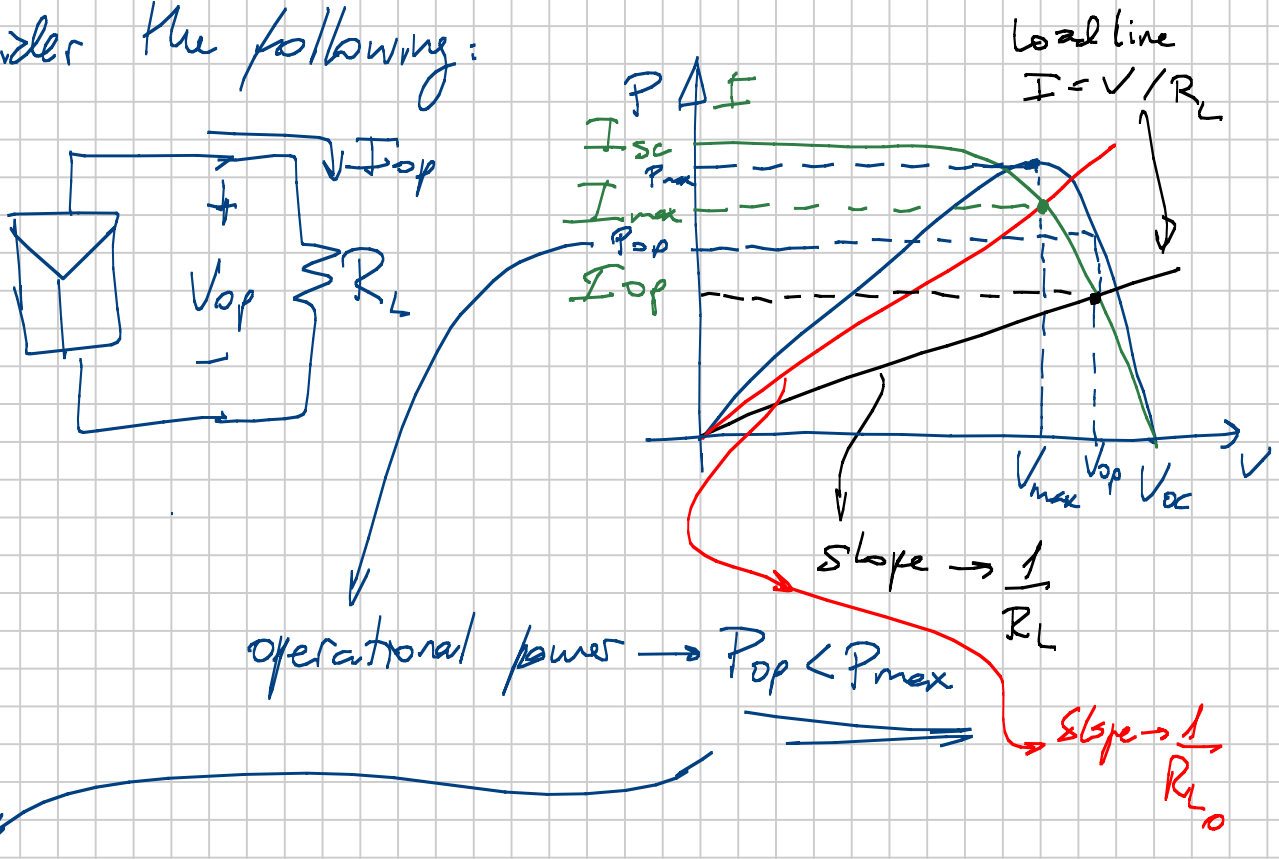


# MPPPT

The maximum power point (MPPPT) problem in PV applications

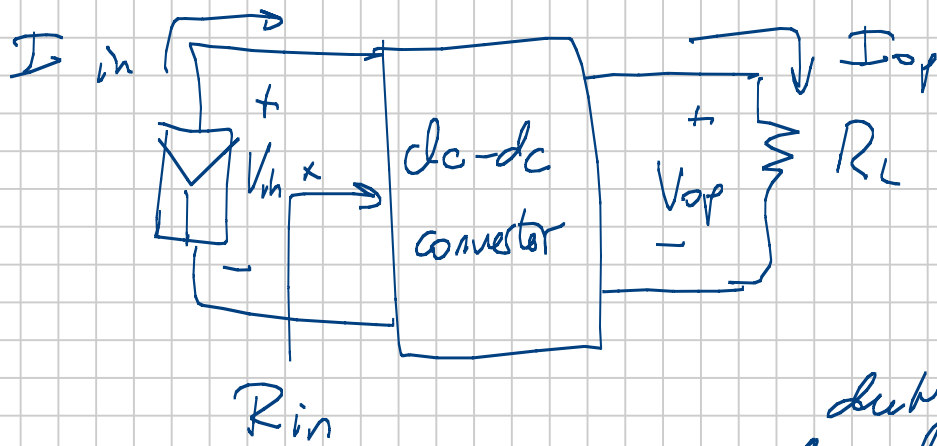
Consider the following:



But I want to operate the PV panel at its maximum.

