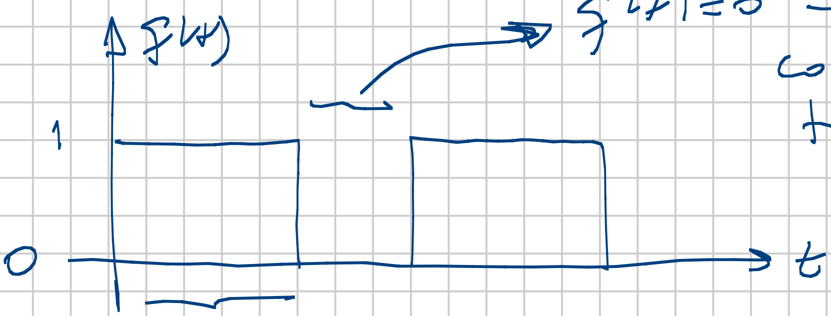
Power electronic circuits modeling

- Switched model

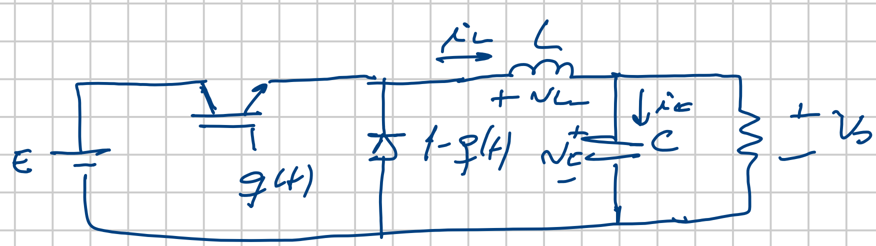
Switching function $g(t)$:



$\to g(t)=0 \Rightarrow$ Switch is commanded to be OFF

$\hookrightarrow g(t)=1 \Rightarrow$ switch is commanded to be ON

- Consider a buck converter operating in continuous conduction mode (CCM) $\to$ i.e., $i(t)>0 \;\; \forall t \geq 0$

