EE394V Rectifiers

Rectifiers $\rightarrow$ ac to dc
Inverters $\rightarrow$ dc to ac

With rectifiers and inverters it's all about managing harmonics $\rightarrow$ creating harmonics thanks to nonlinear nature of power electronics circuit $\rightarrow$ Filtering out unwanted harmonics.

Good references:
- Dr. Mohsen's power electronics book
- Dr. Vreesh's """

Rectifiers

Typically, they can be single-phase or 3-phase.
- The most simple topologies use diodes.
- The most common rectifier circuits convert ac-voltage to dc current.

Let's start with the single-phase circuit.

Half wave:

![Half wave rectifier circuit diagram]

$N_s$ $\rightarrow$ $V_c$
Since I want dc output, what's my dc component \( V_{dc} \) in \( N_L(t) \)?

\[
V_{dc} = \frac{1}{T} \int_{0}^{T/2} \sin \left( \frac{2\pi t}{T} \right) dt = -\frac{T}{2\pi} \left. \frac{1}{T} \cos \left( \frac{2\pi t}{T} \right) \right|_{0}^{T/2} =
\]

\[
= -\frac{1}{2\pi} \left( \cos \frac{T}{2} - \cos 0 \right) = \frac{1}{T}
\]

Let's see a full-wave rectifier.
Doubles the input frequency

**dc component:**

\[ V_{dc} = \frac{1}{T_0} \int_0^{T_0} \frac{1}{2} \sin \left( \frac{2\pi}{T_0} t \right) dt = \frac{T}{2\pi T_0} \left( \cos \left( \frac{2\pi}{T_0} T_0 \right) - 1 \right) = \frac{2}{T_0} \]

So the dc component is double than in the previous case.

What about the harmonic content?

Remember that \( V_{rms}^2 = V_{dc}^2 + V_{harms}^2 \)

In the 1/2 bridge case,

\[ V_{rms}^2 = \frac{1}{T} \int_0^{T/2} \sin^2 \left( \frac{2\pi}{T} t \right) dt = \frac{T}{4} \left( \frac{T}{2} - \frac{T}{4\pi} \sin \frac{4\pi t}{T} \right)^2 \]
\[ \frac{1}{T} \left( \frac{T}{4} - 0 - \frac{T}{8\pi} \sin 2\pi + 0 \right) = \frac{1}{4} \]

So \[ V_{\text{rms}} = \frac{1}{4} - \left( \frac{1}{11} \right)^2 \approx 0.15 \]

For the full bridge case:

\[ V_{\text{rms}} = \frac{1}{T_0} \left( \frac{T_0}{2} - 0 - \frac{T_0}{8\pi} \sin \left( \frac{4\pi}{8\pi} T_0 \right) - 0 \right) = \frac{1}{2} \]

\[ V_{\text{rms}} = \frac{1}{2} - \left( \frac{2}{8\pi} \right)^2 \approx 0.095 \]

So in the full bridge not only there is a smaller harmonic content but it is at a double frequency. Here, it is easier (but not easy) to filter than in the half-bridge case.

How do we filter it? Well, if we want to keep the dc component only then we need a low-pass filter. The simplest one is just one capacitor \\
\[ V(t) \]
But, we still need a large capacitor. Let's see the waveforms.

\[ V_c(t) \]

\[ N(t) \]

\[ V_c(t) \]

\[ i_c(t) \]

The capacitor needs to hold the load for a long time.

- Capacitor discharges
- Capacitor charges

\[ i_{in} = I_L + i_c \]

\[ i_c = C \frac{dV_c}{dt} = C \frac{d(V_{in} \sin(\omega t))}{dt} \]

\[ i_c = |i_{peak} \cos(\omega t)| \]

What is the output voltage ripple?

