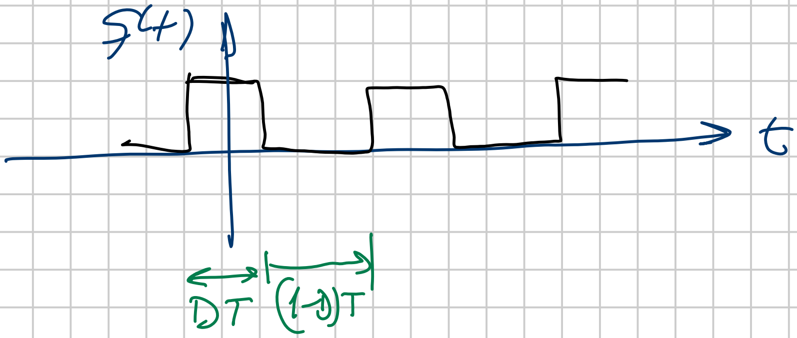
Inverters

Note Title 11/2/2008

Understanding the switching function is essential for analyzing inverters behavior.

For $f(t)$ as:

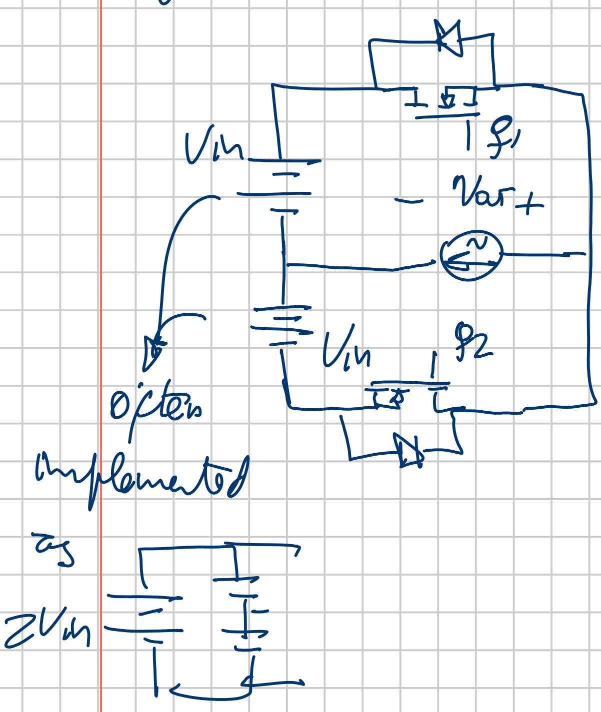


$$f(t) = D + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n} \cos(n\omega_0 t - n\phi_0)$$

D → average value

$$f(t)|_{rms} = \sqrt{D}$$

Like we did with the rectifiers let's assume first a single-phase output half bridge inverter



From KVL $q_1 + q_2$ cannot be more than 1

From KCL $q_1 + q_2$ cannot be less than 1

Hence, $q_1 + q_2 = 1$

When $q_1 = 1$, $V_{out} = V_{in}$

When $q_2 = 1$, $V_{out} = -V_{in}$

$V_{out} = q_1(t) V_{in} - q_2(t) V_{in}$

$$V_{out} = (2q_1 - 1) V_{in}$$

$$\underbrace{\langle V_{out} \rangle}_{\text{average}} = (2D_1 - 1) V_{in}$$

So if the load is inductive which requires $\langle V_L \rangle = 0$, then $\langle V_{out} \rangle = 0 \implies 2D_1 = 1$

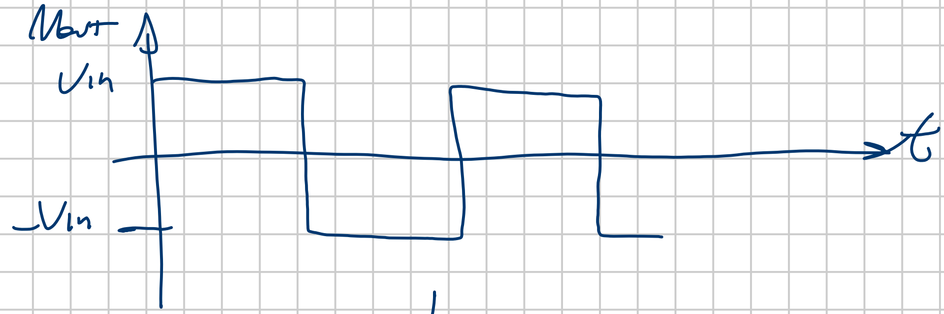
$$D_1 = \frac{1}{2}$$

and since $q_1 + q_2 = 1 \rightarrow D_1 + D_2 = 1$

$$D_2 = D_1 = \frac{1}{2}$$

Since we want an ac output, then let's assume that $D_1 = D_2$ regardless of whether or not the load is inductive.

then



2-level output

square wave

$$V_{out} = \frac{4V_{in}}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n} \cos(n\omega_{sw} t - n\phi)$$

it is obtained from $V_{out} = (2q_1 - 1)V_{in}$

and $q_1(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{2}\pi n\right)}{n} \cos(n\omega_0 t - n\phi_0)$

