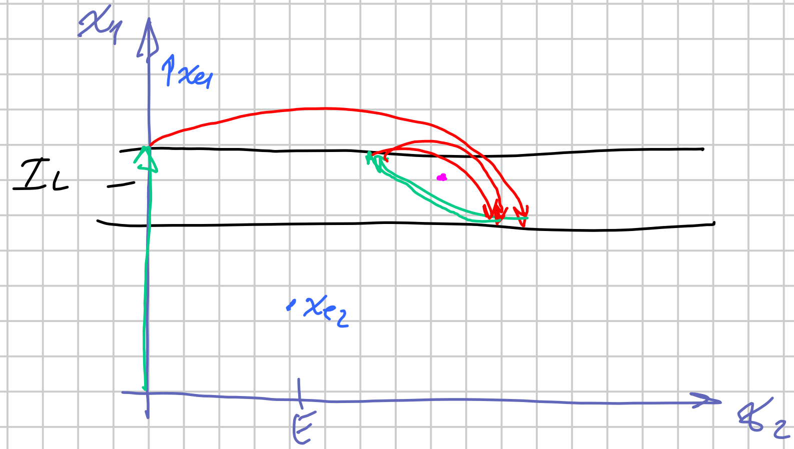
a) Consider that the first control action is $g(t)=0$



b) Now consider that the first control action is $g(t)=1$



Now, lets attemt to regulate inductor current



Now, we achieved the desired goal

"Rules" for histeresis control:

1) The switching surfaces (curves or lines) must separate the equilibrium points

2) The switches must act in opposition to the natural evolution toward an equilibrium point

3) The switch action must have a direct action on the regulated state variable

4) The desired operating point must be between switching surfaces

5) A dead band should be provided to avoid chattering

PI cont:

$$d = k_i \int e\,dt + k_p e$$

↳ Provides a similar action but faster than Integral term

"Speed and gains"

average

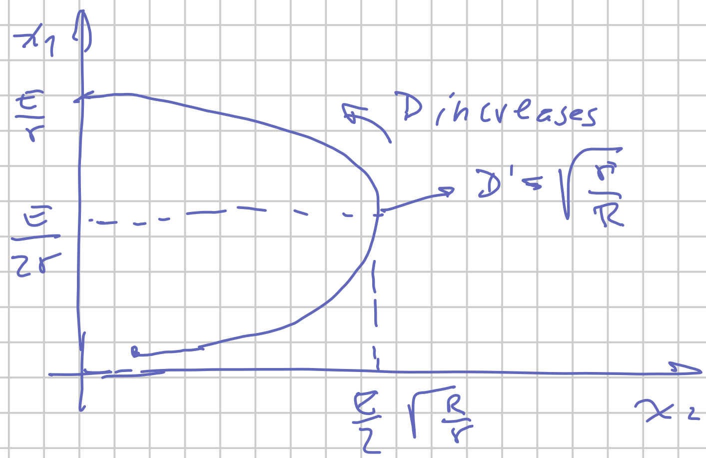
$$\begin{cases} L\dot{x}_1 = r x_1 - d' x_2 + E \\ C\dot{x}_2 = d' x_1 - \frac{x_2}{R} \end{cases}$$

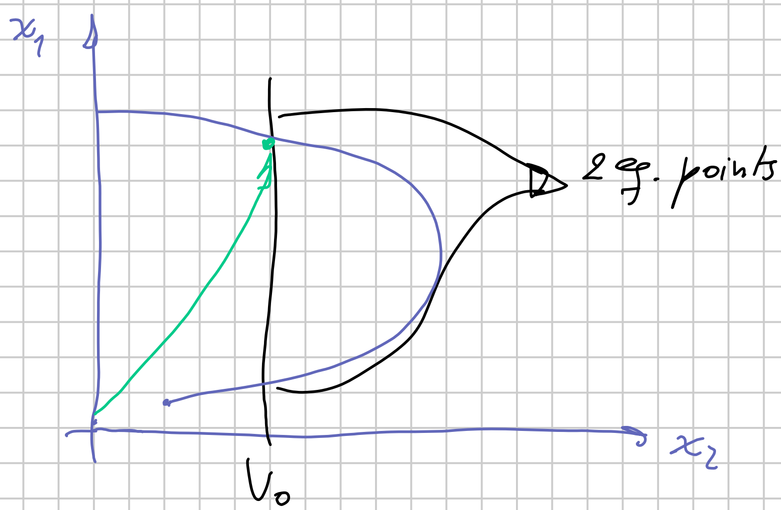
Suppose $d' = D'$

$$x_1 = \frac{1}{r}\left(E - D' x_2\right)$$

$$x_1 = \frac{1}{D'}\frac{x_2}{R} \longrightarrow D' = \frac{x_2}{x_1 R}$$

$$x_2 = \sqrt{-x_1^2 R r + E R x_1}$$





PI controller for boost converter

$$\begin{cases} L\dot{x}_1 = r x_1 - d' x_2 + E \\ C\dot{x}_2 = d' x_1 - \frac{x_2}{R} \end{cases}$$

PI controller $\rightarrow$ $d = k_i \int e\,dt + k_p e$

k_i and k_p have to be small (slow controller) or d should be limited to a max value of $\simeq 0.85 - 0.9$

Otherwise the trajectory goes to the equilibrium point with higher

Next: A more challenging case $\rightarrow$ constant power loads