The maximum power point (MPP) problem in PV applications

Consider the following:

But I want to operate the PV panel at its maximum.

So I use a dc-dc converter to ‘track’ the PV panel into believing it has a load resistance $R_{\text{L}_{\text{ref}}}$ connected to its terminal instead of the actual load resistance $R_{\text{L}}$. 
Since $R_{in} = f(R_L \text{ and } d)$, the idea is to find the duty cycle that makes $R_{in} = R_{lo}$.

So consider a buck converter

\[ V_{op} = V_{in}D \]
\[ I_{op} = \frac{I_{in}}{D} \]
\[ R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{op}}{D} \cdot I_{op} = \frac{R_L}{D^2} \]
\[ \text{For } R_{in} = R_{lo} = \frac{V_{max}}{I_{max}} \rightarrow D = \sqrt{\frac{R_L}{R_{lo}}} \]

Since $0 < D < 1$ and $R_L = R_{lo}D^2$

\[ R_L < R_{lo} \]

So a buck converter can only achieve the MPP for $R_{lo} > R_L$. 
Another problem of buck converters

It can only achieve the MP in average

Consider now a boost converter:

\[ V_{\text{out}} = \frac{V_{\text{in}}}{1-D} \]

\[ I_{\text{out}} = I_{\text{in}} (1-D) \]

\[ R_L = \frac{V_{\text{out}}}{I_{\text{out}}} \]

\[ R_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}} = \frac{V_{\text{out}} (1-D)}{I_{\text{out}} (1-D)} = R_L (1-D)^2 \]
For  \( R_{in} = R_{L0} \) \( \rightarrow R_{L0} = R_L (1-D)^2 \)

\[
\begin{align*}
\text{Since} \quad 0 < D_0 < 1 \\
\Rightarrow \quad 0 < (1-D) < 1
\end{align*}
\]

So the boost converter can only achieve the MPP for \( R_L > R_{L0} \)

At least the boost converter input current is not switched so the the MPP is achieved almost exactly.

Consider the 50\% duty cycle or such boost converters

\[
\begin{align*}
V_{out} &= \frac{D \cdot V_{in}}{1-D} \\
I_{out} &= \frac{(1-D) \cdot I_{in}}{D} \\
R_{in} &= \frac{V_{in}}{I_{in}} = \frac{(1-D)^2}{D^2} \quad V_{out} \quad \frac{V_{out}}{I_{out}} = \frac{(1-D)^2}{D^2} \quad R_L
\end{align*}
\]
For \( R_{in} = R_{so} \rightarrow D = D_s \)

\[
R_{in} = \frac{(1-D_s)^2}{D_s} R_L
\]

So ideally, the Sepic, Cuk, or buck-boost converters can achieve the MPP for all \( R_L \) between 0 and \( \infty \) (can \( R_L \) be actually, 0 or \( \infty \)? Do we have other constraints?)

Although the three of them seem equivalent, they are not — the buck-boost have switched input current so it can only achieve the MPP in average.

The output of a Cuk converter is inverted.

Additional discussion on limitations when implementation MPP can be gained from.

---

**Maximum Power Point Tracking Feasibility in Photovoltaic Energy-Conversion Systems**

Sairaj V. Dhople, Ali Davoudi, Gerald Nilles, and Patrick L. Chapman

Grainger Center for Electric Machinery and Electromechanics
Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801, USA
sdhople2@illinois.edu
So the question now is how do we control the dc-dc converter to achieve the MPP?

There are several methods for this. First, let's explore the problem from a classical mathematical approach. That is, if I am looking for the maximum power point, then I am looking for the point where \( \frac{dP}{dV} = 0 \).

For the next discussion, I am considering my paper:

Analysis of Classical Root Finding Methods Applied to Digital Maximum Power Point Tracking for Sustainable Photovoltaic Energy Generation

Seunghyun Chun, Student member, IEEE, and Alexis Kwasinski, Member, IEEE

Figure 6: Maximum Power Point for different curves of a PV module. Figure 7: Irradiance effect on P-V Characteristic at Constant Temperature (25°C).