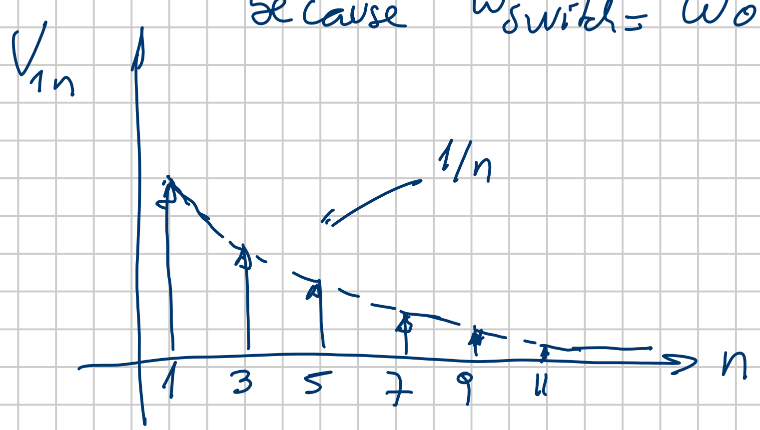
$D = \frac{1}{2}$

Usually we are mostly concerned with the fundamental

$$V_{1out} = \frac{4V_{in}}{\pi}$$

Issues: 1) Output is fixed $\rightarrow$ No voltage regulation

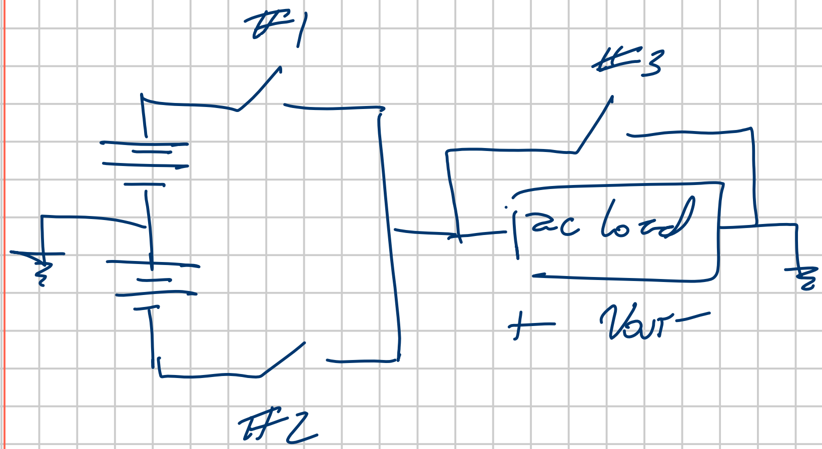
2) Harmonics too close to fundamental. And fundamental frequency is usually low because $\omega_{switch} = \omega_{out}$

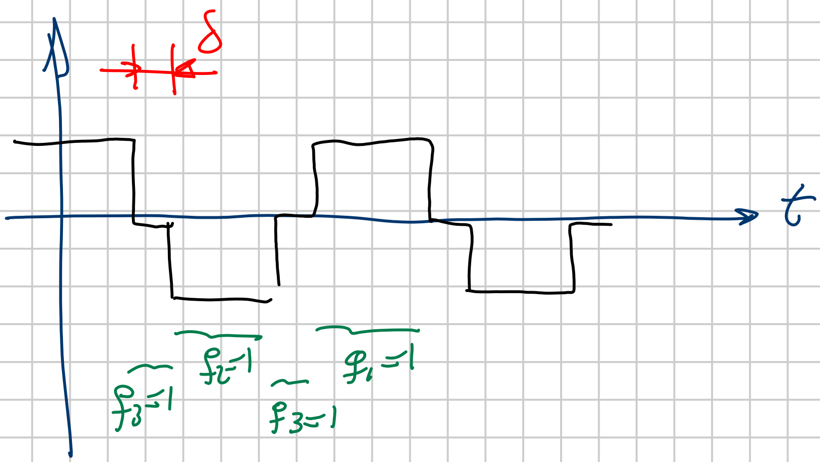


$\longleftarrow$ Spectrum for V_{out}

How can we have output voltage regulation?

One solution is the following one:



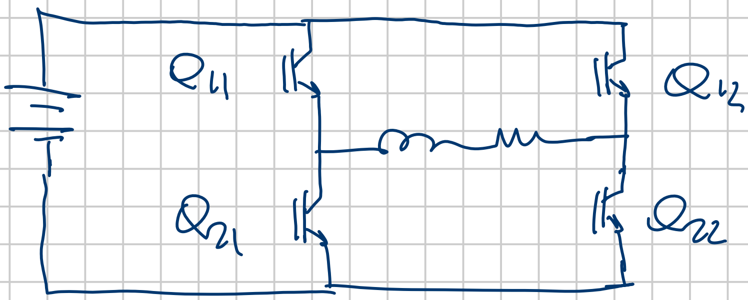


$$V_{OUT} = \frac{2V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi/2)}{n}\left[\cos(n\omega_{sw}t)+\cos(n\omega_{sw}t-n\delta)\right]$$

The fundamental component is

$$V_{OUT_1} = \underbrace{\frac{4V_{in}}{\pi}\cos\frac{\delta}{2}}_{V_{OUT_1}}\cos\left(\omega_{sw}t-\frac{\delta}{2}\right)$$

I can achieve the same behavior with a full bridge inverter:



From KVL & KCL:

$$\begin{cases} f_{11} + f_{21} = 1 \\ f_{12} + f_{22} = 1 \end{cases}$$

If Q_{11} and Q_{21} or Q_{12} and Q_{22} are simultaneously on I have shoot-through

