EE394V Rectifiers

Note Title 11/2/2008

Rectifiers $\longrightarrow$ ac to dc

Inverters $\longrightarrow$ dc to ac

With rectifiers and inverters it's all about managing harmonics $\longrightarrow$ creating harmonics thanks to nonlinear nature of power electronics circuits

$\longrightarrow$ Filtering out unwanted harmonics.

Good references: Dr Mohan's power electronics book

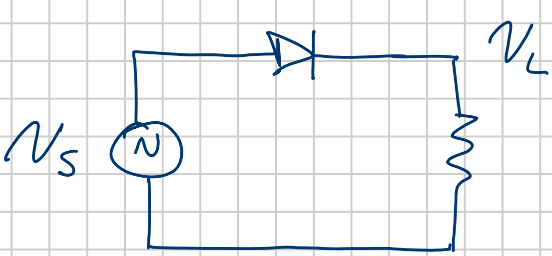
Dr Krein's " " "

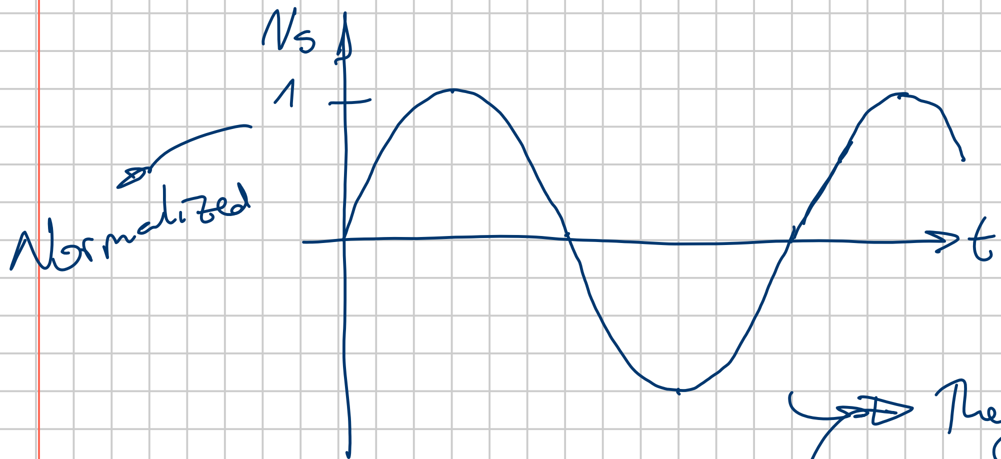
Rectifiers

- Typically they can be single-phase or 3-phase
- the most simple topologies use diodes.
- The most common rectifier circuits convert <u>ac-voltage</u> to <u>dc current</u>

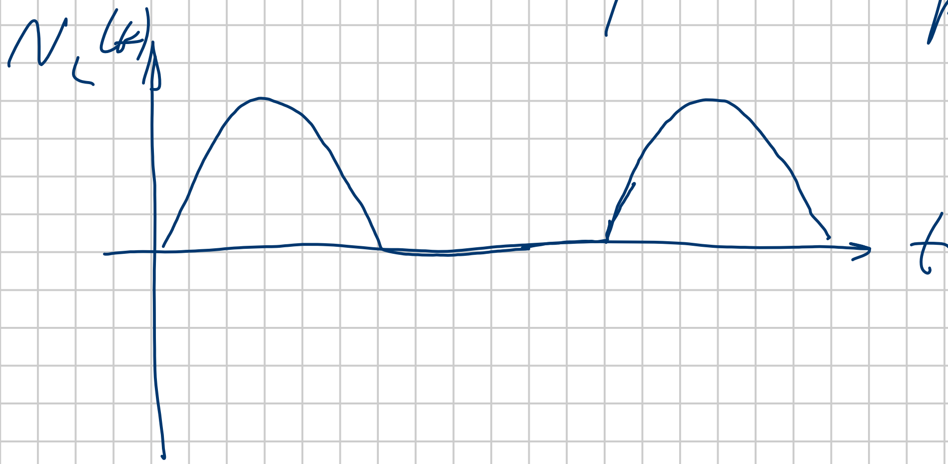
Let's start with the single-phase circuits.

Half wave:





They have the same period

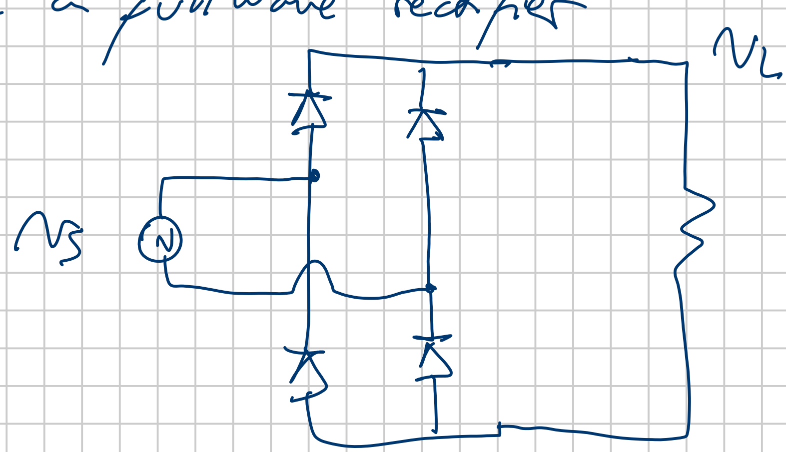


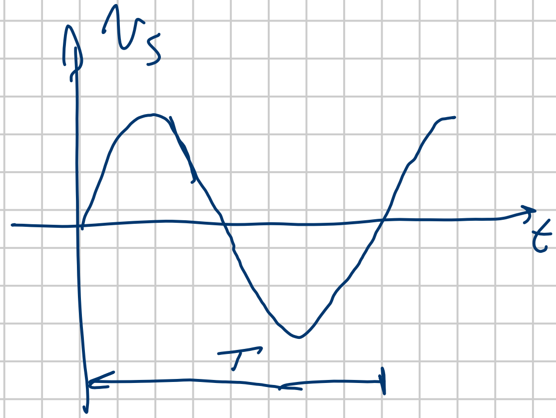
Since I want dc output, what's my dc component V_{dc} in $v_L(t)$?