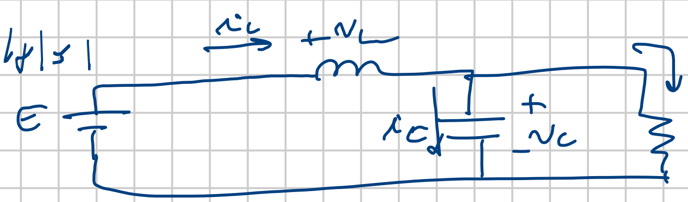


$i_L = x_1$

$v_C = x_2$

$\to$ I assume ideal components

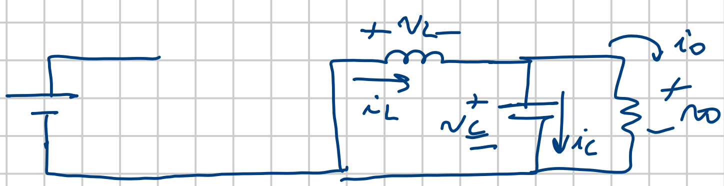
When $g(t)=1$



$i_o = \dfrac{v_C}{R}$

$$\begin{cases} L\dot{x}_1 = E - x_2 \\ C\dot{x}_2 = x_1 - x_2/R \end{cases}$$

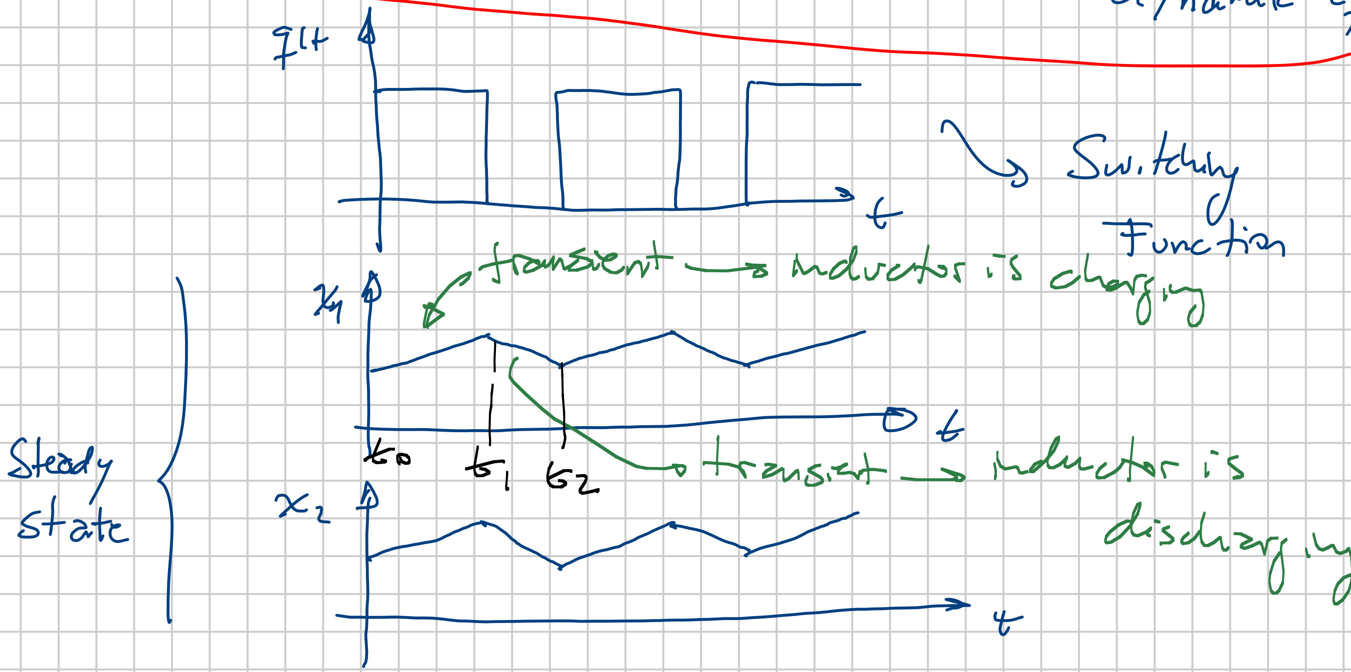
When $g(t)=0$



$$\begin{cases} L\dot{x}_1 = -x_2 \\ C\dot{x}_2 = x_1 - x_2/R \end{cases}$$

Hence

$$\boxed{\begin{cases} L\dot{x}_1 = g(t)E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases} \quad (1) \longrightarrow \text{Switched system dynamic eqs.}}$$



- Note that $g(t)$ is nonlinear. So power electronics circuits are nonlinear circuits. Because of $g(t)$ in (1) I cannot use Fourier, Laplace or identify impedances.

- Steady state is a succession of transient states

That is $x_1(t_0) \neq x_1(t_1)$ and $x_1(t_1) \neq x_1(t_2)$ but

transient   transient

$$x_1(t_0) = x_1(t_2)$$

steady state

Equilibrium points $\longrightarrow$ are those points where $\dot{x}_1 = 0$ and $\dot{x}_2 = 0$ (e.g. "velocity" $\dot{x}$ is zero)

For $g(t)=1 \rightarrow \begin{cases} 0 = E - x_2 \longrightarrow x_{2_{01}} = E \\ 0 = x_1 - x_2/R \longrightarrow x_{1_{01}} = E/R \end{cases}$

For $g(t)=0 \rightarrow \begin{cases} 0 = -x_2 \longrightarrow x_{2_{02}} = 0 \\ 0 = x_1 - x_2/R \longrightarrow x_{2_{02}} = 0 \end{cases}$

In matrix form (1) & (2) can be written as:

$$(1)\ \begin{cases} L\dot{x}_1 = g(t)E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases} \longrightarrow \mathcal{E}\dot{x} = Ax + Bu$$

circuit structure

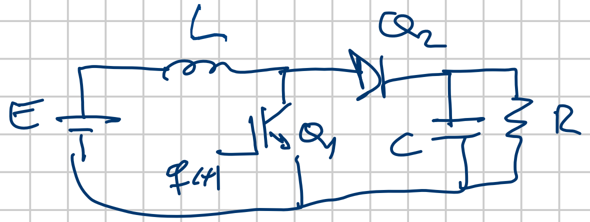
"inertia" $\leftarrow \mathcal{E} = \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix}, \quad A = \begin{pmatrix} 0 & -1 \\ 1 & -1/R \end{pmatrix}$

Based on control input $\longrightarrow B = \begin{pmatrix} E \\ 0 \end{pmatrix}, \quad u = g(t)$

or

Based on power input $\longrightarrow B = \begin{pmatrix} g(t) \\ 0 \end{pmatrix}, \quad u = E$

- For a boost converter:



$g'(t) = 1 - g(t)$

$g(t)=1 \quad \begin{cases} L\dot{x}_1 = E \\ C\dot{x}_2 = -\dfrac{x_2}{R} \end{cases}$