So I use a dc-dc converter to "trick" the PV panel into believing it has a load resistance  $R_{L0}$  connected to its terminal instead of the actual load resistance  $R_L$





Since  $R_{in} = f(R_L \text{ and } d)$  the idea is to find the duty cycle that makes  $R_{in} = R_{Lo}$

So consider a buck converter

$$\begin{aligned} V_{op} &= V_{in} D \\ I_{op} &= \frac{I_{in}}{D} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} R_L = \frac{V_{op}}{I_{op}}$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{op}}{D} \frac{1}{I_{op} D} = \frac{R_L}{D^2}$$

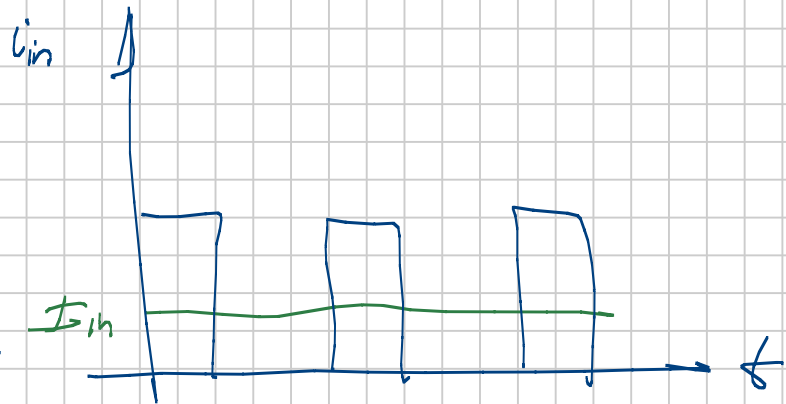
$$\text{For } R_{in} = R_{Lo} = \frac{V_{max}}{I_{max}} \rightarrow D_o = \sqrt{\frac{R_L}{R_{Lo}}}$$

Since  $0 < D < 1$  and  $R_L = R_{Lo} D_o^2$

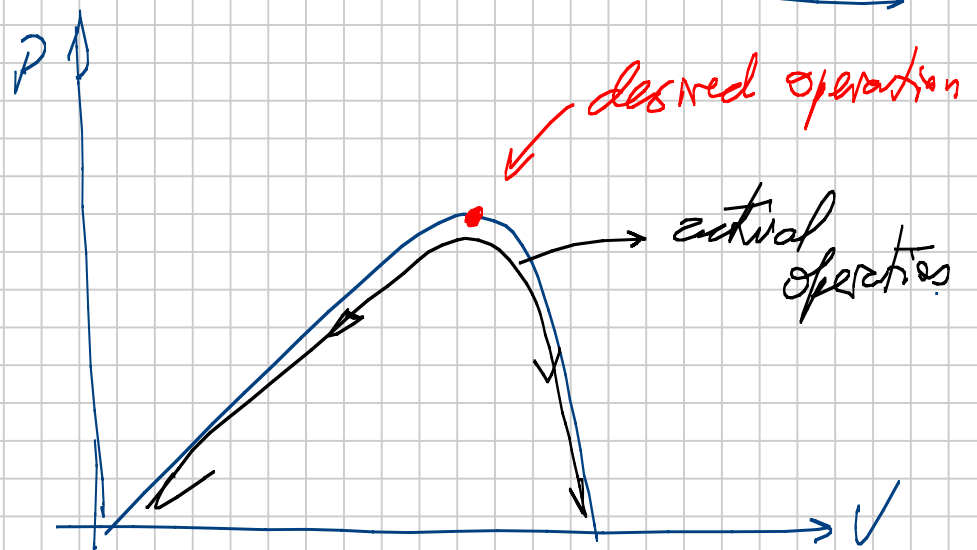
$$R_L < R_{Lo}$$

So a buck converter can only achieve the MPP for  $R_{Lo} > R_L$

Another problem of buck converters



It can only achieve the MP in average



Consider now a boost converter.

$$V_{out} = \frac{V_{in}}{1-D}$$

$$I_{out} = I_{in}(1-D)$$

$$R_L = \frac{V_{out}}{I_{out}}$$

$$R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{out}(1-D)}{I_{out}(1-D)} = R_L(1-D)^2$$

$$\text{For } R_{in} = R_{Lo} \longrightarrow R_{Lo} = R_L (1-D)^2$$

$$\text{Since } 0 < D_0 < 1$$

$$0 < (1-D) < 1$$

So the boost converter can only achieve the MPP for  $R_L > R_{Lo}$

At least the boost converter input current is not switched so the the MPP is achieved almost exactly

— Consider the SEPIC, Cuk or Buck-boost converters

$$V_{out} = \frac{D V_{in}}{1-D}$$

$$I_{out} = \frac{(1-D) I_m}{D}$$

$$R_L = \frac{V_{out}}{I_{out}}$$

$$R_m = \frac{V_m}{I_m} = \frac{(1-D)^2}{D^2} \frac{V_{out}}{I_{out}} = \frac{(1-D)^2}{D^2} R_L$$

For  $R_{in} = R_{Lo} \rightarrow D = D_0$

$$R_{Lo} = \frac{(1-D_0)^2}{D_0^2} R_L$$

$\Sigma$ , ideally, the SEPIC, Cuk, or buck boost converters can achieve the MPP for all  $R_L$  between 0 and  $\infty$ . (Can  $R_L$  be, actually, 0 or  $\infty$ ? Do we have other constraints?)