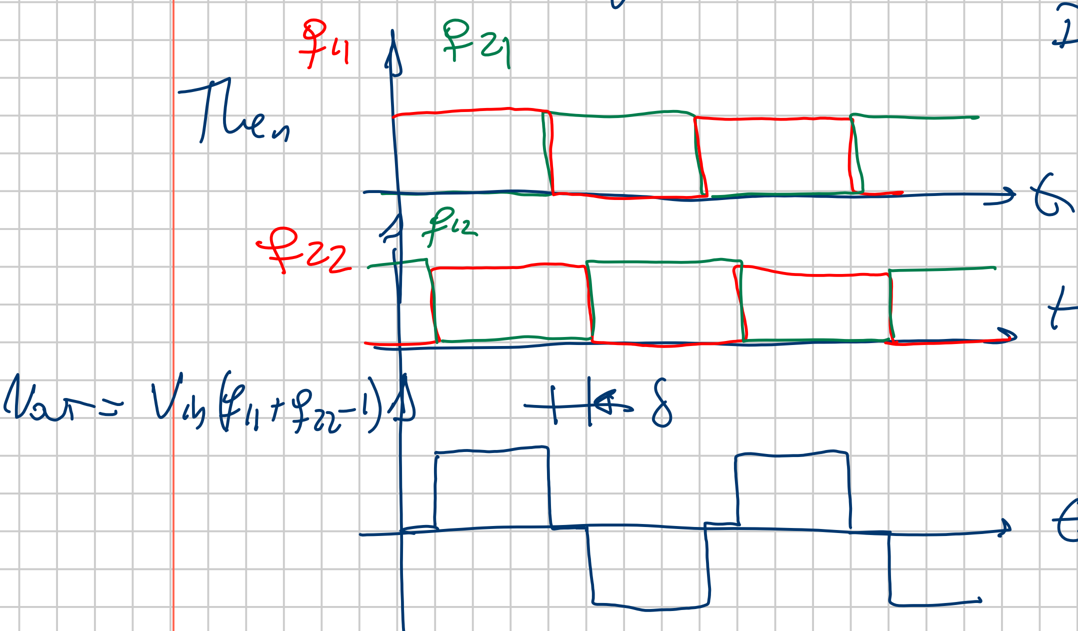
$$\begin{cases} Q_{11}, Q_{22} \text{ ON} \longrightarrow V_{out} = V_{in} \\ Q_{12}, Q_{21} \text{ ON} \longrightarrow V_{out} = -V_{in} \\ Q_{11}, Q_{12} \text{ ON} \longrightarrow V_{out} = 0 \\ Q_{21}, Q_{22} \text{ ON} \longrightarrow V_{out} = 0 \end{cases}$$

$$V_{out} = f_{11} V_{in} - f_{12} V_{in} = (f_{11} - f_{12}) V_{in} = (f_{11} + f_{22} - 1) V_{in}$$

To avoid having dc we need $D_{11} = D_{21} = 1/2$

$D_{12} = D_{22} = 1/2$

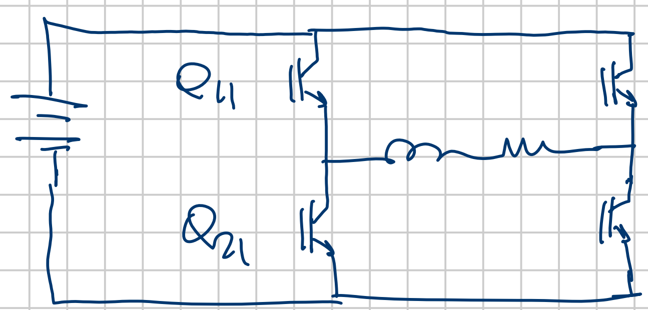
Then



So I have addressed issue #1 above. But this approach does not address issue #2. Can I address both simultaneously? Yes, with pulse-width modulation (PWM):

PWM

Let's consider the inverter seen before



and let's operate it only with the following 2 states:

State 1: Q_{11} ON and Q_{22} ON $\longrightarrow$ $V_{OUT} = V_{in}$
State 2: Q_{12} ON and Q_{21} ON $\longrightarrow$ $V_{OUT} = -V_{in}$

S_1 last for $d \cdot T_S$ and S_2 lasts for $(1-d)T_S$ where d is the duty cycle for a given switching period and T_S is the switching period.

Although d stays fixed during each switching period it can change from switching period to switching period

My goal is to obtain an ac output in which the fundamental has a frequency of $\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

So lets define k as

$$k = \frac{T_0}{T_s} \longrightarrow k >> 1$$

Let's consider now one switching interval $\hat{T_s}$.
During that particular switching interval d is fixed and equals: $d_{11} = D|_{\hat{T_s}}$

Then, the average output voltage for that switching interval $\hat{T_s}$ is:

$$\langle V_{OUT} \rangle|_{\hat{T_s}} = D|_{\hat{T_s}} V_{in} + (1 - D|_{\hat{T_s}})(-V_{in}) =$$
$$= V_{in}(2D|_{\hat{T_s}} - 1)$$

This expression comes from $V_{OUT} = (\varsigma_{11} + \varsigma_{22} - 1) V_{in}$. If $\varsigma_{11} = \varsigma_{22}$ then

$$\boxed{V_{OUT} = (2\varsigma_{11} - 1) V_{in}} \quad (1)$$

$$\downarrow \text{average}$$

$$\langle V_{OUT} \rangle = (2D_{11} - 1) V_{in}.$$

So from $\langle V_{OUT} \rangle|_{\hat{T_s}} = V_{in}(2D|_{\hat{T_s}} - 1)$ Let's assume, as I said before, that the duty cycle changes from switching cycle to switching cycle in the following particular way:

$$d_1(t) = \frac{1}{2} + \frac{1}{2} m(t)$$

where $m(t) = m \cos(\omega_0 t)$

modulation signal

modulation index

$$\boxed{m \triangleq \frac{V_{OUT1}}{V_{in}}}$$

desired fundamental amplitude

where $m \leq 1$ $\forall t$ so $|m(t)| \leq 1$ and

$$|d_1(t)| \leq \frac{1}{2} \quad \forall t$$

So

$$d_1(t) = \frac{1}{2} + \frac{1}{2} m \cos(\omega_0 t)$$

and

$$\langle V_{OUT} \rangle (t) = V_{in}\, m \cos(\omega_0 t)$$

The problem here is that I said "Although d stays fixed during each switching period it can change from switching period to switching period"

And in the above equation d changes continuously with time. So the correct form is

$$D|_{T_S} = \frac{1}{2} + \frac{1}{2} m \cos(\omega_0 t(nT_S))$$

As $T_S \to 0$ ($f_{SW} \to \infty$) then $D|_{T_S} \to d(t)$

and $t(nT_S) \to t$.