$g(t)=0 \quad \begin{cases} L\dot{x}_1 = E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases}$

$$\boxed{\begin{cases} L\dot{x}_1 = E - g'(t)x_2 \\ C\dot{x}_2 = g'(t)x_1 - \dfrac{x_2}{R} \end{cases}}$$

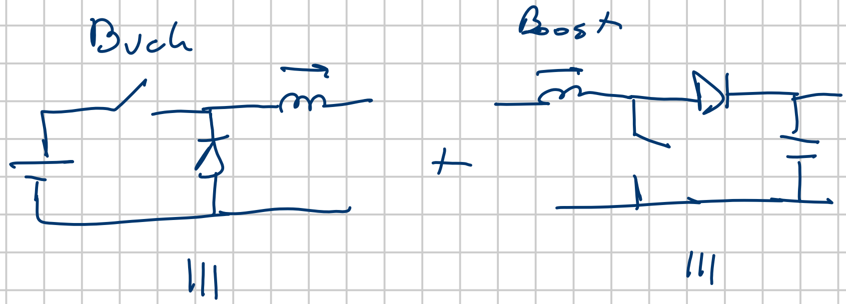
switched model

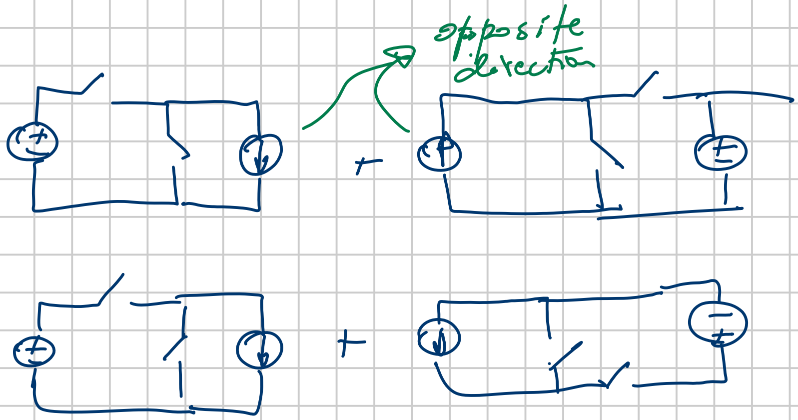
Equilibrium points:

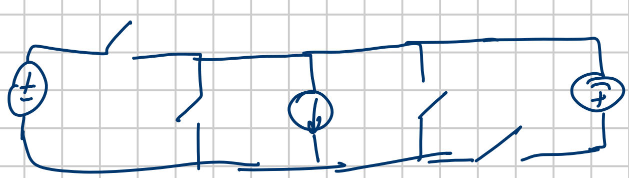
$$f(t)=1 \longrightarrow \begin{cases} 0 = E \ (?) \\ 0 = -\dfrac{x_2}{R} \end{cases} \longrightarrow$$ There is no equilibrium. If I leave the switch closed $x_1 \to \infty$

$$f(t)=0 \longrightarrow \begin{cases} 0 = E - x_2 & \longrightarrow x_{2_{eq}} = E \\ 0 = x_1 - \dfrac{x_2}{R} & \longrightarrow x_{1_{eq}} = \dfrac{E}{R} \end{cases}$$

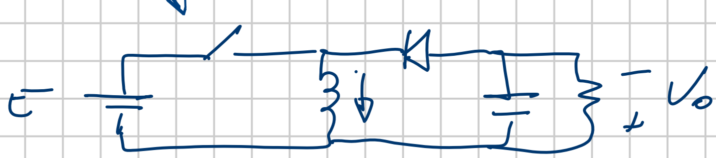
- Buck-boost converter







Redundant switches



$$\begin{cases} L\dot{x}_1 = \xi(t)\,E - \xi'(t)\,x_2 \\ C\dot{x}_2 = \xi'(t)\,x_1 - \dfrac{x_2}{R} \end{cases}$$

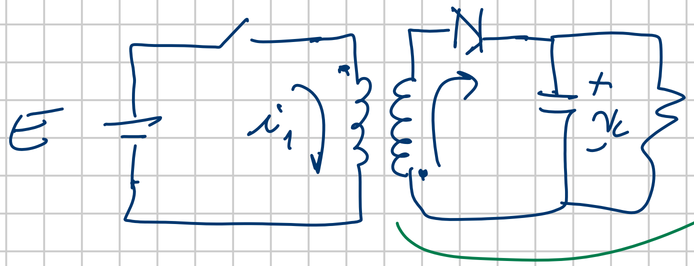
Equilibrium points:

$\xi(t) = 1 \longrightarrow \begin{cases} 0 = E \quad (?) \\ x_{2_{01}} = 0 \end{cases}$ → Analogous to the same condition in the boost converter

$\xi(t) = 0 \longrightarrow \begin{cases} 0 = -x_2 \longrightarrow x_{2_{02}} = 0 \\ 0 = x_1 - \dfrac{x_2}{R} \longrightarrow x_{1_{02}} = 0 \end{cases}$

- Fly back converter

From the buck-boost converter let's split the inductor in two coupled inductors



Not a transformer ↓ they are 2 coupled inductors

$$\frac{d\phi}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2}$$

$\phi = A_L\,(i_1 N_1 + i_2 N_2)$ ← general form

↳ $A_L = \dfrac{1}{\mathcal{R}}$ → Reluctance

permeance

$\phi\,\mathcal{R} = \mathcal{F}$

↳ Ni

↳ ohm's law for a magnetic circuit

So ϕ plays the role of an inductor current

$$\begin{cases} \dfrac{d\phi}{dt} = q\dfrac{E}{N_1} - q'\dfrac{V_c}{N_2} \\ C\dfrac{dV_c}{dt} = \dfrac{q'\phi}{A_L N_2} - \dfrac{1}{R}V_c \end{cases}$$

$\phi \mathcal{R} = Ni$

$L = N\dfrac{\phi}{i} = \dfrac{N^2}{\mathcal{R}}$

$\dfrac{N}{L} = \dfrac{\mathcal{R}}{N} = \dfrac{1}{A_L N}$

$i_2 \neq 0$ when $i_1 = 0$

So when the diode is conducting we have that

$\phi = A_L i_2 N_2 \longrightarrow i_2 = \dfrac{\phi}{A_L N_2}$

This is why it can't be considered a transformer

Otherwise the general form should be valid.

If $\phi = x_1$ and $V_c = x_2$ $\longrightarrow$

$$\begin{cases} \dot{x}_1 = q\dfrac{E}{N_1} - q'\dfrac{x_2}{N_2} \\ C\dot{x}_2 = q'\dfrac{x_1}{A_L N_2} - \dfrac{x_2}{R} \end{cases}$$

Fast average model

Fast average operator $\rightarrow$ $\boxed{\bar{f}(t) = \dfrac{1}{T_{sw}}\displaystyle\int_t^{t+T_{sw}} f(t)\,dt}$

This is a linear operator

- An operator is a machine

What kind of machine is this?

- If I apply laplace transform on both sides I obtain that $\bar{F}(s)$ is proportional to $\dfrac{F}{s}$

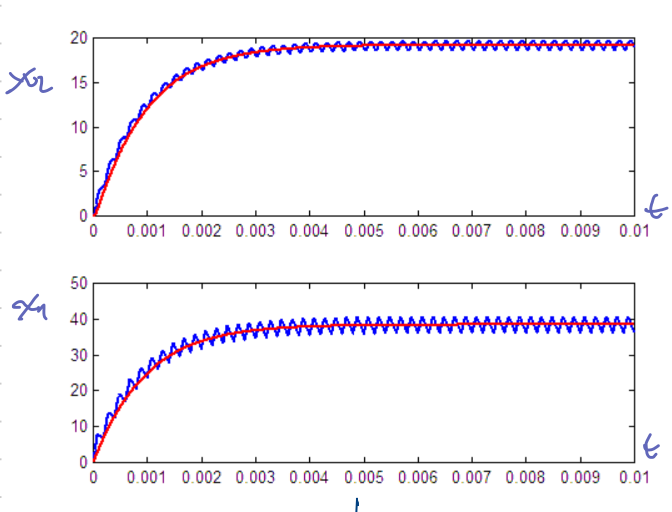
- Since $\frac{F}{s}$ is indicative of a low-pass filter the fast average operator acts as a low-pass filter