Although the three of them seem equivalent, they are not  $\rightarrow$  the buck-boost have switched input current so it can only achieve the MPP in average.  
 $\rightarrow$  The output of a Cuk converter is inverted.

Additional discussion or limitations when implementing MPPT can be gained from:

## Maximum Power Point Tracking Feasibility in Photovoltaic Energy-Conversion Systems

Sairaj V. Dhople, Ali Davoudi, Gerald Nilles, and Patrick L. Chapman  
Grainger Center for Electric Machinery and Electromechanics  
Department of Electrical and Computer Engineering  
University of Illinois at Urbana-Champaign  
Urbana, Illinois 61801, USA  
sdhople2@illinois.edu

So the question now is how do we control the dc-dc converter to achieve the MPP?

There are several methods for this. First, let's explore the problem from a classical mathematical approach. That is, if I am looking for the maximum power point then it is a looking for the point where  $\frac{dP}{dV} = 0$

For the next discussion I am considering this paper:

## Analysis of Classical Root Finding Methods Applied to Digital Maximum Power Point Tracking for Sustainable Photovoltaic Energy Generation

Seunghyun Chun, *Student member, IEEE*, and Alexis Kwasinski, *Member, IEEE*

$$\frac{dP}{dV}$$

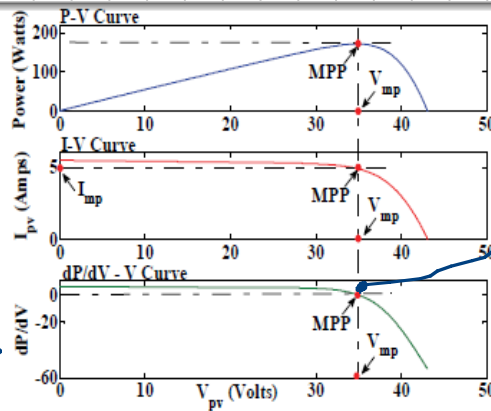


Figure 6: Maximum Power Point for different curves of a PV module.  
Figure 2: Irradiance effect on P-V Characteristic at Constant Temperature (25°C).

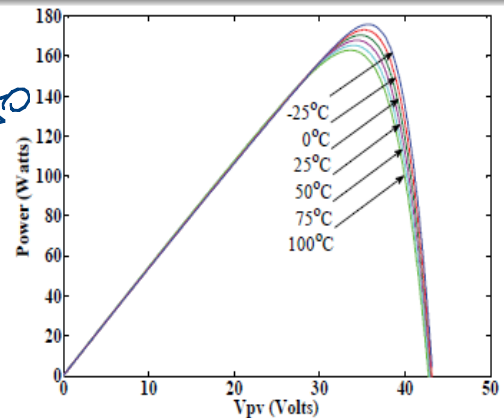


Figure 4: Temperature Effect on P-V Characteristic at constant irradiance (1000W/m²).

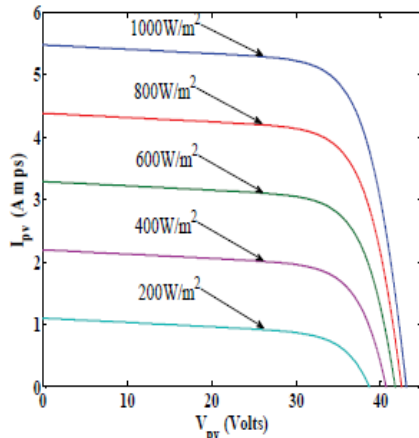


Figure 3: Irradiance effect on I-V Characteristic at Constant Temperature (25°C).

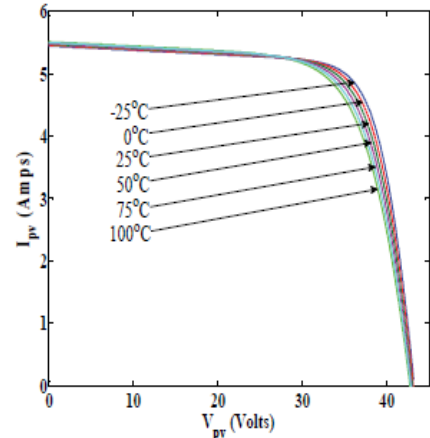


Figure 5: Temperature Effect on I-V Characteristic at constant irradiance (1000W/m²).

If we need to find the point where  $\frac{dP}{dV} = 0$ , then

the MPP controller just need to find a root:

Some methods to find a root:

2) Newton-Raphson

Let  $f(V) = \frac{dP}{dV}$  and  $V^*$  be the voltage at the MPP

then we perform iterations of-

$$V_{n+1} = V_n - \frac{f(V_n)}{f'(V_n)}$$

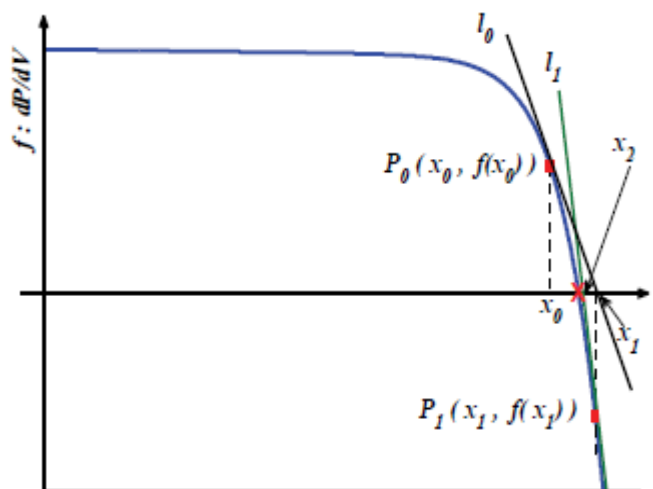
until

$$|f(V_i)| < \epsilon$$

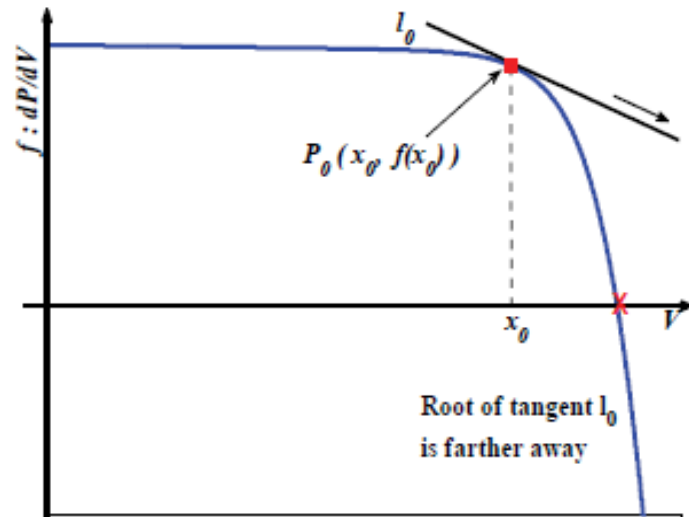
↳ tolerance

then  $V_i$  is considered to be  $V^*$

In a graphic way:



Problem  $\rightarrow$  convergence:



b) Secant Method

The algorithm is now:

$$V_{n+1} = V_n - f(V_n) \left( \frac{V_n - V_{n-1}}{f(V_n) - f(V_{n-1})} \right)$$

So I consider 2 steps now  
So convergence is faster

But it is not ensured

c) Bisection method





(i) Given a well-defined function  $f(x)$ , choose a lower value  $x_l$  and an upper value  $x_u$ . These two points define an interval  $[x_l, x_u]$  that must include the root  $x^*$  of  $f(x)$ . That is,  $f(x)$  has opposing signs in  $x_l$  and  $x_u$ , e.g.  $f(x_l)f(x_u) < 0$ .

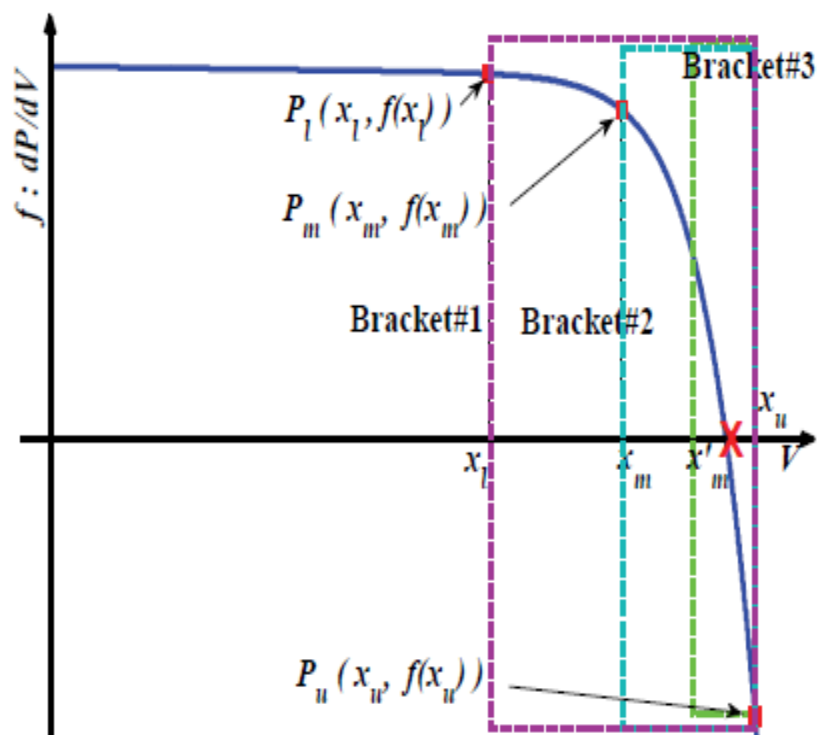
(ii) Approximate the root to the midpoint  $x_m$  of the interval  $[x_l, x_u]$ . That is

$$x_m = \frac{x_u + x_l}{2} \quad (6)$$

(iii) If  $f(x_l)f(x_m) < 0$  then set  $x_u = x_m$  and repeat the previous step. If  $f(x_l)f(x_m) > 0$  then set  $x_l = x_m$  and repeat the previous step. If  $|f(x_m)| \leq \epsilon$  (where  $\epsilon$  is the tolerance) then take  $x_m$  as the root or approximation.

