EE411 Sinusoidal steady state analysis

Note Title 7/15/2008

Analysis is only for steady state, <u>NOT</u> for transient.

Steps → 1) Transform the circuit to the frequency domain
2) Solve the problem using the techniques discussed with dc circuits (nodal analysis, etc)
3) Transform back to the time domain.

$$\begin{tikzcd}
\text{Time domain} \arrow[r, "\mathcal{F}"] \arrow[d] & \text{frequency domain} \arrow[r, "\mathcal{F}^{-1}"] \arrow[d] & \text{time domain} \arrow[d] \\
\text{differential equation} \arrow[r, "\mathcal{F}"] & \text{algebraic equation} \arrow[r, "\mathcal{F}^{-1}"] & \text{time response}
\end{tikzcd}$$

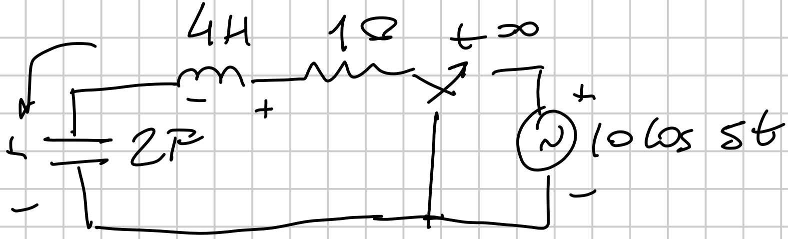
↑ analysis in steady state does not require initial conditions

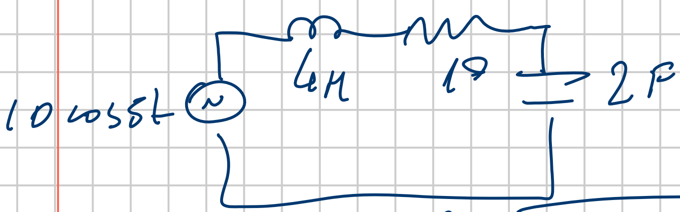
algebraic equation → phasors

algebraic equation → same techniques than with dc circuits → instead of resistances we have impedances

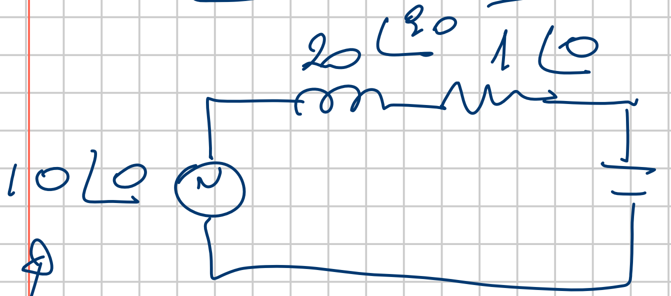
phasors → tricky part → phasors are vectors. they have magnitude and phase. → So avoid the common mistake of operating with them as if they were real numbers

Circuit used in 2nd order system analysis →





But frequency information still persist here



$\frac{1}{10}\angle -90 = 0.1\angle -90$

frequency information is lost here

$\mathbb{I}\,\mathbb{Z} = \mathbb{V}_s = 10\angle 0$

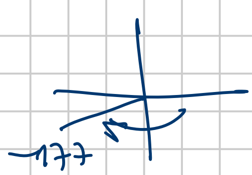
$\mathbb{Z} = 20\angle 90 + 1\angle 0 + 0.1\angle -90 =$

$= 19.92\angle 87.12$

$\mathbb{I} = \frac{10\angle 0}{19.92\angle 87.12} = 0.5\angle -87.12$

$\mathbb{V}_c = \mathbb{I}\,\mathbb{Z}_c = 0.1\angle -90 \;\; 0.5\angle -87.12 =$

$= 0.05\angle -177.12$



Steady state →

$v_c(t)_{ss} = 0.05\cos(5t - 177.12) = A\cos(5t) + B\sin(5t)$

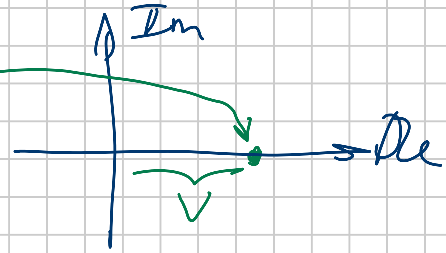
$A = 0.05\cos(177.12) \cong -0.05$

$B = 0.05\sin(177.12) \cong 0.0025$

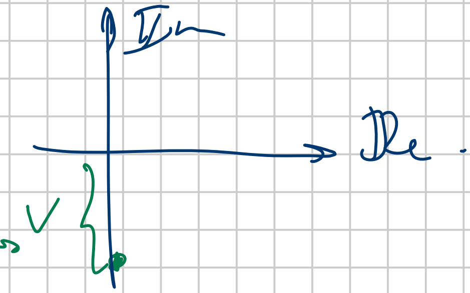
$v_c(t)_{ss} = -0.05\cos(5t) + 0.0025\sin(5t)$