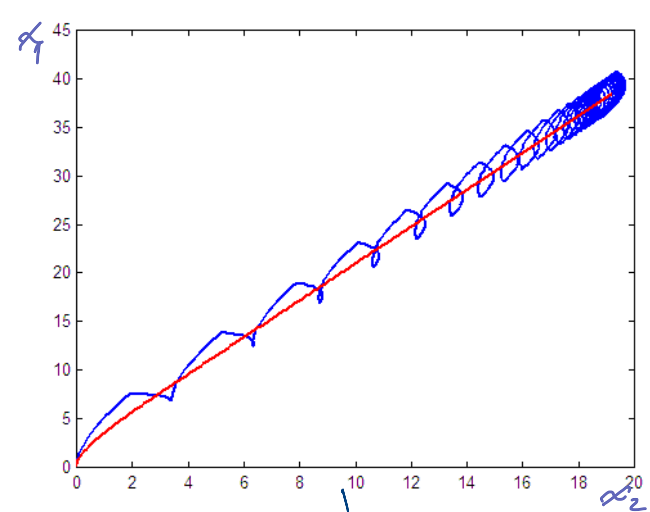
So when I apply the fast average operator to the switching function I obtain the instantaneous duty cycle $\bar{d}(t)$

$$q(t) \longrightarrow \bar{d}(t) = \frac{1}{T_{sw}} \int_{t}^{t+T_{sw}} q(t)\, dt$$

From (1)

$$\begin{cases} L\dot{x}_1 = q(t)E - x_2 \\ C\dot{x}_2 = x_1 - \dfrac{x_2}{R} \end{cases} \xrightarrow[\text{operator}]{\text{Fast average}} \begin{cases} L\dot{\bar{x}}_1 = \bar{d}(t)E - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases} \quad (2)$$





time domain

state space
(phase portrait)

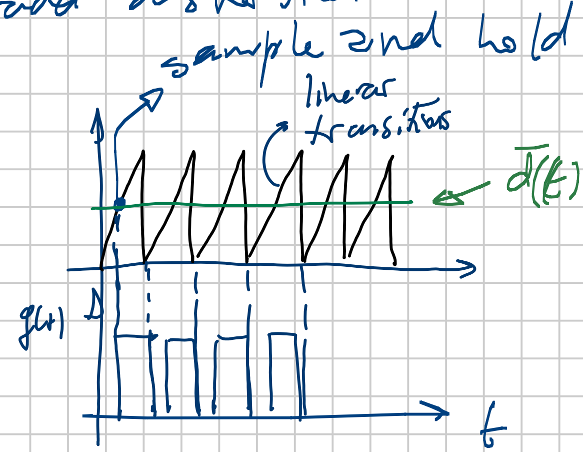
Blue → switched model
Red → Fast average model

Simulations performed with Simulink with a buck converter with $E = 48V$, $R = 0.5\Omega$, $L = 500\mu H$, $C = 100\mu F$

Note that in order to realize the switching function we sample an instantaneous duty cycle signal with linear transitions that do not add distortion

$$\bar{d}(t) = \frac{1}{T}\int_{\tau}^{\tau+T} g(t)\,dt$$

generated with PWM from



Like earlier I can represent the fast average model in a matrix form.

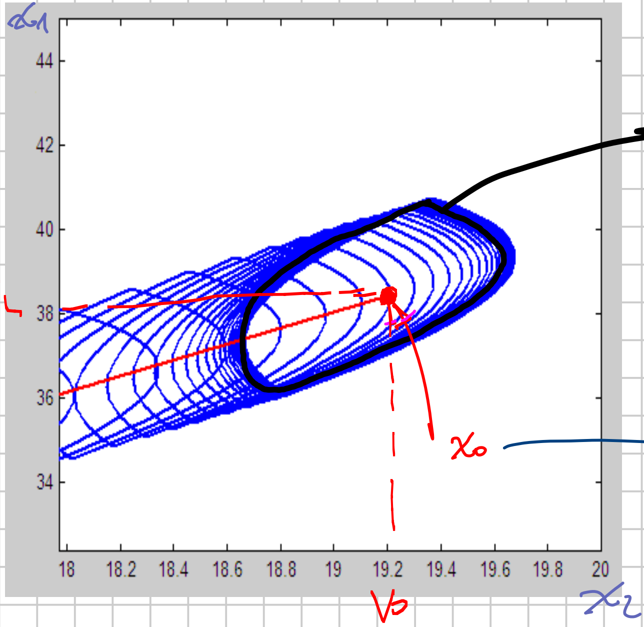
$$(2)\begin{cases} L\dot{\bar{x}}_1 = \bar{d}(t)E - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \frac{\bar{x}_2}{R} \end{cases} \longrightarrow M\dot{\bar{x}} = A\bar{x} + B\bar{u}$$