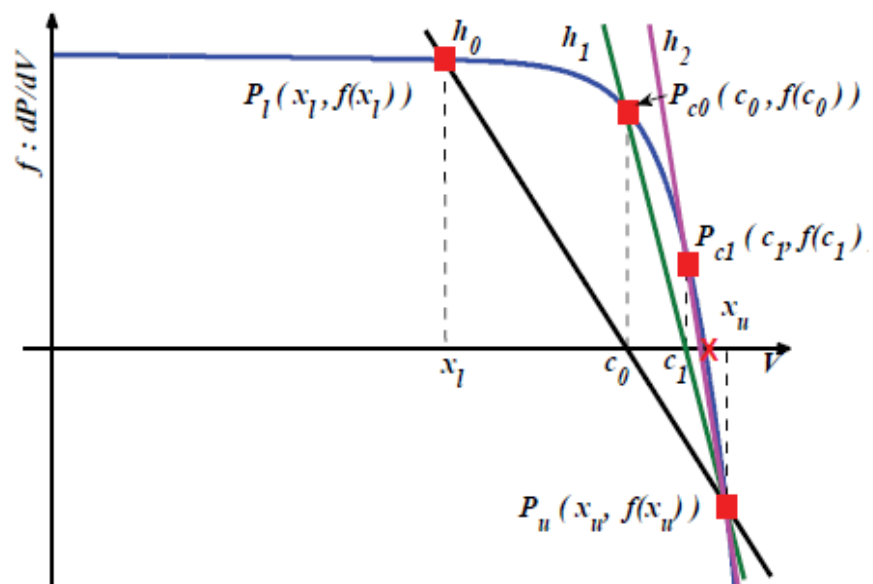
The BSM convergence rate is slower than the SM. Yet, with the BSM root convergence is guaranteed.

*SM = Secant method*



## d) Regula Falsi:

- (i) Given a continuous function  $f(x)$  find initial points  $x_l$  and  $x_u$ , such that  $x_l \neq x_u$  and  $f(x_l)f(x_u) < 0$ . Hence, according to the intermediate value theorem the root of  $f(x)$  is located inside the interval  $[x_l, x_u]$ .
- (ii) Calculate the approximate value for the root  $c_i$  with (7)
- (iii) If  $|f(c_i)| \leq \varepsilon$  (where  $\varepsilon$  is the tolerance) then it is considered that the root has been reached and that  $c_i$  is the root. Else, if  $f(c_i) \cdot f(x_u) < 0$  then let  $x_l = c_i$ , else if  $f(c_i) \cdot f(x_l) < 0$  then let  $x_u = c_i$ . These changes yield a smaller interval.
- (iv) Iterate steps (ii) and (iii) until the root is reached.



## e) Modified Regula Falsi:

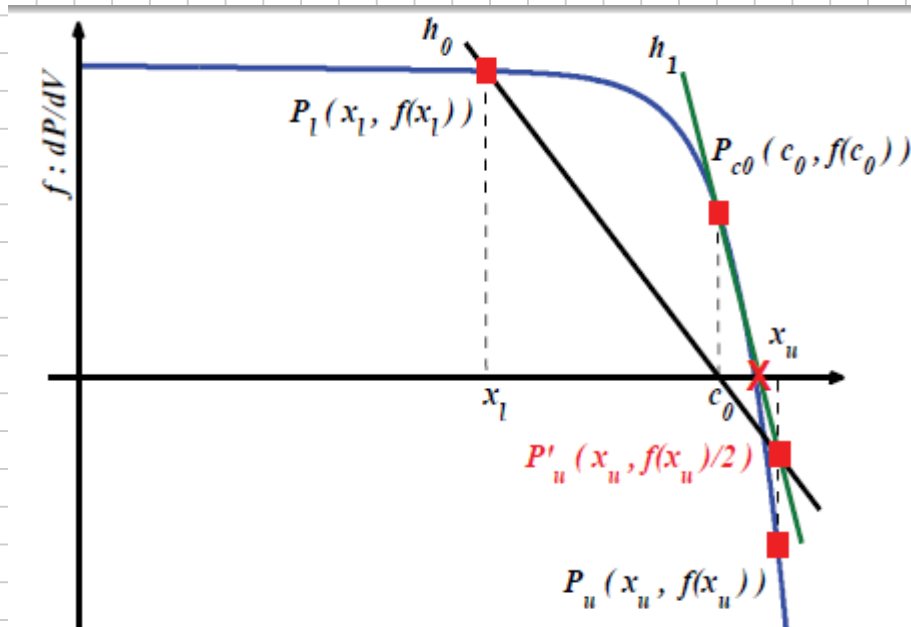
We replace in method (d) step (ii) by:

- (ii) If  $f(x_l)f(x_u) < 0$  and  $f(x_l) > 0$  then  $f(x_u)$  is replaced in (7) by  $f_p(x_u) = f(x_u)/2$  and  $f_p(x_l) = f(x_l)$

$$c_i = \frac{x_l \cdot f_p(x_u) - x_u \cdot f_p(x_l)}{f_p(x_u) - f_p(x_l)} = \frac{x_l \cdot f(x_u) \cdot 0.5 - x_u \cdot f(x_l)}{0.5 \cdot f(x_u) - f(x_l)}, \quad (8)$$

If  $f(x_l) \cdot f(x_u) < 0$  and  $f(x_l) < 0$  then  $f(x_l)$  is replaced in (7) by  $f_p(x_l) = f(x_l)/2$  and  $f_p(x_u) = f(x_u)$

$$c_i = \frac{x_l \cdot f_p(x_u) - x_u \cdot f_p(x_l)}{f_p(x_u) - f_p(x_l)} = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l) \cdot 0.5}{f(x_u) - 0.5 \cdot f(x_l)}, \quad (9)$$



How do we implement all these methods  $\rightarrow$  digitally.