Note $V\sin(\omega t) = V\cos(\omega t - 90) \longrightarrow \mathbf{V} = V\angle{-90}$

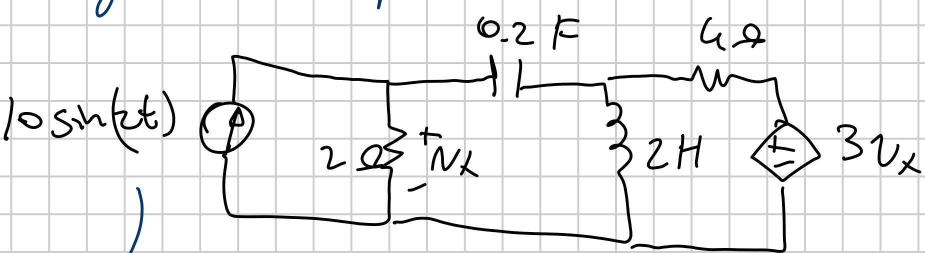
$v(t) = V\cos(\omega t)$



$v(t) = V\sin(\omega t)$

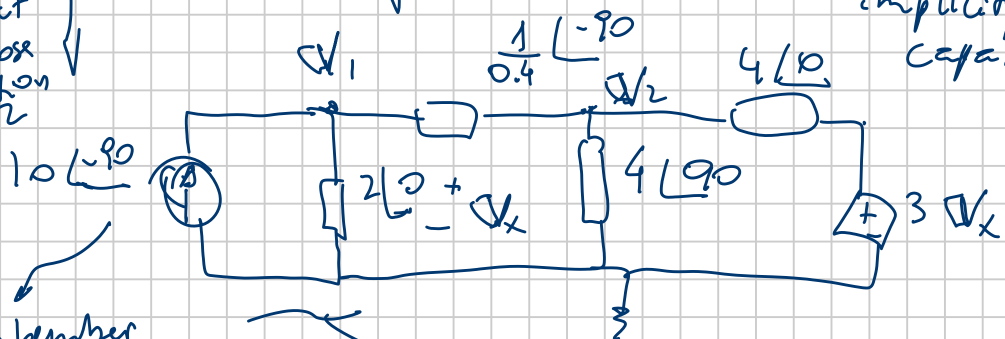


e.g. practice problem 10.1

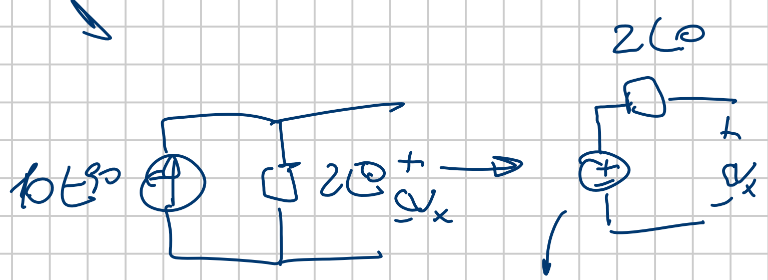


Information about $\omega = 2$ still survives implicitly within the capacitors and inductors impedance

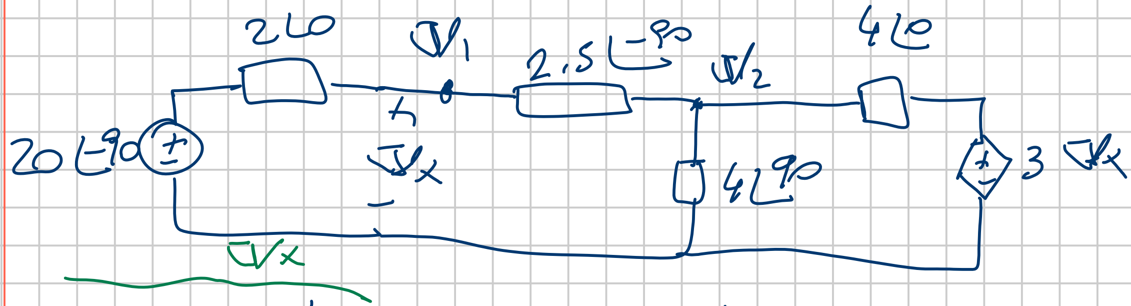
Notice that we lose information about $\omega = 2$



remember that phase of phasors is with respect to cos function



$2\angle 0 \times 10\angle{-90} = 20\angle{-90}$



#1

$$20\angle{-90} - 2\angle 0\, I_{m_1} - 2.5\angle{-90}\, I_{m_1} - 4\angle 90\,(I_{m_1} - I_{m_2}) = 0$$