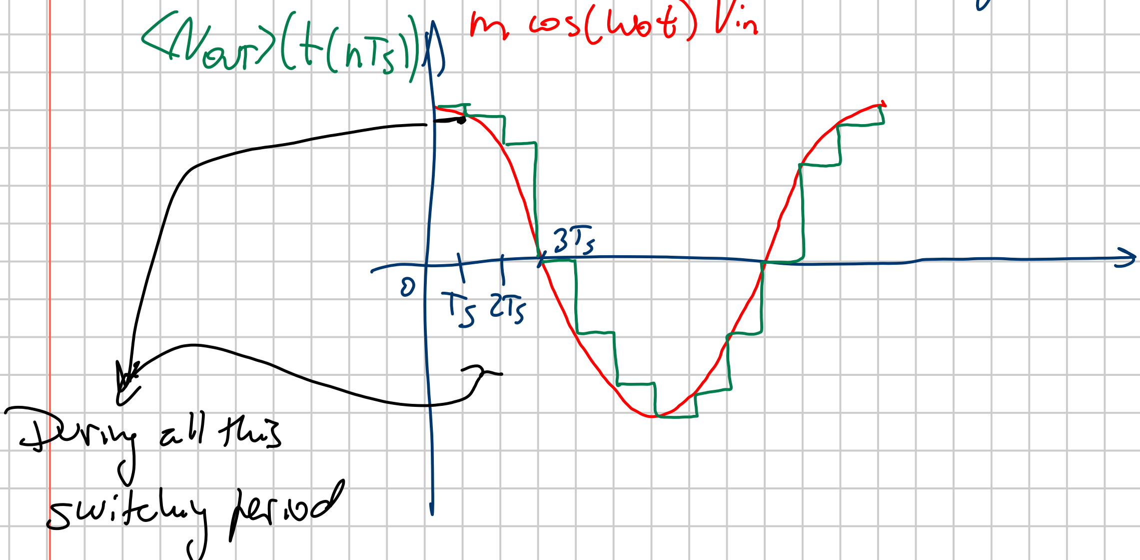
So, how do I take $t(nT_S)$. In other words, how do I sample the modulation signal?

There are 2 main approaches for sampling $m(t)$.

1) Uniform PWM (UPWM)

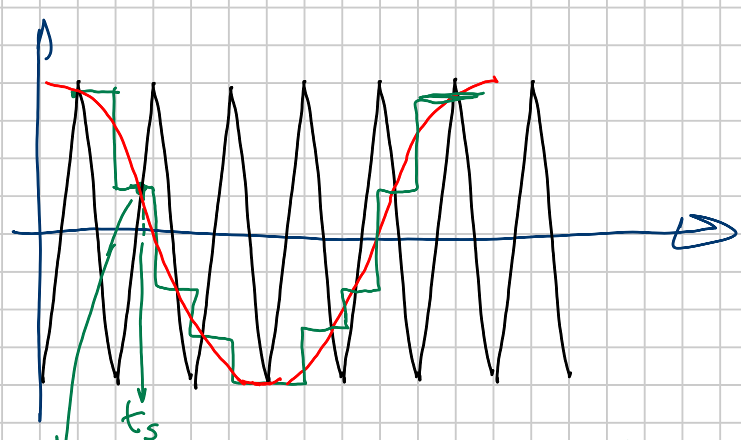
I sample every T_s seconds, usually at the start of each switching period.

This is easy to do with digital implementation



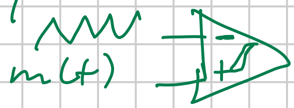
$$D\big|_{T_s} = \frac{1}{2} + \frac{1}{2} m \cos(\omega_0 T_s)$$

2) Natural PWM (NPWM) $\longrightarrow$ Sample is given by a triangle waveform



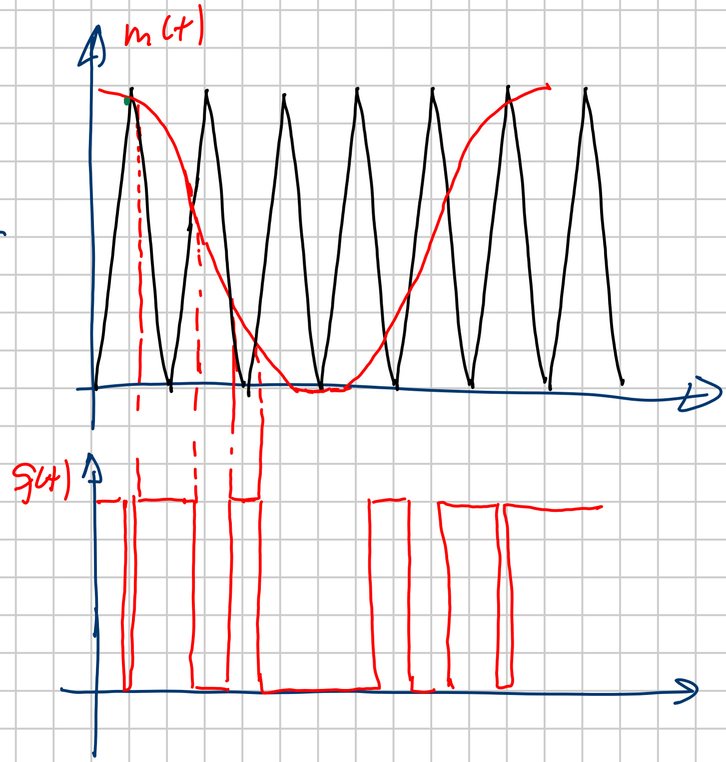
$\rightarrow$ to find t_s I need to solve the trascendental equation $c_t(t_s) = m(t_s)$