Figure 4: Temperature Effect on P-V Characteristic at constant irradiance (1000W/m²).

Figure 3: Irradiance effect on I-V Characteristic at Constant Temperature (25°C).

Figure 5: Temperature Effect on I-V Characteristic at constant irradiance (1000W/m²).
If we need to find the point where \( \frac{df}{dx} = 0 \), the MPPT controller just needs to find a root. Some methods to finding a root:

1. **Newton-Raphson**

Let \( f(v) = \frac{df}{dv} \) and \( v^* \) be the voltage at the MPPT.

Then, we perform iterations of

\[
v_{n+1} = v_n - \frac{f(v_n)}{f'(v_n)}
\]

until \( |f(v_i)| < \varepsilon \)

Then \( v_i \) is considered to be \( v^* \).

In a graphic way:

![Graph](image-url)
Problem → convergence

b) Secant method

The algorithm is now:

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \left( \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \right) \]

So I consider 2 steps now
So convergence is faster
But it is not ensured

c) Bisection method
(i) Given a well-defined function $f(x)$, choose a lower value $x_l$ and an upper value $x_u$. These two points define an interval $[x_l, x_u]$ that must include the root $x^*$ of $f(x)$. That is, $f(x)$ has opposing signs in $x_l$ and $x_u$, e.g. $f(x_l)f(x_u) < 0$.

(ii) Approximate the root to the midpoint $x_m$ of the interval $[x_l, x_u]$. That is

$$x_m = \frac{x_u + x_l}{2} \quad (6)$$

(iii) If $f(x_l)f(x_m) < 0$ then set $x_u = x_m$ and repeat the previous step. If $f(x_l)f(x_m) > 0$ then set $x_l = x_m$ and repeat the previous step. If $|f(x_m)| \leq \varepsilon$ (where $\varepsilon$ is the tolerance) then take $x_m$ as the root or approximation.

The BSM convergence rate is slower than the SM. Yet, with the BSM root convergence is guaranteed.

---

SM = Secant method
Given a continuous function \( f(x) \) find initial points \( x_i \) and \( x_u \) such that \( x_i \neq x_u \) and \( f(x_i)f(x_u) < 0 \). Hence, according to the intermediate value theorem the root of \( f(x) \) is located inside the interval \([x_i, x_u]\).

(ii) Calculate the approximate value for the root \( c_i \) with (7).

(iii) If \( |f(c_i)| \leq \varepsilon \) (where \( \varepsilon \) is the tolerance) then it is considered that the root have been reached and that \( c_i \) is the root. Else, if \( f(c_i) \cdot f(x_i) < 0 \) then let \( x_i = c_i \), else if \( f(c_i) \cdot f(x_u) < 0 \) then let \( x_u = c_i \). These changes yield a smaller interval.

(iv) Iterate steps (ii) and (iii) until the root is reached.

(8) \[
\frac{c_i}{f_p(x_i) - f_p(x_u)} = \frac{x_i \cdot f(x_u) \cdot 0.5 - x_u \cdot f(x_i)}{0.5 \cdot f(x_u) - f(x_i)}.
\]
If \( f(x_l) \cdot f(x_u) < 0 \) and \( f(x_u) < 0 \) then \( f(x_l) \) is replaced in (7) by \( f_p(x_l) = f(x_l)/2 \) and \( f_p(x_u) = f(x_u) \)

\[
c_i = \frac{x_l \cdot f_p(x_u) - x_u \cdot f_p(x_l)}{f_p(x_u) - f_p(x_l)} = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - 0.5 \cdot f(x_l)} \quad (9)
\]

How do we implement all these methods digitally

Other methods.

Several methods are summarized here.