$\downarrow$

If $\bar{d}(t)$ is constant and equal to D, then

$$\begin{cases} L\dot{\bar{x}}_1 = DE - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \frac{\bar{x}_2}{R} \end{cases} \xrightarrow[\dot{\bar{x}}_1 = 0,\ \dot{\bar{x}}_2 = 0]{\text{eq. point}} \begin{cases} 0 = DE - V_0 \\ 0 = I_L - \frac{V_0}{R} \end{cases}$$

$$\text{eq. point} \longrightarrow x_0 = \begin{pmatrix} I_L \\ V_0 \end{pmatrix} = \begin{pmatrix} DE/R \\ DE \end{pmatrix}$$



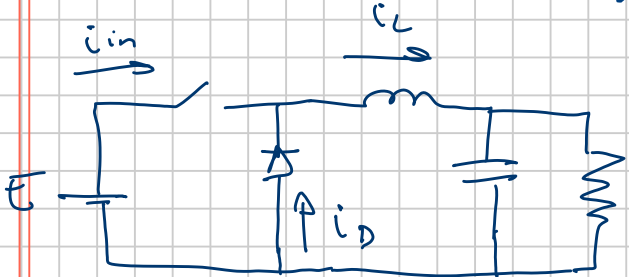
Limitcycle

(1) does not lead to an equilibrium point

Equilibrium point only achieved in a weighted average sense

$$\bar{x}_e = \begin{pmatrix} DE/R \\ DE \end{pmatrix} = \underbrace{x_{e01}}_{\begin{pmatrix} E/R \\ E \end{pmatrix}} D + (1-D)\, \underbrace{x_{e02}}_{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

What if we are not in CCM and $i_L = 0$ for part of the period (we are in discontinuous conduction mode - DCM)



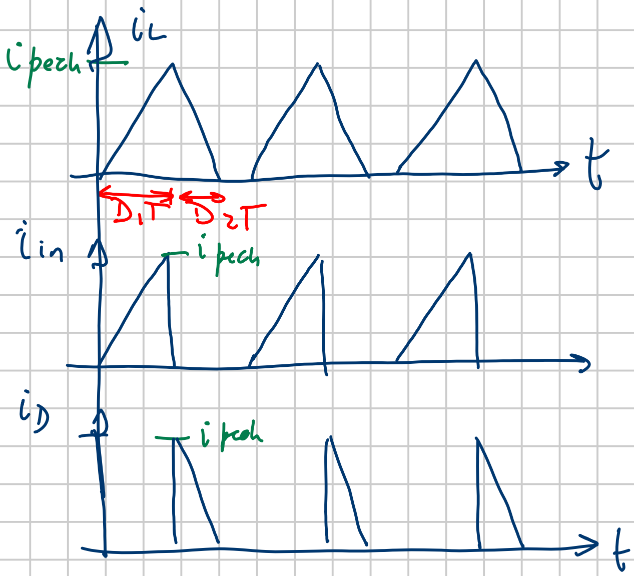
When $Q_1 = ON$,

$$V_L = V_{in} - V_{OUT} = L\frac{di}{dt} = L\frac{i_{peak}}{D_1 T}$$

then $i_{peak} = \frac{D_1 T}{L}(E - V_{OUT})$ (*)

Also $P_{in} = \frac{1}{T}\int_0^T V_{in}\, i_{in}\, dt =$

$$= E\, \underbrace{\frac{1}{T}\int_0^T i_{in}\, dt}_{\langle i_{in} \rangle}$$



Now,

$$\langle i_{in} \rangle = \frac{1}{T}\, \underbrace{\frac{D_1 T\, i_{peak}}{2}}_{\hookrightarrow \text{area of triangle}} = \frac{D_1\, i_{peak}}{2} \quad (**)$$

And, since $P_{in} = P_{out} \longrightarrow E\langle i_{in}\rangle = \frac{V_{out}^2}{R}$

$\downarrow$ in (*) and (**)

$$\frac{D_1^2 T}{2L}\left(E - V_{out}\right) V_{in} = \frac{V_{out}^2}{R}$$

$$V_{out} = -\frac{D_1^2 E R T}{4L} + DE\sqrt{\frac{RT}{2L} + \frac{R^2 T^2 D_1^2}{16L^2}}$$

A complete average model for a buck converter in DCM is

$$L\dot{\bar{x}}_1 = \bar{d}E - \frac{L\,2\,\bar{x}_1\bar{x}_2}{\bar{d}T(E - \bar{x}_2)}$$

$$C\dot{\bar{x}}_2 = \bar{x}_1 - \frac{\bar{x}_2}{R}$$

- For the boost converter

$$\begin{cases} L\dot{x}_1 = E - \mathfrak{f}'(t)\,x_2 \\ C\dot{x}_2 = \mathfrak{f}'(t)\,x_1 - \frac{x_2}{R} \end{cases}$$