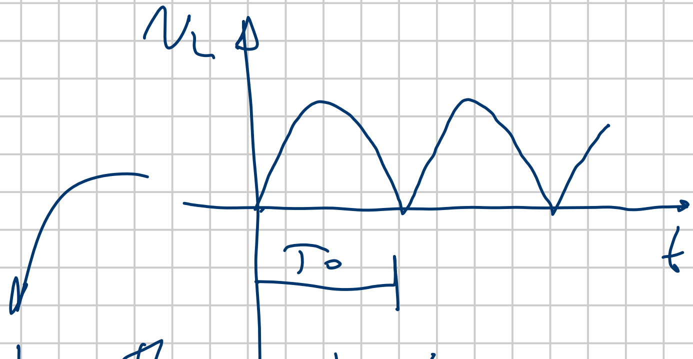
$$V_{dc} = \frac{1}{T}\int_0^{T/2} \sin\frac{2\pi}{T}t\,dt = -\frac{T}{2\pi}\,\frac{1}{T}\cos\frac{2\pi}{T}t\Big|_0^{T/2} =$$

$$= -\frac{1}{2\pi}\left(\cos\pi - \cos 0\right) = \frac{1}{\pi}$$

Let's see a full wave rectifier







Doubles the input frequency

dc component:

$$V_{dc} = \frac{1}{T_0}\int_0^{T_0} \sin\left(\frac{2\pi}{T}t\right)dt = \frac{T}{2\pi T_0}\left(-\cos\left(\frac{2\pi}{T}t\right)\right)\Bigg|_0^{T_0} =$$

$$= -\frac{1}{\pi}\left(\cos\left(\frac{2\pi}{2T_0}T_0\right) - 1\right) = \frac{2}{\pi}$$

So the dc component is double than in the previous case

What about the harmonic content:

Remember that $V_{RMS}^2 = V_{dc}^2 + V_{h\,RMS}^2$

In the 1/2 bridge case

$$V_{RMS}^2 = \frac{1}{T}\int_0^{T/2} \sin^2\left(\frac{2\pi}{T}t\right)dt = \frac{1}{T}\left(\frac{t}{2} - \frac{T}{4\cdot 2\pi}\sin\frac{4\pi}{T}t\right)\Bigg|_0^{T/2} =$$

$$= \frac{1}{T}\left(\frac{T}{4} - 0 - \frac{T}{8\pi}\sin 2\pi + 0\right) = \frac{1}{4}$$

So $V_{h\,RMS}^2 = \frac{1}{4} - \left(\frac{1}{\pi}\right)^2 \cong 0.15$

For the full bridge case:

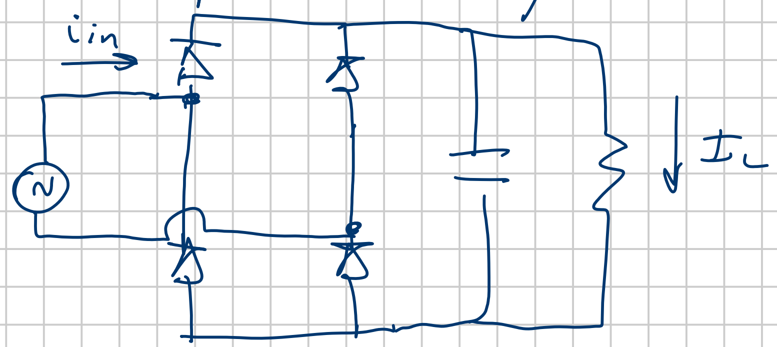
$$V_{RMS_h}^2 = \frac{1}{T_0}\int_0^{T_0} \sin^2\left(\frac{2\pi}{T}t\right)dt = \frac{1}{T_0}\left(\frac{t}{2} - \frac{T}{4 \cdot 2\pi}\sin\frac{4\pi}{T}t\right)\Bigg|_0^{T_0} =$$

$$= \frac{1}{T_0}\left(\frac{T_0}{2} - 0 - \frac{T}{8\pi}\sin\left(\frac{4\pi}{2T_0}T_0\right) - 0\right) = \frac{1}{2}$$

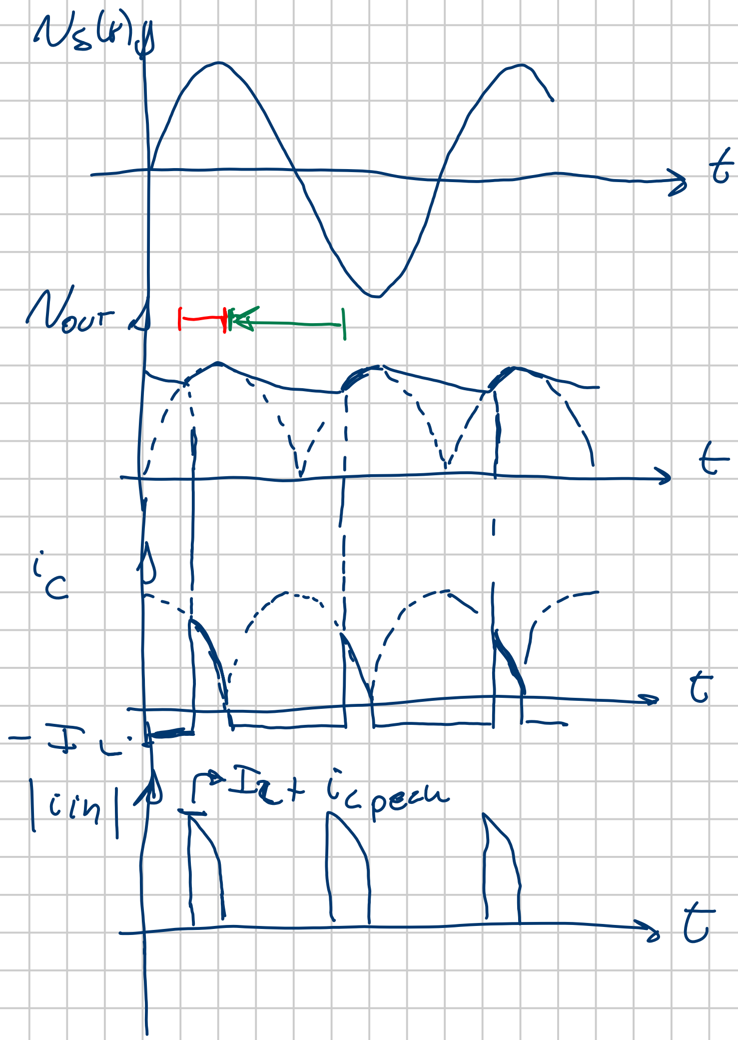
$$V_{h\,rms}^2 = \frac{1}{2} - \left(\frac{2}{\pi}\right)^2 \cong 0.095$$