This is difficult when it is implemented digitally but it is easy when it is implemented analogically

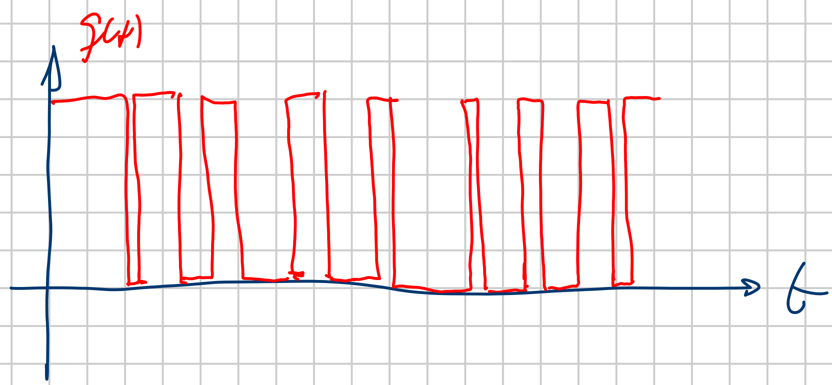


m(t) has, in fact, an offset of 1/2 with respect to the red curve above. So $g(t)$ looks something like the following.

If m > triangle g=1
If m < triangle g=0

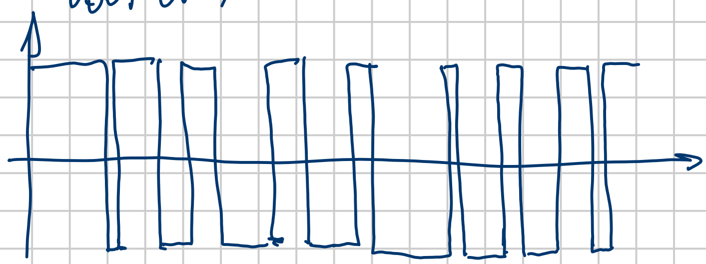


Typically $T_s << T_0$ so $g(t)$ looks closer to the following



And since $V_{out} = (2 g_{11}(t) - 1) V_{in}$

if $d_{11}(t) = \frac{1}{2} + \frac{1}{2} m(t)$



$$v_{OUT}(t) = \underbrace{\left(2D|_{\hat{T}_s} - 1\right)}_{\langle v_{OUT}\rangle|_{\hat{T}_s}(t)} V_{in} + \frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\pi D|_{\hat{T}_s}\right)}{n}\cos(n\omega_{sw}t)$$

If $T_s \to 0$ then

$$v_{out}(t) = \left(2d_{11}(t) - 1\right)V_{in} + \frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi d_{11})}{n}\cos(n\omega_{sw}t)$$

$$v_{out}(t) = m(t)V_{in} + \frac{4V_{in}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(n\pi\left(\frac{1}{2}+\frac{m(t)}{2}\right)\right)}{n}\cos(n\omega_{sw}t)$$