Comparison of Photovoltaic Array Maximum Power Point Tracking Techniques

Trishan Esram, Student Member, IEEE, and Patrick L. Chapman, Senior Member, IEEE
### TABLE III
**Major Characteristics of MPPT Techniques**

<table>
<thead>
<tr>
<th>MPPT Technique</th>
<th>PV Array Dependent?</th>
<th>True MPPT?</th>
<th>Analog or Digital?</th>
<th>Periodic Tuning?</th>
<th>Convergence Speed</th>
<th>Implementation Complexity</th>
<th>Sensed Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hill-climbing/P&amp;O</td>
<td>No</td>
<td>Yes</td>
<td>Both</td>
<td>No</td>
<td>Varies</td>
<td>Low</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>IncCond</td>
<td>No</td>
<td>Yes</td>
<td>Digital</td>
<td>No</td>
<td>Varies</td>
<td>Medium</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>Fractional $V_{OC}$</td>
<td>Yes</td>
<td>No</td>
<td>Both</td>
<td>Yes</td>
<td>Medium</td>
<td>Low</td>
<td>Voltage</td>
</tr>
<tr>
<td>Fractional $I_{SC}$</td>
<td>Yes</td>
<td>No</td>
<td>Both</td>
<td>Yes</td>
<td>Medium</td>
<td>Medium</td>
<td>Current</td>
</tr>
<tr>
<td>Fuzzy Logic Control</td>
<td>Yes</td>
<td>Yes</td>
<td>Digital</td>
<td>Yes</td>
<td>Fast</td>
<td>High</td>
<td>Varies</td>
</tr>
<tr>
<td>Neural Network</td>
<td>Yes</td>
<td>Yes</td>
<td>Digital</td>
<td>Yes</td>
<td>Fast</td>
<td>High</td>
<td>Varies</td>
</tr>
<tr>
<td>RCC</td>
<td>No</td>
<td>Yes</td>
<td>Analog</td>
<td>No</td>
<td>Fast</td>
<td>Low</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>Current Sweep</td>
<td>Yes</td>
<td>Yes</td>
<td>Digital</td>
<td>Yes</td>
<td>Slow</td>
<td>High</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>DC Link Capacitor Droop Control</td>
<td>No</td>
<td>No</td>
<td>Both</td>
<td>No</td>
<td>Medium</td>
<td>Low</td>
<td>Voltage</td>
</tr>
<tr>
<td>Load $I$ or $V$ Maximization</td>
<td>No</td>
<td>No</td>
<td>Analog</td>
<td>No</td>
<td>Fast</td>
<td>Low</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>$dP/dV$ or $dP/dI$ Feedback Control</td>
<td>No</td>
<td>Yes</td>
<td>Digital</td>
<td>No</td>
<td>Fast</td>
<td>Medium</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>Array Reconfiguration</td>
<td>Yes</td>
<td>No</td>
<td>Digital</td>
<td>Yes</td>
<td>Slow</td>
<td>High</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>Linear Current Control</td>
<td>Yes</td>
<td>No</td>
<td>Digital</td>
<td>Yes</td>
<td>Fast</td>
<td>Medium</td>
<td>Irradiance</td>
</tr>
<tr>
<td>$I_{SC}$ &amp; $V_{OC}$ Computation</td>
<td>Yes</td>
<td>Yes</td>
<td>Digital</td>
<td>Yes</td>
<td>N/A</td>
<td>Medium</td>
<td>Irradiance, Temperature</td>
</tr>
<tr>
<td>State-based MPPT</td>
<td>Yes</td>
<td>Yes</td>
<td>Both</td>
<td>Yes</td>
<td>Fast</td>
<td>High</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>OCC MPPT</td>
<td>Yes</td>
<td>No</td>
<td>Both</td>
<td>Yes</td>
<td>Fast</td>
<td>Medium</td>
<td>Current</td>
</tr>
<tr>
<td>BFV</td>
<td>Yes</td>
<td>No</td>
<td>Both</td>
<td>Yes</td>
<td>N/A</td>
<td>Low</td>
<td>None</td>
</tr>
<tr>
<td>LRCM</td>
<td>Yes</td>
<td>No</td>
<td>Digital</td>
<td>No</td>
<td>N/A</td>
<td>High</td>
<td>Voltage, Current</td>
</tr>
<tr>
<td>Slide Control</td>
<td>No</td>
<td>Yes</td>
<td>Digital</td>
<td>No</td>
<td>Fast</td>
<td>Medium</td>
<td>Voltage, Current</td>
</tr>
</tbody>
</table>