So in the full bridge not only there is a smallest harmonic content but it is at a double frequency. Hence, it is easier (but not easy) to filter than in the 1/2-bridge case.

How do we filter it? Well, if we want to keep the dc component only then we need a low-pass filter. The simplest one is just one capacitor:



But, we still need a "large" capacitor. Let's see the waveforms.



The capacitor needs to hold the load for a long time

— capacitor discharges

— capacitor charges

$i_{in} = I_L + i_c$

$i_c = C\frac{dv}{dt} = \left|C\frac{d(\sin \omega t)}{dt}\right|$

$i_c = |\omega C \cos \omega t|$

What is the output voltage ripple?

Well, from

$$i_c = C\frac{dv_c}{dt}$$

$$\Downarrow$$

$$I_c \cong C\frac{\Delta V_c}{\Delta t}$$

Hence,

$$\Delta V_C \simeq \frac{I_L \Delta t}{C}$$

So $$\Delta V_C \simeq \frac{I_L (t_d - t_c)}{C}$$

If C is large enough $t_c \simeq 0$ and $t_d \simeq T/2$

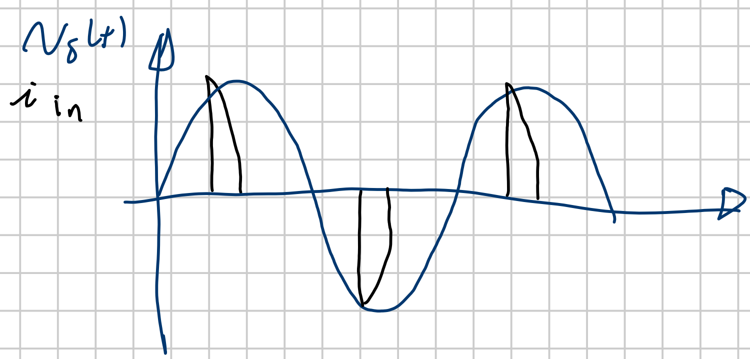
so

$$\Delta V_C \simeq \frac{I_L \, T/2}{C}$$

$$\boxed{\Delta V_C \simeq \frac{I_L}{2 f C}}$$

↳ Since f is the line frequency (at most a few hundred hertz in eturbines), I usually need a "large" capacitor for "small" ripples

The current and voltage at the source are:

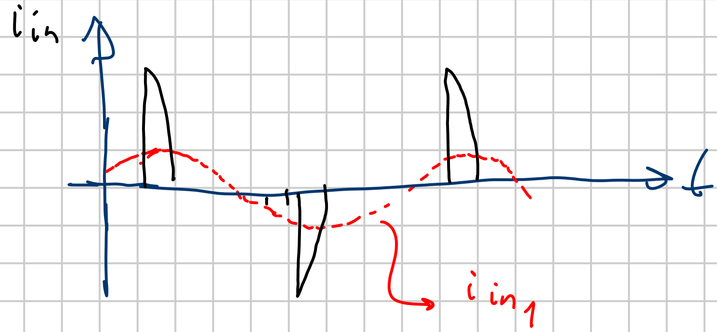


What is the power factor as seen by the source

Remember

$$pf = \frac{P}{V_{rms} I_{rms}}$$

For $P = V_{s\,rms} I_{1\,rms} \cos\varphi_1$



$$I_{in\,rms} = I_{in_{1rms}}^2 + \underbrace{I_{in_h}^2}$$

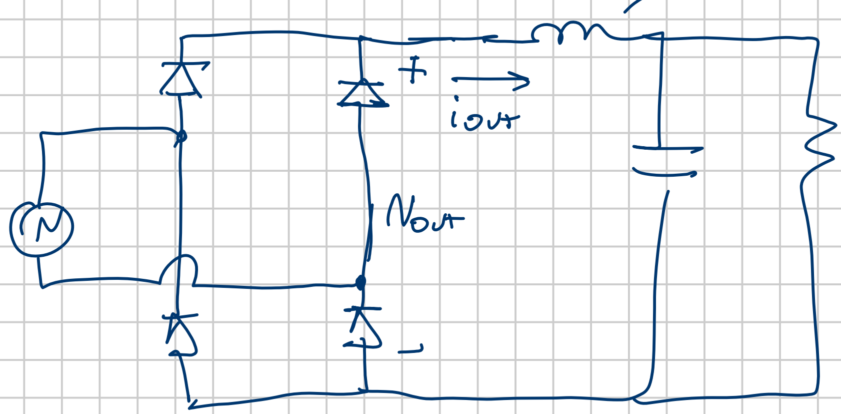
Tends to be high when compared to $I_{h_1}^2$ because it is formed by pulses (higher content of higher harmonics)

So $$pf = \frac{\cancel{V_{s\,rms}} I_{1\,rms} \cos\varphi_1}{\cancel{V_{s\,rms}} \left(I_{in_{1\,rms}}^2 + I_{in_h}^2 \right)} < 1$$

And in reality is quite low

This is not good for the source

What can we do to improve this?