$\downarrow$ But I cannot replace $\mathfrak{f}(t)$ by $\bar{d}(t)$ as I did with the buck converter without some clarification:

Fast average issue $\longrightarrow \frac{1}{T_{sw}}\int_t^{t+T_{sw}} \mathfrak{f}'(t)\,x_i\,dt \neq \frac{1}{T_{sw}^2}\underbrace{\int_t^{t+T_{sw}} \mathfrak{f}'(t)\,dt}_{\bar{d}(t)}\underbrace{\int_t^{t+T_{sw}} x_i\,dt}_{\bar{x}}$

That is, the fast average operator applied to $\mathfrak{f}'(t)\,x_i$ is not necessarily $\bar{d}'(t)\,\bar{x}$

e.g.

Since in the switched model the state variables follow linear transitions consider that

$$x_i = a t$$

Then

$$\overline{x_i q(t)} = \frac{1}{T_{sw}} \int_t^{t+T_{sw}} a t \, q(t) dt = \frac{A}{2} D \left(2t + \underline{\underline{D T_{sw}}} \right)$$

↳ Fast average of the product $x_i q(t)$

Now consider

$$\bar{x}_i \, d(t) = \left(\frac{1}{T_{sw}} \int_t^{t+T_{sw}} A t \right) \left(\frac{1}{T_{sw}} \int_t^{t+T_{sw}} q(t) dt \right) = \frac{DA}{2} \left(2t + \overline{\overline{T_{sw}}} \right)$$

This is the only difference

For T_{sw} very small there is no problem

So, for high switching frequency $\left(f_{sw} = \frac{1}{T_{sw}} \right)$.

$$\begin{cases} L \dot{\bar{x}}_1 = E - \bar{d}' \bar{x}_2 \\ C \dot{\bar{x}}_2 = \bar{d}' \bar{x}_1 - \frac{\bar{x}_2}{R} \end{cases}$$

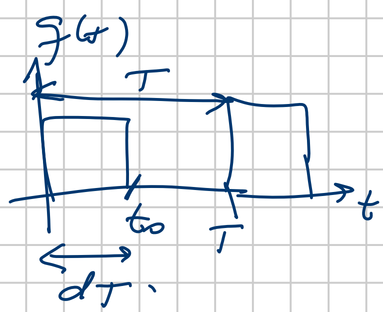
But how "high" is a "high" switching frequency?

Consider a switched linear system

$$\dot{x} = \left(A_1 q(t) + A_2 (1 - q(t)) \right) x$$

$$|||$$

$$\dot{x} = \begin{cases} f_1(t) = A_1 x, & t \in [0, t_0] \\ f_2(t) = A_2 x, & t \in [t_0, T] \end{cases}$$



The exact solutions for $f_1(t)$ and $f_2(t)$ are

A_1 and A_2 are matrices so this function is the exponential of a matrix

$$x(t) = e^{A_1 t} x(0) \qquad t \in [0, t_0)$$
$$x(t) = e^{A_2 (t-t_0)} x(t_0) \qquad t \in [t_0, T)$$

which is calculated as $e^{\mathcal{A}_0} = \sum_{k=0}^{\infty} \frac{1}{k!} \mathcal{A}_0^k$

$t_0 = dT$

$$x(T) = e^{A_2(T-t_0)} x(t_0) = e^{A_2(T-dT)} x(t_0)$$

Since $x(t)$ is continuous at t_0 $(x(t_0^-) = x(t_0^+))$ then

$$x(T) = e^{A_2(T-dT)} x(t_0) = e^{A_2(1-d)T} e^{A_1 dT} x(0)$$

$t_0 = dT$

$x(t_0) = e^{A_1 t_0} x(0)$

Now let's call $A_2(1-d)T = \mathcal{A}_2$ (it's a matrix) and

$A_1 dT = \mathcal{A}_1$ (another matrix)

So $\quad x(T) = e^{\mathcal{A}_2} e^{\mathcal{A}_1} \quad (3)$

Before continuing let's see some useful properties of the function of the exponential of a matrix:

$$e^{0} = I$$
$$e^{a\mathcal{A}} e^{b\mathcal{A}} = e^{(a+b)\mathcal{A}}$$
$$e^{\mathcal{A}} e^{-\mathcal{A}} = I$$