- Hill climbing and Perturb and Observe
- Perturb the voltage
- Increment duty cycle which changes the voltage
- $dV > 0 \rightarrow dP < 0$
- $dV < 0 \rightarrow dP > 0$
Whenever \( dp > 0 \) → perturbation is kept the same
\( dp < 0 \) → it is reversed

Process is repeated until reaching MTP

Problem: oscillates around MTP

Solution: choose a smaller step size (smoother perturbation)

Problem: smaller step size means slower convergence.

Another problem: can fail in rapidly changing weather conditions

- Expected behavior
- Real behavior
- Expected behavior: \( dv > 0 \) → \( dp < 0 \) so perturbation is reversed
- Actual necessary behavior: \( dv > 0 \) → \( dp > 0 \) so
Incremental conductance

Notice that:
\[ \frac{dP}{dV} = 0 \quad \text{at MPP} \]
\[ \frac{dP}{dV} < 0 \quad \text{right of MPP} \]
\[ \frac{dP}{dV} > 0 \quad \text{left of MPP} \]

Now:
\[ \frac{dP}{dV} = \frac{d}{dV} \left( \frac{I}{V} \right) = \frac{I}{V} \frac{dV}{dV} + V \frac{dI}{dV} = \frac{1}{V} I + V \frac{dI}{dV} \]

So
\[ \frac{dI}{dV} = -\frac{I}{V} \quad \text{at MPP} \]
\[ \frac{dI}{dV} < -\frac{I}{V} \quad \text{right of MPP} \]
\[ \frac{dI}{dV} > -\frac{I}{V} \quad \text{left of MPP} \]

Approach #1:

Consider:
\[ e = \frac{I}{V} + \frac{dI}{dV} \]

\[ e > 0 \quad \text{at MPP} \]

\[ e > 0 \quad \rightarrow \text{so we can use a P \& I controller to make } e \to 0 \]
Approach #2

Inputs: $V(t)$, $I(t)$

$\Delta I = I(t) - I(t-\Delta t)$
$\Delta V = V(t) - V(t-\Delta t)$

- $\Delta V = 0$
  - No
  - $\Delta I/\Delta V = -I/V$
    - No
    - $\Delta I/\Delta V > -I/V$
      - Yes
        - Increment $V_{ref}$
      - No
        - Decrement $V_{ref}$
  - Yes
    - $\Delta I = 0$
      - Yes
        - Increment $V_{ref}$
      - No
        - Decrement $V_{ref}$

$I(t-\Delta t) = I(t)$
$V(t-\Delta t) = V(t)$

Return

- Ripple cancellation control

Input to the dc-dc converter always has some ripple

This ripple is used to drive the converter to the PV module.
Consider a boost converter. When $d > 0$, $i > 0$, $V < 0$.

\[ d(t+1) = \kappa \int \hat{p} i \, dt \]

Because like error signal, $\hat{p} i = 0$.

\[ V_{\text{max}} \quad V_{\text{op}} \quad V_{\text{oc}} \quad \hat{p} > 0 \quad \hat{p} < 0 \]

\[ i > 0 \quad \hat{p} < 0 \quad \hat{p} > 0 \quad \hat{p} < 0 \]

\[ \hat{p} V_{\text{oc}} \quad \hat{p} i > 0 \]