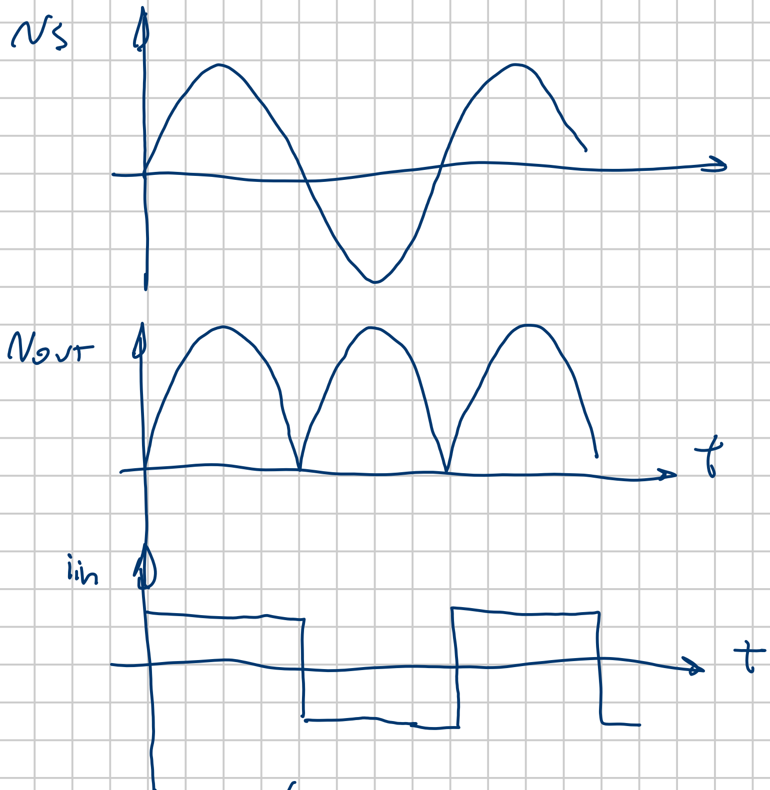


we can add this inductor in here.

it provides a current interface

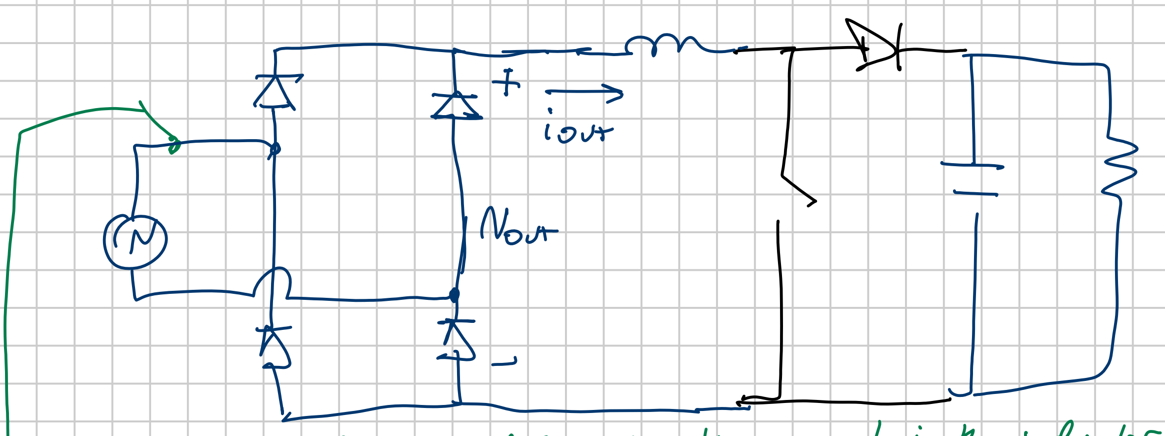
The inductor combined with the capacitor form a 2nd order low-pass filter. So the output is more constant with a smaller capacitor.

With a "large" inductor i_{OUT} is approximately constant an equal to I_L. Then



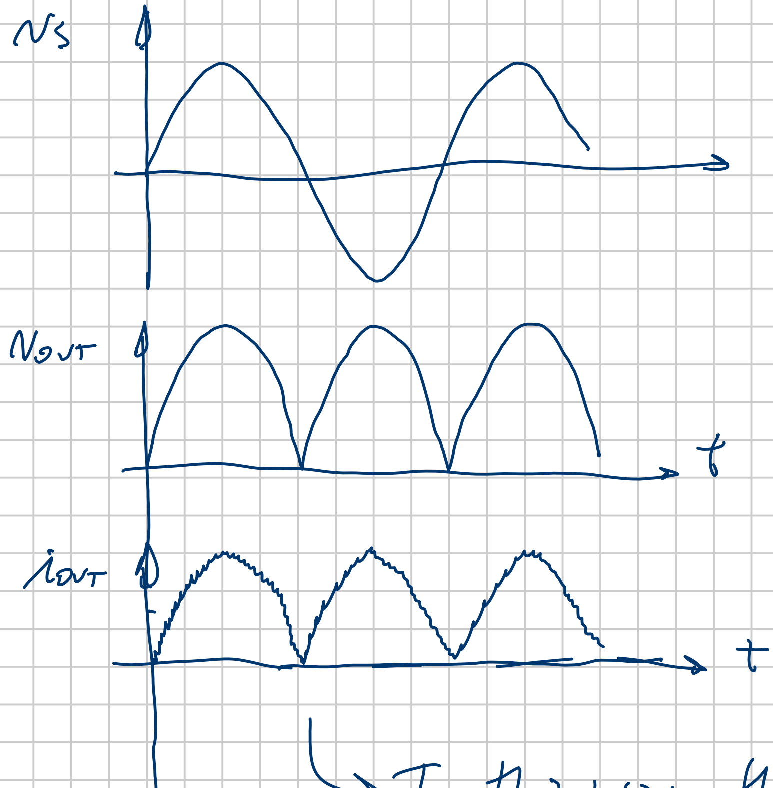
↳ Now i_{in} has a higher fundamental with respect to the harmonic content so the power factor is better but it can be still low.