$$2.5\angle 36.89\, I_{m_1} - 4\angle 90\, I_{m_2} = 20\angle{-90}$$

#2

$$-4\angle 90\,(I_{m_2} - I_{m_1}) - 4\angle 0\, I_{m_2} - 60\angle{-90} + 6\angle 0\, I_{m_1} = 0$$

$$-7.21\angle 33.7\, I_{m_1} + 5.65\angle 45\, I_{m_2} = -60\angle{-90}$$

$$\begin{cases} 2.5\angle 36.89\, I_{m_1} - 4\angle 90\, I_{m_2} = 20\angle{-90} \\ -7.21\angle 33.7\, I_{m_1} + 5.65\angle 45\, I_{m_2} = -60\angle{-90} \end{cases}$$

$$I_{m_1} = \frac{\begin{vmatrix} 20\angle{-90} & -4\angle 90 \\ -60\angle{-90} & 5.65\angle 45 \end{vmatrix}}{\begin{vmatrix} 2.5\angle 36.89 & -4\angle 90 \\ -7.21\angle 33.7 & 5.65\angle 45 \end{vmatrix}} = \frac{178.93\angle{-153.47}}{20.59\angle{-29.08}}$$

$$I_{m_1} = 8.69\angle{-124.4}$$

$$V_1 = V_x = \underbrace{20\angle{-90}}_{V_s} - \underbrace{2\angle 0 \quad 8.69\angle{-124.4}}_{R\, I_{m1}} = 11.33\angle{-29.95}$$

$$v_1(t) = 11.33\cos(2t - 29.95)$$

$$V_2 = V_1 - I_{m1}\, 2.5\angle{-90}$$

$$V_2 = 11.33\angle{-29.95} - 8.69\angle{124.4}\; 2.5\angle{-90} =$$

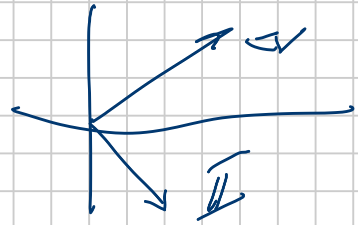
$$= 11.33\angle{-29.95} - 21.725\angle{-214.4} =$$

$$= 33.03\angle{-32.877}$$

$$v_2(t) = 33.03 \cos(2t - 32.877)$$

Leading and lagging:

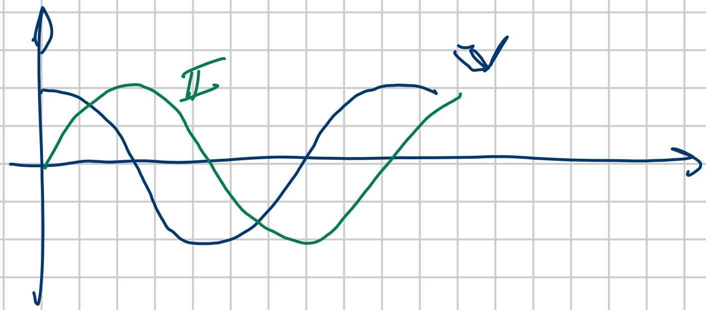
$\rightarrow \quad I = \frac{V}{\omega L \angle 90} = V(\omega L) \angle -90$



I lags V

↓

the voltage is leading.



(with a capacitor the voltage is lagging

With phasors, steady state analysis with sinusoids is like dc analysis

↳ Instead of R we use Z

So we can apply the same techniques: Norton equivalent
Thevenin "
Superposition
Source transformation
Nodal analysis
Mesh "

Operational amplifiers:

$V_1 = 0\angle 0$

$$I = \frac{V_i}{Z_1} = -\frac{V_o}{Z_2}$$

$$\longrightarrow \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

If $Z_2 = R_2$, $Z_1 = R_1$ $\quad\Big|\quad$ $\frac{V_o}{V_i} = -\frac{R_2}{R_1}$ $\longrightarrow$ inverting amplifier

If $Z_2 = \frac{1}{\omega C}\angle -90$, $Z_1 = R$ $\quad\Big|\quad$ $\frac{V_o}{V_i} = \frac{1}{\omega C R}\angle -90$

If $V_i = V_i \angle \varphi_i$

$\downarrow$

$$V_o = \frac{1}{\omega RC}\angle \varphi_i - 90$$

$\longrightarrow$ Integrator.

$$\int v(t)\,dt \longleftrightarrow \frac{V}{j\omega}$$

If $Z_2 = R$
$Z_1 = \frac{1}{\omega C} \angle -90$

$\frac{V_o}{V_i} = \omega C R \angle 90$

↓

Differentiator