EE 411 Complex numbers

Note Title 8/26/2009

Three forms of representing a complex number

- Rectangular form
- polar form
- Exponential form

Rectangular form $z = x + jy$

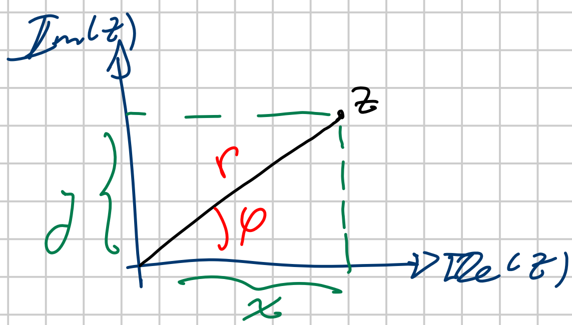
$$j = \sqrt{-1}$$

$$x = \mathrm{Re}(z)$$

$$y = \mathrm{Im}(z)$$

Polar form

$$z = |z| \angle \varphi = r \angle \varphi$$



$$r = \sqrt{x^2 + y^2}$$

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

$$z = x + jy = r\angle\varphi = r\cos\varphi + jr\sin\varphi$$

Exponential form

$$z = re^{j\varphi}$$

→ complex function

Operations

Complex conjugate $\rightarrow$ $z^* = x - jy = r\angle -\varphi = re^{-j\varphi}$

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$z_1 z_2 = r_1 r_2 \angle \varphi_1 + \varphi_2$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \varphi_1 - \varphi_2$$

$$z_1 z_1^* = r_1^2 \angle 0 = x_1^2 + y_1^2$$

$$\frac{z_1}{z_2} = \frac{z_1 z_2^*}{r_2^2} \qquad \rightarrow \qquad \frac{1}{j} = -j$$

Euler's formula $\rightarrow$ $e^{j\theta} = \cos\theta + j\sin\theta$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$