Well, from

\[ i_c = C \frac{dV_c}{dt} \]

\[ I_c \approx C \frac{\Delta V_c}{\Delta t} \]
Hence, \[ \Delta V_C \propto \frac{I_L \Delta t}{C} \]

So \[ \Delta V_C \propto I_L \left( t_d - t_c \right) \]

If \( C \) is large enough, \( t_c \approx 0 \) and \( t_d \approx T/2 \)

So \[ \Delta V_C \propto I_L \frac{T/2}{C} \]

\[ \Delta V_C \propto \frac{I_L}{2fC} \]

Since \( f \) is the line frequency (at most a few hundred Hertz in practice), I usually need a large capacitor for small voltage ripples.

The current and voltage at the source are:

![Graph showing current and voltage](image)

What is the power factor as seen by the source?

Remember \[ \text{Pf} = \frac{P}{\text{Vrms} \times \text{Irms}} \]
For $P = V_{rms} I_{rms} \cos \phi$, 

\[ i_{in} \]

\[ I_{in \text{ rms}} = I_{in \text{ base}}^2 + I_{in h}^2 \]

- tends to be high when compared to $I_{in h}^2$
  - because it is formed by pulses (higher content of higher harmonics)

So \[ \text{pf} = \frac{V_{rms} I_{rms} \cos \phi}{V_{rms} (I_{in \text{ base}}^2 + I_{in h}^2)} \leq 1 \]

- And in reality
  - is quite low

This is not good for the source

What can we do to improve this?

We can add

This inductor in here.

It provides a current interface
The inductor combined with the capacitor form a 2nd order low-pass filter. So the output is more constant with a smaller capacitor, with a "large" inductor $i_{out}$ is approximately constant on equal to $i_L$. Then

\[ V_S \]

\[ V_{out} \]

\[ i_{in} \]

Now $i_{in}$ has a higher fundamental with respect to the harmonic content so the power factor is better but it can be still low.

One other solution to use a boost converter after the rectifier

$V_S$ act as a reference for the current in the inductor
The current in the inductor is controlled to follow the voltage in the source. The output capacitor takes care of the last filter stage.

\[ V_s \]

\[ V_o \]

\[ I_o \]

\[ I_o' \]

\[ \text{In this way the PF} \approx 1. \]

Let's see some additional issues in rectifiers:

I can draw it as:
One issue is that I cannot regulate my output (i.e., if \( V_\text{in} \) changes, \( V_\text{out} \) changes). One solution is to use SCRs instead of diodes:

An SCR conducts when it is forward biased and it has been triggered by injecting an adequate current pulse at the gate.

\[ \text{So I can now change } V_{\text{out}} \text{ by changing } \alpha \]
Another issue is that most DC sources that require a rectifier (e.g., microturbine) have a current interface.
So I actually have

![Diagram showing electrical components and waveforms]

So how do the curves look with a current input and voltage output?

This is the most likely scenario.
3-phase rectifier

\[ N_{\text{in}} \]

\[ N_{\text{out}} \]

\[ V_{\text{out}} = \text{constant} \]

\[ t \]

It yield a bad power factor

Needs to be compensated

\( \text{e.g. dc link capacitor and a boost converter} \)
\[ V_{out} = \text{constant} \]

\[ V_{zn}, V_{bn}, V_{an} \]

\[ V_{zb}, V_{bc}, V_{cz} \]

\[ V_{eb}, V_{bc}, V_{cz} \]

Note: Unfiltered signal frequency is 6 times the line frequency (easier to filter harmonics).
Generalized $m$-phase rectifiers

Not a great power factor either

It is also not as simple to compensate than in the single phase case

Full bridge rectifier

\[ V_{out} = \frac{m \sqrt{2} V \sin \left( \frac{\pi f}{m} \right)}{f} \cos \phi \]

for $m > 2$
\[ V_{\text{out}} = \frac{\sqrt{2}}{\pi} V_{\text{rms}} \sin \left( \frac{\pi}{m} \right) \left[ \cos \alpha_m - \cos \alpha_n \right] \]

for \( m \geq 2 \)

Special case → With diodes \( \alpha_n = 0 \), \( \alpha_m = 180^\circ \) \( m \)

\[ V_{\text{out}} = 2\frac{\sqrt{2}}{\pi} V_{\text{rms}} \sin \left( \frac{\pi}{m} \right) \]