One other solution to use a boost converter after the rectifier



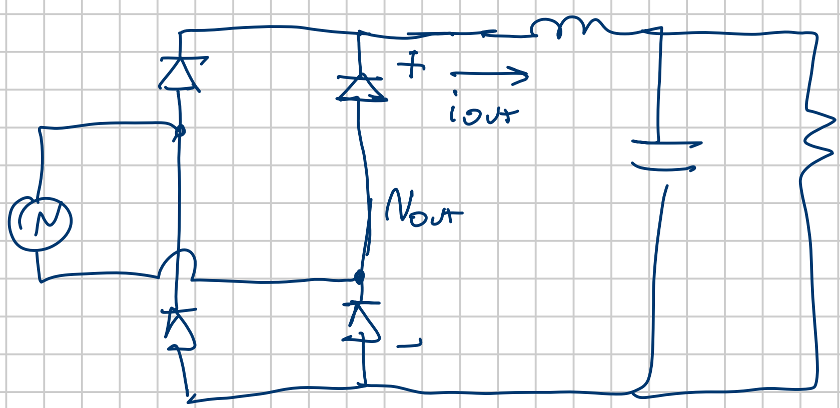
V_s act as a reference for the current in the inductor

The current in the inductor is controlled to follow the voltage in the source. The output capacitor takes care of the last filter stage.

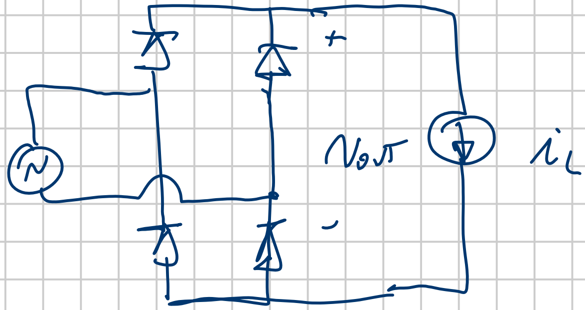


In this way the $pf \simeq 1$.

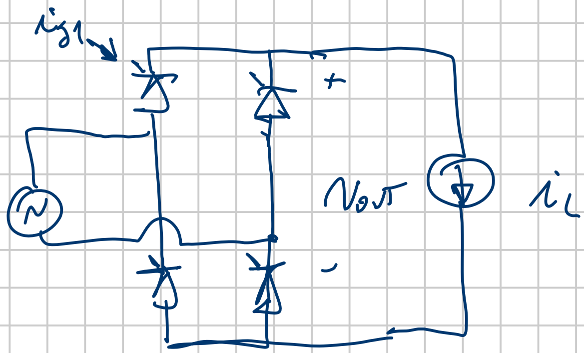
Let's see some additional issues in rectifiers

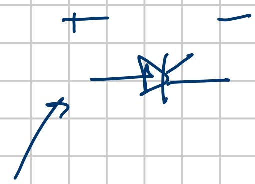


I can draw it as:

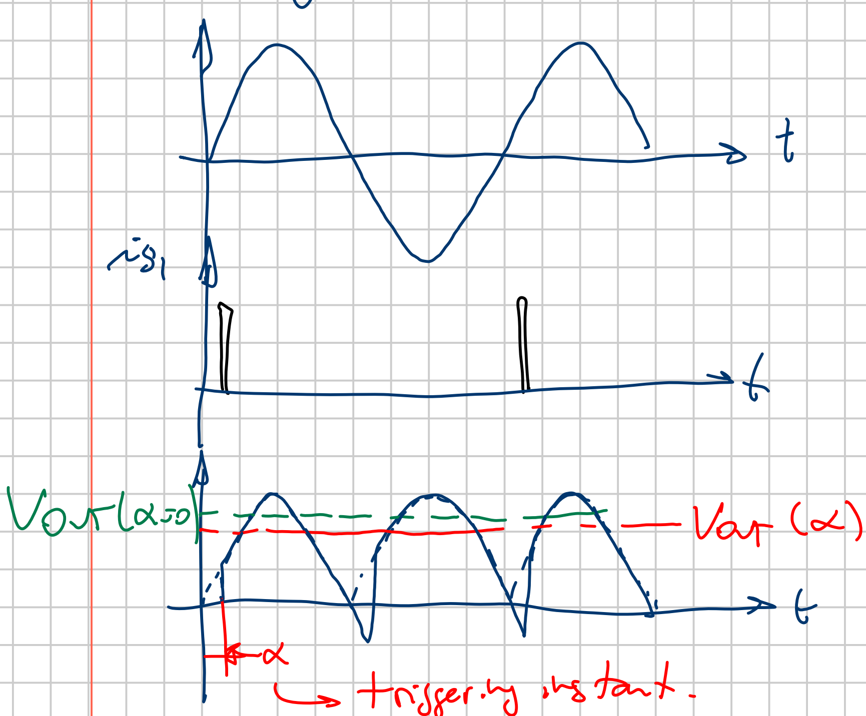


One issue is that I cannot regulate my output (i.e., if Vs changes, Vout changes). One solution is to use SCRs instead of diodes:



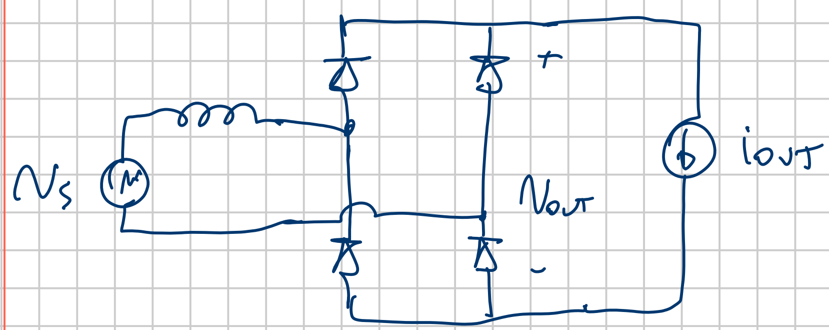


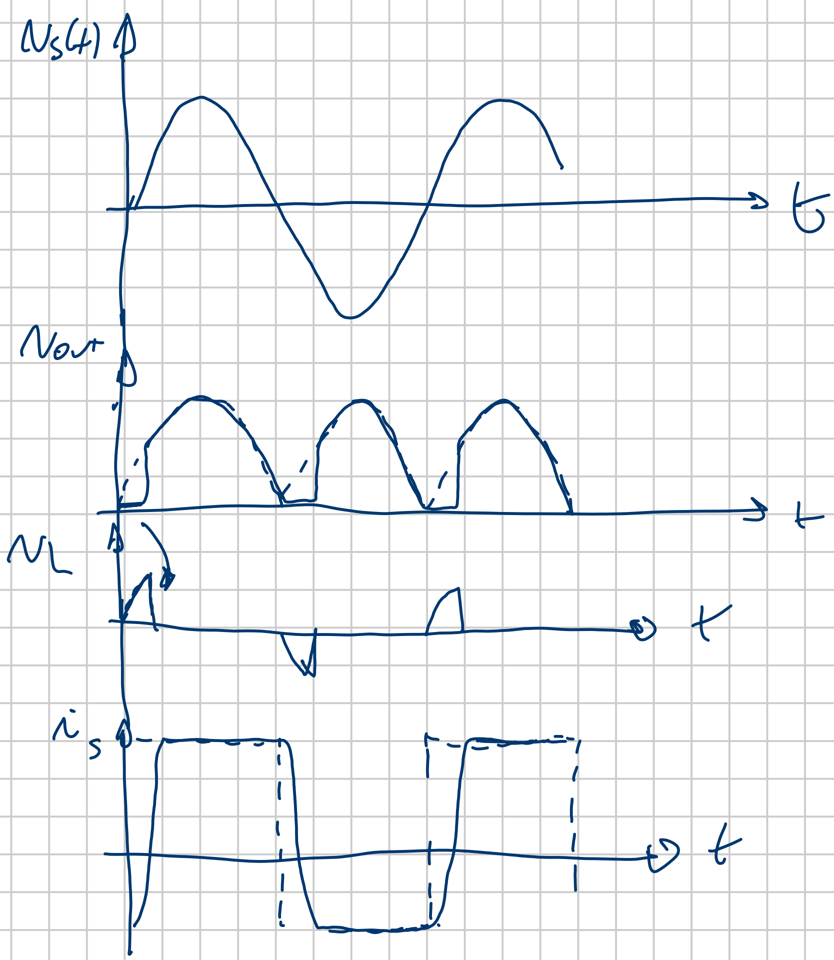
An SCR conducts when it is forward biased <u>and</u> it has been triggered by injecting an adequate current pulse at the gate.



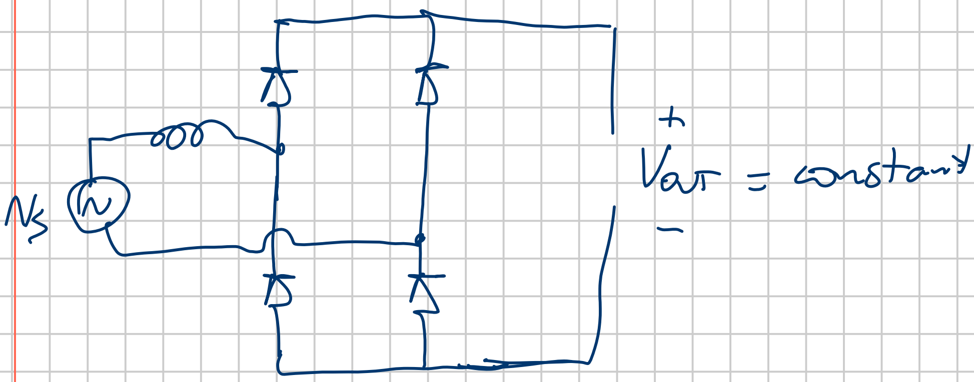
→ So I can now change Vout by changing α

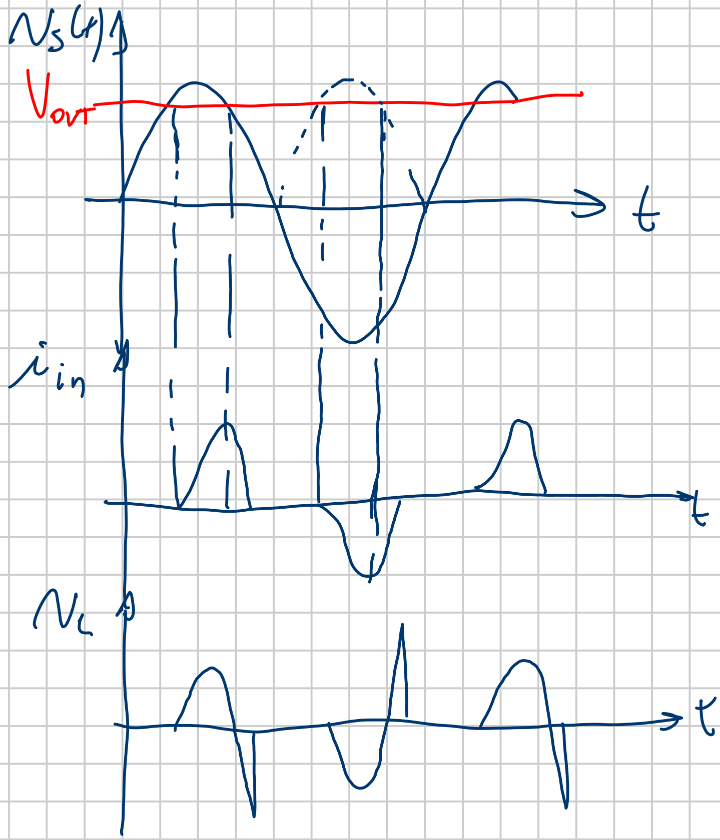
Another issue is that most DG sources that require a rectifier (e.g. microturbine) have a current interface
So I actually have





So how do the curves look with a current input and voltage output?
↳ This is the most likely scenario