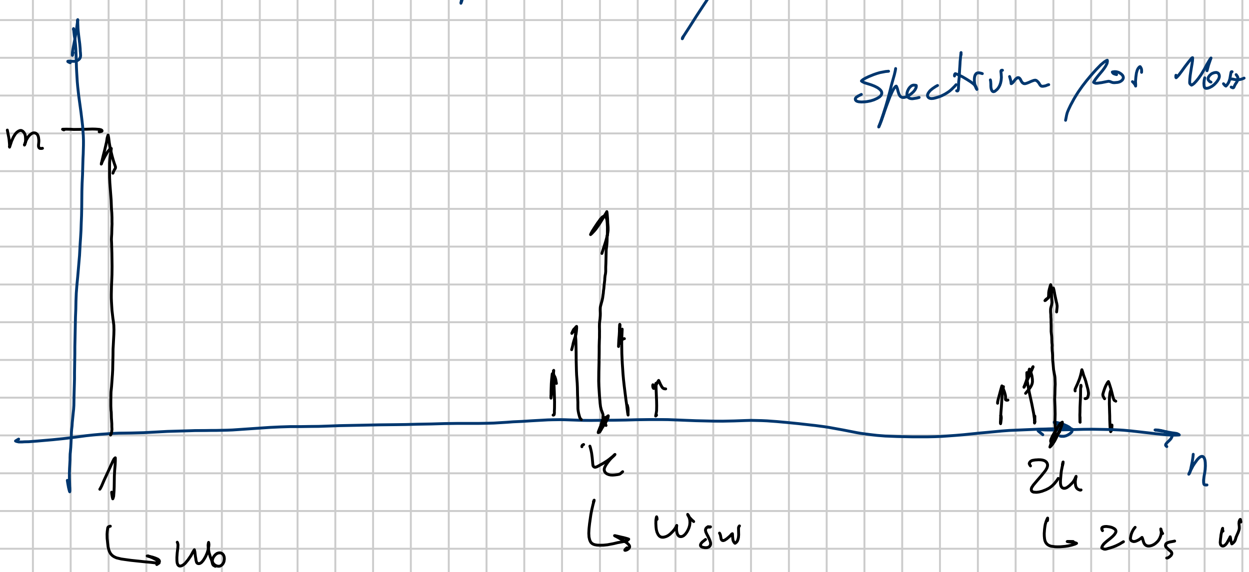
Fundamental

$v_{out_1}(t) = m V_{in}\cos\omega_0 t$

harmonic content

For T_s small (k large) both NPWM and UPWM yields the same spectrum for v_{OUT}

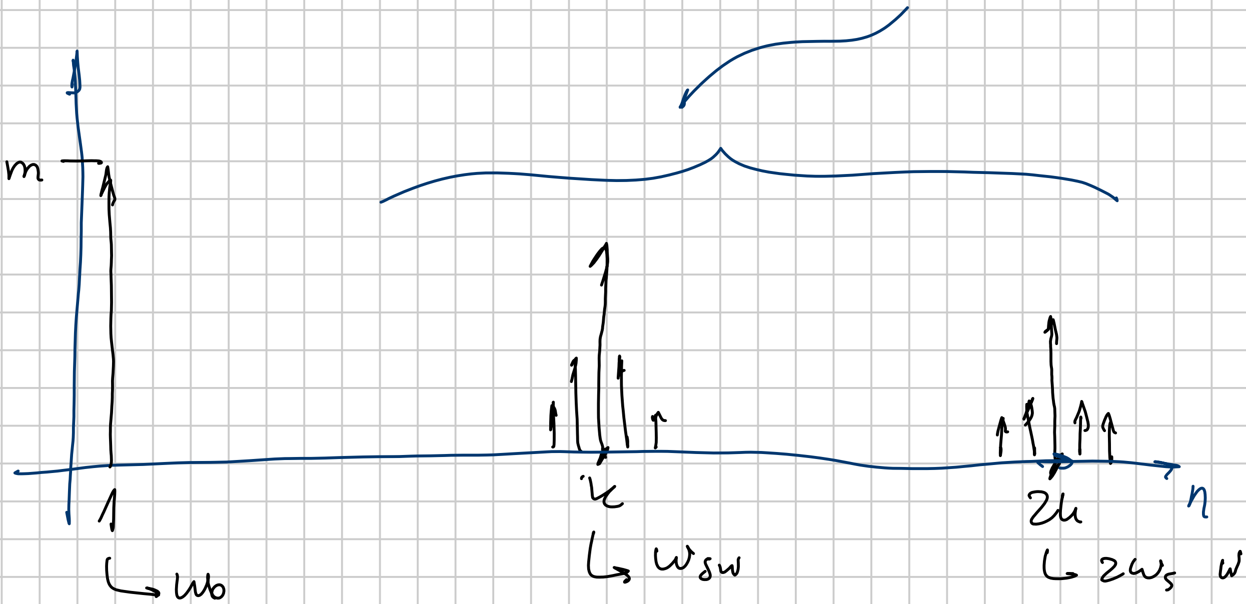
Spectrum for v_{out}



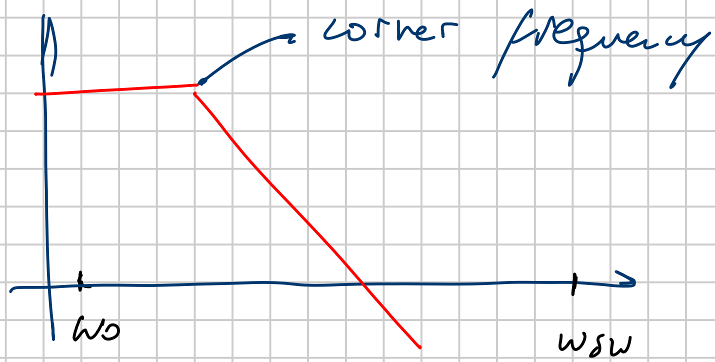
In reality → NPWM → it is more difficult to implement but doesn't have harmonics around the fundamental (wo)

→ UPWM → it's easier to implement but for low k it yields harmonics around ω_0

How do I filter the unwanted harmonics?

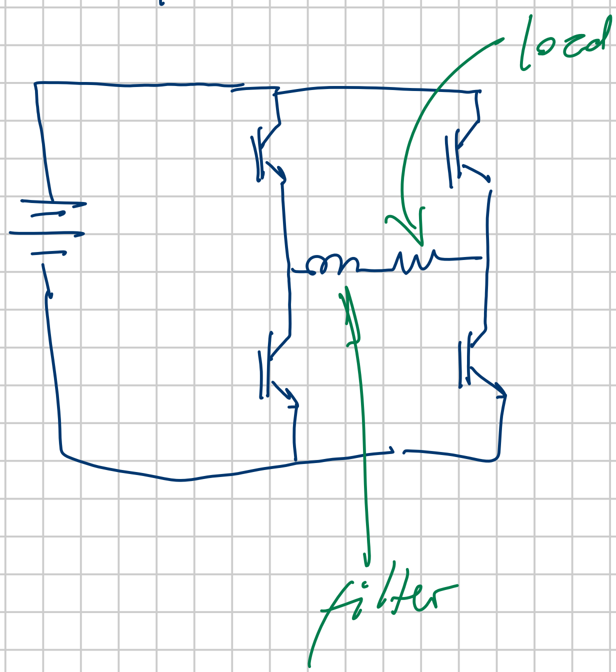


Answer: I use a low-pass filter:



The advantage is that wsw is away from wo, so I can reduce the size of my filter components by choosing a higher corner frequency

So the filter looks something like this.