Other methods.

Several methods are summarized here.)

## Comparison of Photovoltaic Array Maximum Power Point Tracking Techniques

Trishan Eeram, *Student Member, IEEE*, and Patrick L. Chapman, *Senior Member, IEEE*

TABLE III  
MAJOR CHARACTERISTICS OF MPPT TECHNIQUES

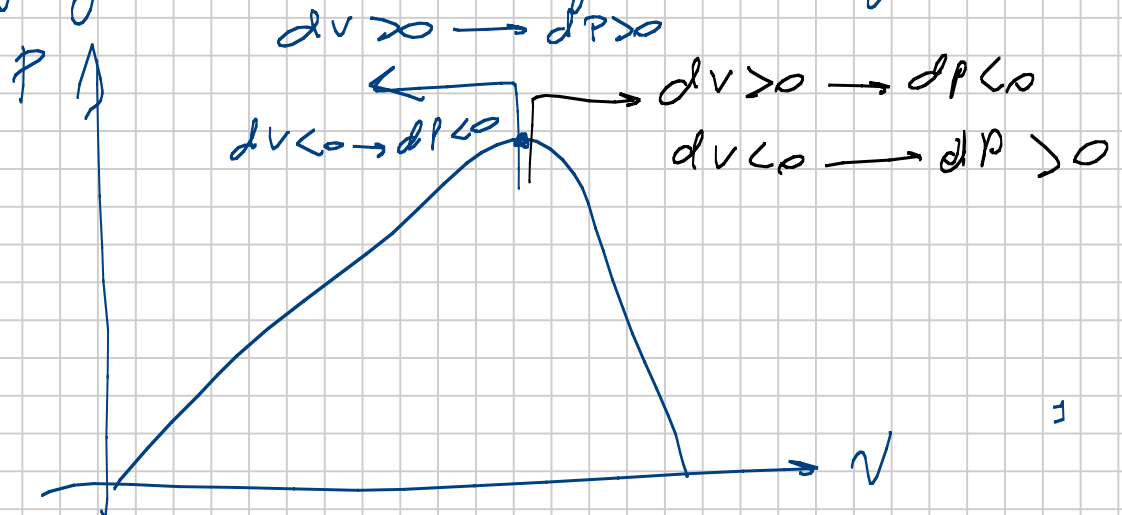
MPPT Technique	PV Array Dependent?	True MPPT?	Analog or Digital?	Periodic Tuning?	Convergence Speed	Implementation Complexity	Sensed Parameters
Hill-climbing/P&O	No	Yes	Both	No	Varies	Low	Voltage, Current
IncCond	No	Yes	Digital	No	Varies	Medium	Voltage, Current
Fractional $V_{OC}$	Yes	No	Both	Yes	Medium	Low	Voltage
Fractional $I_{SC}$	Yes	No	Both	Yes	Medium	Medium	Current
Fuzzy Logic Control	Yes	Yes	Digital	Yes	Fast	High	Varies
Neural Network	Yes	Yes	Digital	Yes	Fast	High	Varies
RCC	No	Yes	Analog	No	Fast	Low	Voltage, Current
Current Sweep	Yes	Yes	Digital	Yes	Slow	High	Voltage, Current
DC Link Capacitor Droop Control	No	No	Both	No	Medium	Low	Voltage
Load $I$ or $V$ Maximization	No	No	Analog	No	Fast	Low	Voltage, Current
$dP/dV$ or $dP/dI$ Feedback Control	No	Yes	Digital	No	Fast	Medium	Voltage, Current
Array Reconfiguration	Yes	No	Digital	Yes	Slow	High	Voltage, Current
Linear Current Control	Yes	No	Digital	Yes	Fast	Medium	Irradiance
$I_{MPP}$ & $V_{MPP}$ Computation	Yes	Yes	Digital	Yes	N/A	Medium	Irradiance, Temperature
State-based MPPT	Yes	Yes	Both	Yes	Fast	High	Voltage, Current
OCC MPPT	Yes	No	Both	Yes	Fast	Medium	Current
BFV	Yes	No	Both	Yes	N/A	Low	None
LRCM	Yes	No	Digital	No	N/A	High	Voltage, Current
Slide Control	No	Yes	Digital	No	Fast	Medium	Voltage, Current

Hill climbing and Perturb and observe

↓  
perturbation is added to the dc-dc

perturbs the voltage

converted duty cycle → which changes the voltage



Process:

Whenever  $dP > 0 \rightarrow$  perturbation is kept the same  
 $dP < 0 \rightarrow$  " " is reversed