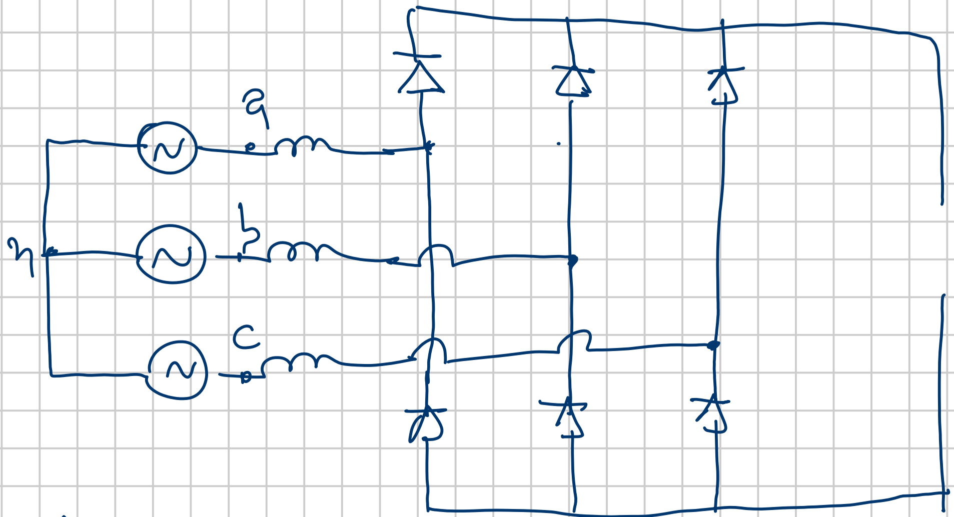


→ it yield a bad power factor

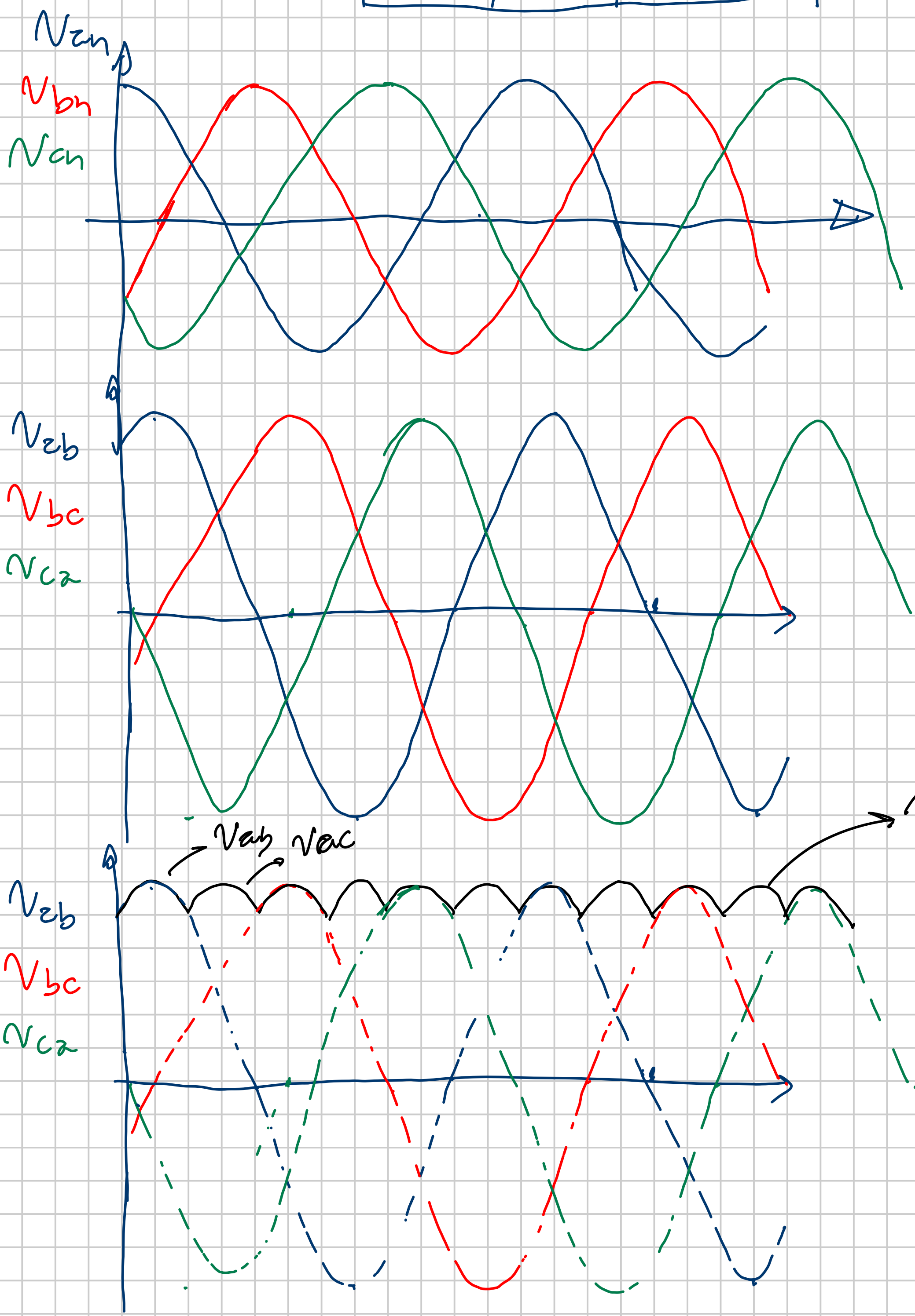
Needs to be compensated

e.g. dc link capacitor and a boost converter

3-phase rectifier

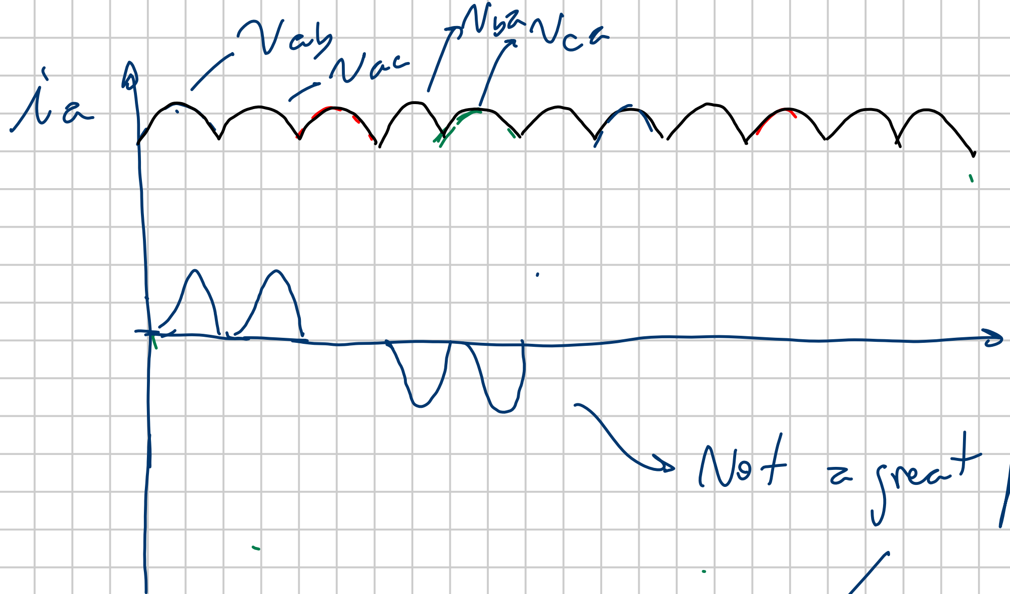


V_{out} = constant



Vout unfiltered

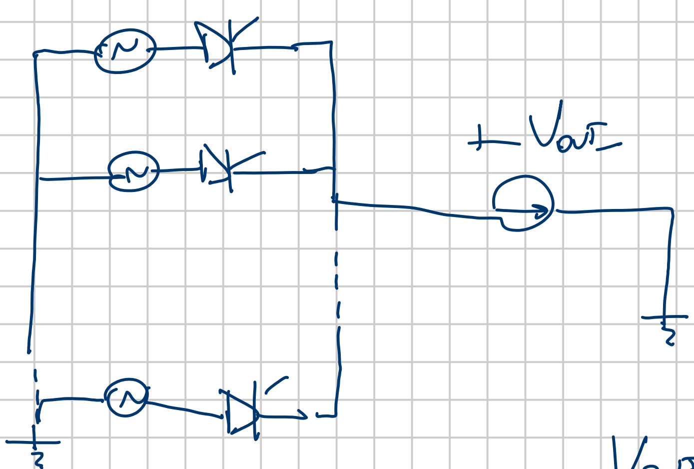
its frequency is 6 times the line frequency (easier to filter harmonics)



Not a great power factor either

it is also not as simple to compensate than in the single phase case