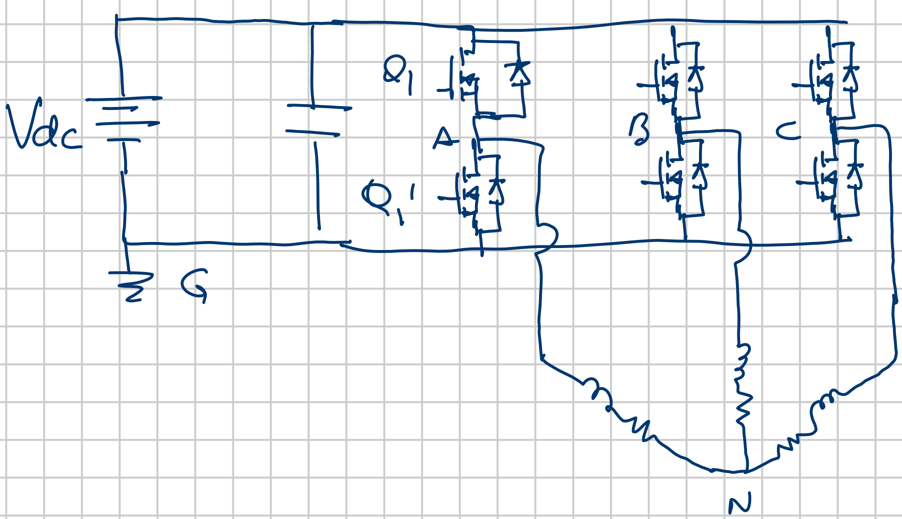
Voltage input → current output

↓

Voltage-source inverter

VSI

3-phase VSI:



If Q_1 and Q_1' are simultaneously on I have shoot-through

↓

I need to add standby time

Possible voltages $V_{AG}, V_{BG}, V_{CG} = \begin{cases} V_{dc} \\ 0 \end{cases}$

$$V_{AB}, V_{BC}, V_{CA} = \begin{cases} +V_{dc} \\ -V_{dc} \\ 0 \end{cases}$$

$$V_{AN}, V_{BN}, V_{CN} = \begin{cases} 2/3\,V_{dc} \\ +1/3\,V_{dc} \\ -1/3\,V_{dc} \\ 2/3\,V_{dc} \end{cases}$$

$$v_{AG} = \begin{cases} V_{dc} & \text{with } Q_1 = on \\ 0 & \text{with } Q_1 = off \end{cases}$$

$$v_{AG_{fund}}(t) = g_{11}(t)\, V_{dc} = \left(\frac{1}{2} + \frac{1}{2} m_a(t)\right) V_{dc}$$

↳ fundamental

In 3 phase systems:

$$m_a(t) = m \cos \omega_0 t$$

$$m_b(t) = m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$m_c(t) = m \cos\left(\omega_0 t + \frac{2\pi}{3}\right)$$

$$v_{AB_{fund}} = v_{AG_{fund}} - v_{BG_{fund}} = \left\{ \frac{1}{2}\, m_a(t) - m_b(t) \right\} V_{dc} =$$

line voltage

$$= \left\{ \frac{1}{2}\, m \left[\cos \omega_0 t - \cos\left(\omega_0 t - \frac{2\pi}{3}\right)\right]\right\} V_{dc}$$

$$= \frac{1}{2}\, m\, V_{dc} \sqrt{3} \cos\left(\omega_0 t + \frac{\pi}{6}\right)$$

$$V_{AB_{fund\ peak}} = \frac{\sqrt{3}}{2}\, m\, V_{dc} \longrightarrow m = \frac{V_{AB\, fund\, peak}}{\sqrt{3}\ \frac{V_{dc}}{2}}$$

$$m = \frac{V_{AB\, fund\, peak}}{\sqrt{3}\ \frac{V_{dc}}{2}} = \frac{\sqrt{2}\, V_{AB\, fund\, RMS}}{\sqrt{3}\ \frac{V_{dc}}{2}} = \frac{\sqrt{2}\, V_{AN\, fund\, RMS}}{\frac{V_{dc}}{2}}$$

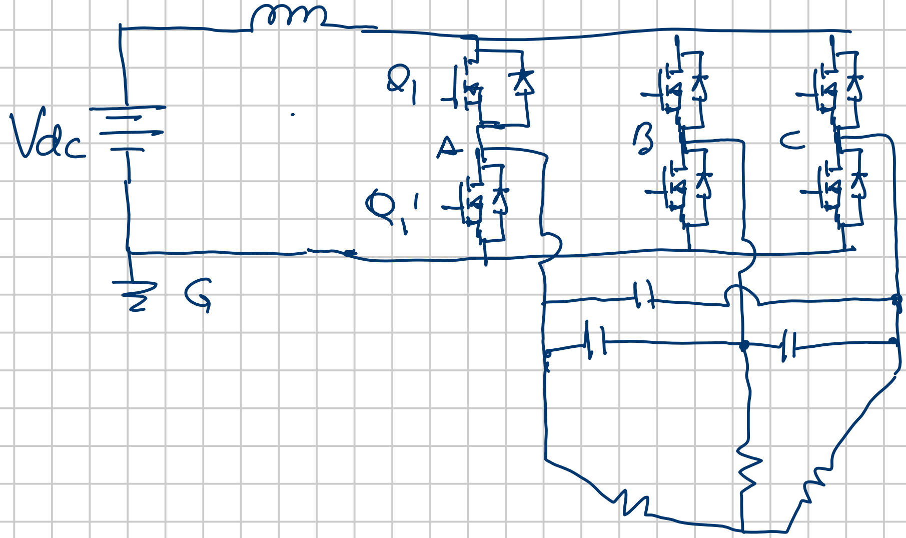
$$\boxed{m = \frac{V_{AN\, fund\, peak}}{V_{dc}/2}}$$