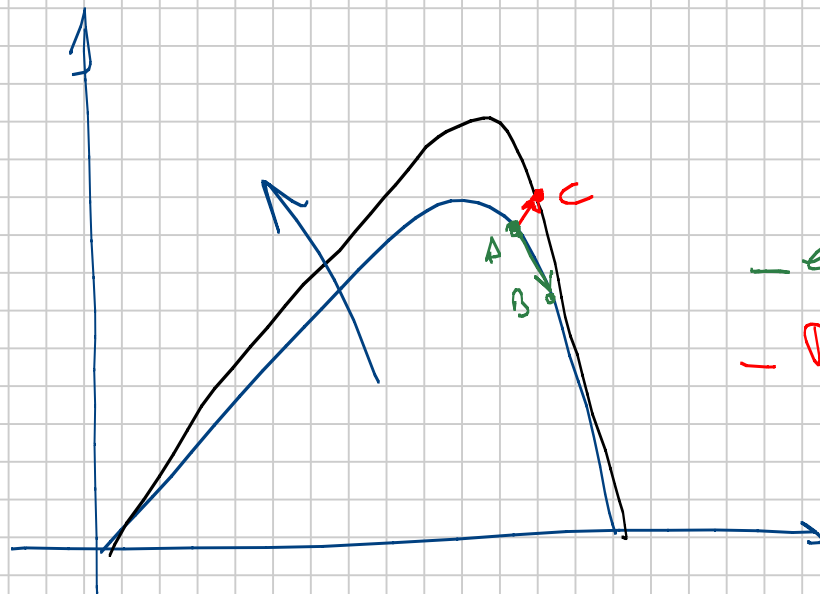
Process is repeated until reaching MPP

Problem: oscillates around MPP

Solution  $\rightarrow$  choose a smaller step size  $L$  smaller perturbation)

Problem  $\rightarrow$  smaller step size means slower convergence.

Another problem  $\rightarrow$  can fail in rapidly changing weather conditions



- expected behavior

- Actual behavior

- expected behavior  $\rightarrow dV > 0 \rightarrow dP < 0$  so perturbation is reversed

- Actual necessary behavior  $\rightarrow dV > 0 \rightarrow dP > 0$  so

perturbation is kept the same



that is  $dV > 0$



So it moves away from MPP

Incremental conductance

Notice that:

$$\left. \begin{array}{l} dP/dV = 0 \quad \text{at MPP} \\ dP/dV < 0 \quad \text{right of MPP} \\ dP/dV > 0 \quad \text{left of MPP} \end{array} \right\}$$

$$\text{Now: } \frac{dP}{dV} = \frac{d(I \cdot V)}{dV} = I + V \frac{dI}{dV} \approx I + V \frac{\Delta I}{\Delta V}$$

$$\text{So } \left\{ \begin{array}{l} \Delta I / \Delta V = -I/V \quad \text{at MPP} \\ \Delta I / \Delta V < -I/V \quad \text{right of MPP} \\ \Delta I / \Delta V > -I/V \quad \text{left of MPP} \end{array} \right.$$

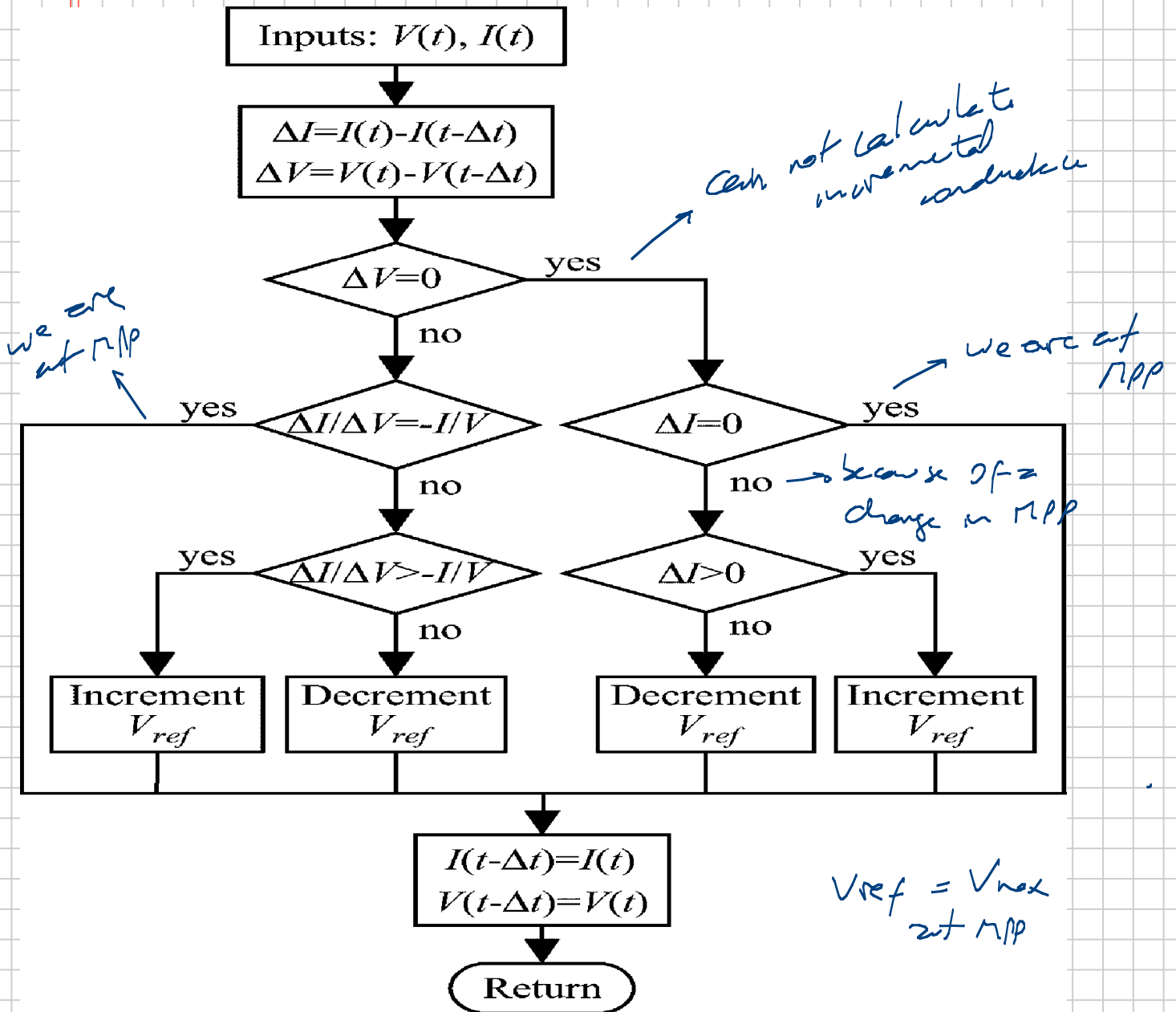
Approach #1:

$$\text{Consider } e = \frac{I}{V} + \frac{dI}{dV}$$



at MPP  $e = 0 \rightarrow$  so we can use a PI controller to make  $e = 0$

# Approach #2

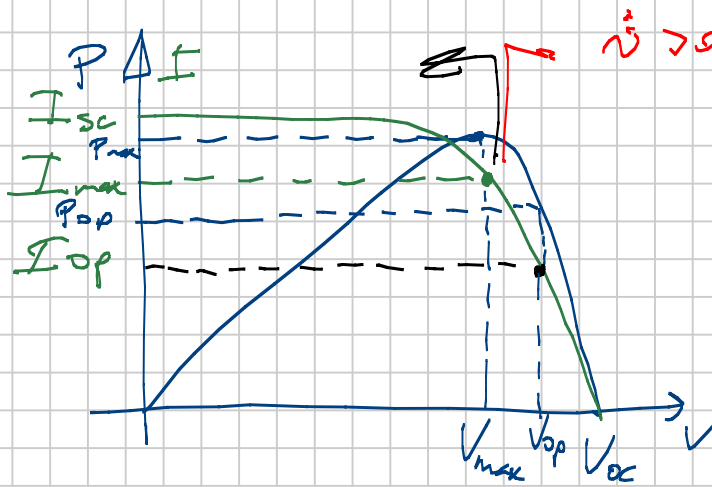


- Ripple correlation control

in to the dc-dc converter always has some ripple

this ripple is used to drive the converter to the PV module MPP

$\dot{V} > 0$  or  $\dot{i} > 0 \rightarrow \dot{P} > 0 \rightarrow \dot{P} > 0$  or  $\dot{P} > 0$

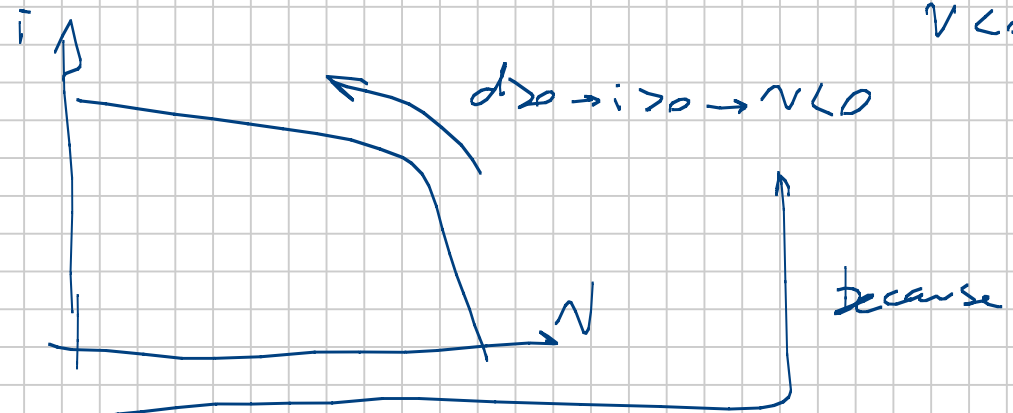


$\dot{V} > 0$  or  $\dot{i} > 0 \rightarrow \dot{P} < 0$

$\dot{P} < 0$  or  $\dot{P} < 0$

Consider a boost converter  $\rightarrow$  when  $d > 0 \rightarrow i > 0$

$\downarrow$   
 $V < 0$



$\int d(t) = -k \int \dot{P} \dot{V} dt$

$\hookrightarrow$  like error signal  $\rightarrow \dot{P} \dot{V} = 0$

at MPP

because

$\dot{P} = 0$  at MPP

or  $d(t) = k \int \dot{P} i dt$