Generalized m-phase rectifiers

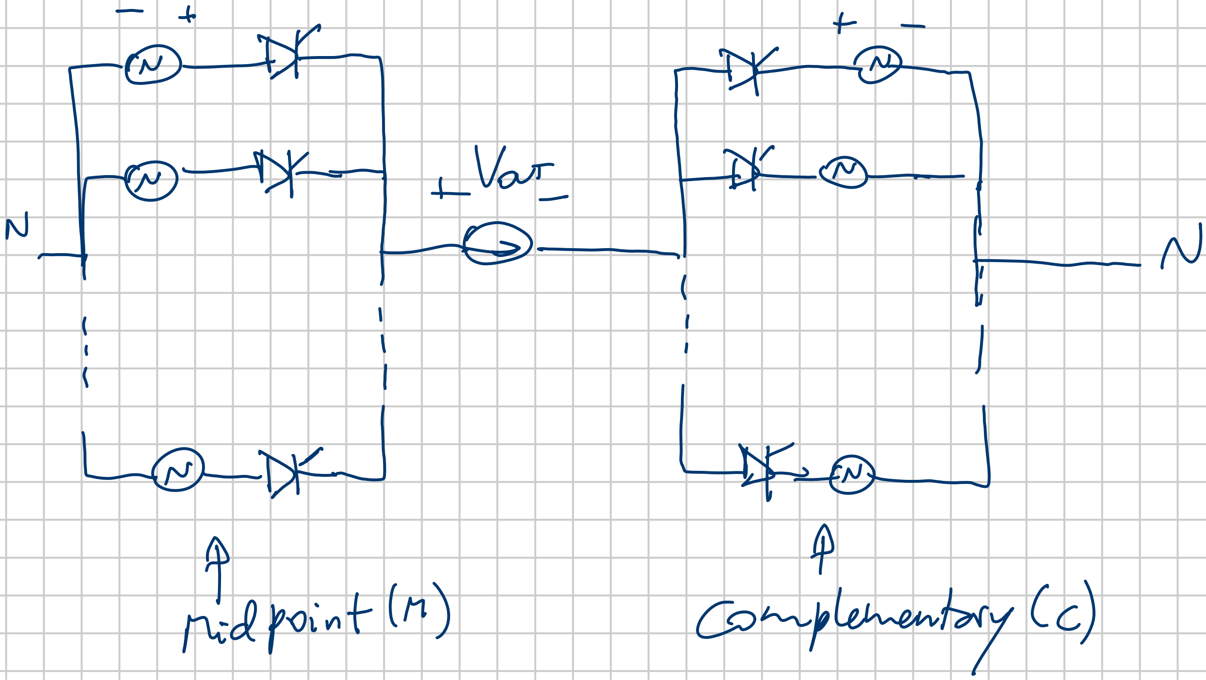


midpoint rectifier

$$V_{out} = \frac{m\sqrt{2}\,V_{s,L-N,RMS}}{\pi}\sin\left(\frac{\pi}{m}\right)\cos\alpha$$

for $m \geq 2$

Full bridge rectifier



$$V_{out} = \frac{\sqrt{2}}{\pi} V_{s\,L-L\,RMS}\; m \sin\left(\frac{\pi}{m}\right)\left[\cos\alpha_M - \cos\alpha_c\right]$$

for $m \geq 2$

Special case $\rightarrow$ With diodes $\alpha_M = 0$, $\alpha_c = 180°$ then

$$V_{out} = \frac{2\sqrt{2}}{\pi} V_{s\,L-L\,RMS}\; m \sin\left(\frac{\pi}{m}\right)$$