→ half the gain than a 1-phase inverter

Still we have a VSI ⇒ Voltage input, current output

Other topologies are:

_ Current source interface (CSI)



← load

Z-Source Inverter

Fang Zheng Peng, *Senior Member, IEEE*

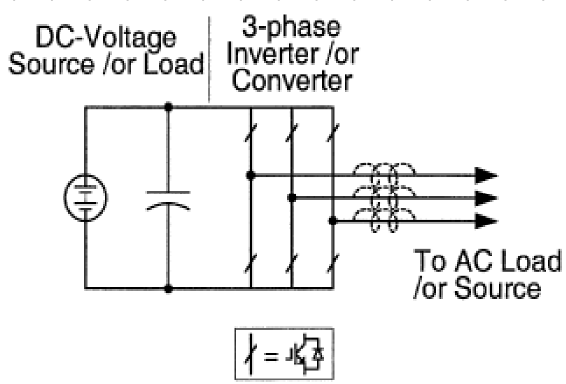
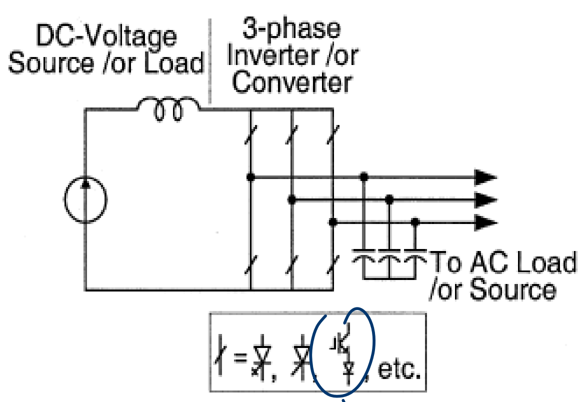


Fig. 1. Traditional V-source converter.

only reduces the input voltage

inconvenient output filter



only boost input voltage

Fig. 2. Traditional I-source converter.

Requires additional diode to block

Impedance source inverter → ZSI

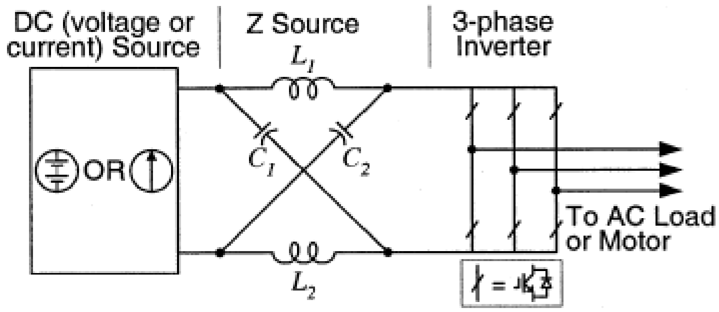


Fig. 4. Z-source converter structure using the antiparallel combination of switching device and diode.

Can both buck and boost voltage

$$V_{out\,fund\,peak\,ph} = mB\frac{V_{dc}}{2}$$

m: mod index

B: boost factor → depends on the ratio between shoot-through and non-shoot-through states

$mB = B_B$ → Buck-Boost factor → Can vary between 0 and 1

2:1 range in fuel cells

typical dc link voltage

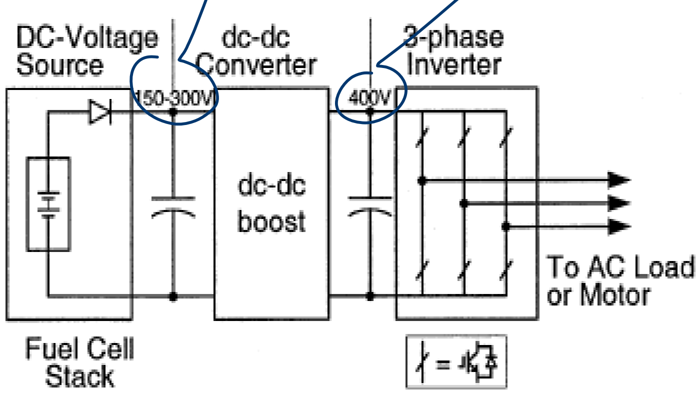


Fig. 6. Traditional two-stage power conversion for fuel-cell applications.

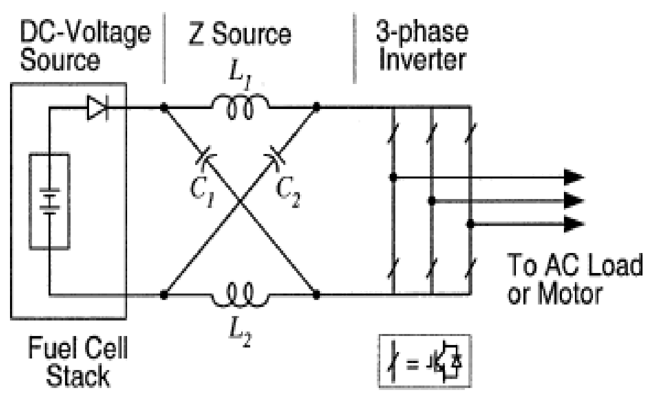


Fig. 7. Z-source inverter for fuel-cell applications.