If $\mathcal{B}$ is invertible then $\rightarrow e^{\mathcal{B}\mathcal{A}\mathcal{B}^{-1}} = \mathcal{B} e^{\mathcal{A}} \mathcal{B}^{-1}$

$$\det(e^{\mathcal{A}}) = e^{tr(\mathcal{A})}$$

↳ trace of $\mathcal{A}$ ⟶ $tr(\mathcal{A}) = \sum_{j=1}^{n} a_{jj}$

$$e^{\mathcal{A}^T} = (e^{\mathcal{A}})^T$$

If $\mathcal{A}$ and $\mathcal{B}$ commute (i.e, $\mathcal{A}\mathcal{B} = \mathcal{B}\mathcal{A}$) then $e^{\mathcal{A}+\mathcal{B}} = e^{\mathcal{A}}e^{\mathcal{B}}$

If $\mathcal{A}$ and $\mathcal{B}$ do not commute we can use Baker-Campbell-Hausdorff formula

↳ If $e^{C} = e^{\mathcal{A}}e^{\mathcal{B}}$ (4)

then $C = \mathcal{A} + \mathcal{B} + \frac{1}{2}[\mathcal{A}, \mathcal{B}] + \frac{1}{12}\left([\mathcal{A},[\mathcal{A},\mathcal{B}]] + [[\mathcal{A},\mathcal{B}],\mathcal{B}]\right) + \cdots$

↳ commutator ⟶ $[\mathcal{A}, \mathcal{B}] = \mathcal{A}\mathcal{B} - \mathcal{B}\mathcal{A}$

↳ So if they commute $[\mathcal{A}, \mathcal{B}] = 0$

and
$$e^{C} = e^{\mathcal{A}+\mathcal{B}}$$

So let's go back to (3) ⟶ $x(T) = e^{\mathcal{A}_2} e^{\mathcal{A}_1}$

Since $\mathcal{A}_1$ and $\mathcal{A}_2$ do not necessarily commute then from (4)

$$\mathcal{A} = AT = \mathcal{A}_1 + \mathcal{A}_2 + \frac{1}{2}[\mathcal{A}_1, \mathcal{A}_2] =$$

$d' = 1-d$
$$= (d A_1 + d' A_2)T + dd'(A_1A_2 - A_2A_1)T^2 + \cdots$$

Now, if T is small (i.e., f_{sw} large) then $T^2 \ll T$ and

$$AT \approx (d A_1 + d' A_2)T$$

and $x(T) = e^{AT}x(0) \approx e^{(d A_1 + d' A_2)T}x(0)$ (4)

solution to $\dot{x} = (\underbrace{d A_1 + d' A_2}_{\text{weighted average of}})x$ $\left\{ \begin{array}{l} \dot{x} = A_1 x \\ \dot{x} = A_2 x \end{array} \right.$

So the fast average model is a good approximation for the switched model if T is small (or f_{sw} is large) so the following approximation is valid

$$e^{A_0} = \sum_{k=0}^{\infty} \frac{1}{k!} A_0^k \approx I + A_0 \longrightarrow \begin{cases} e^{dA_1T} \approx I + dA_1T \\ e^{d'A_2T} \approx I + d'A_2T \end{cases}$$

so let's go back to the buck converter for a quick example

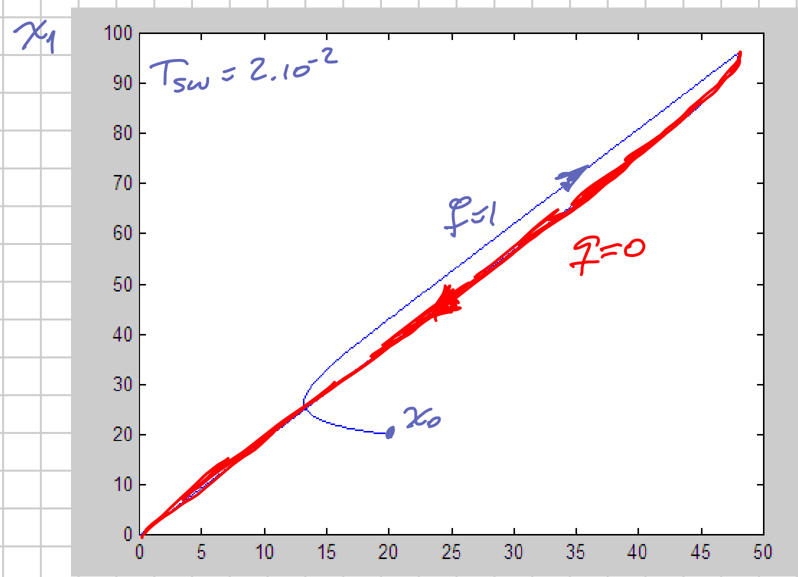
Switched system

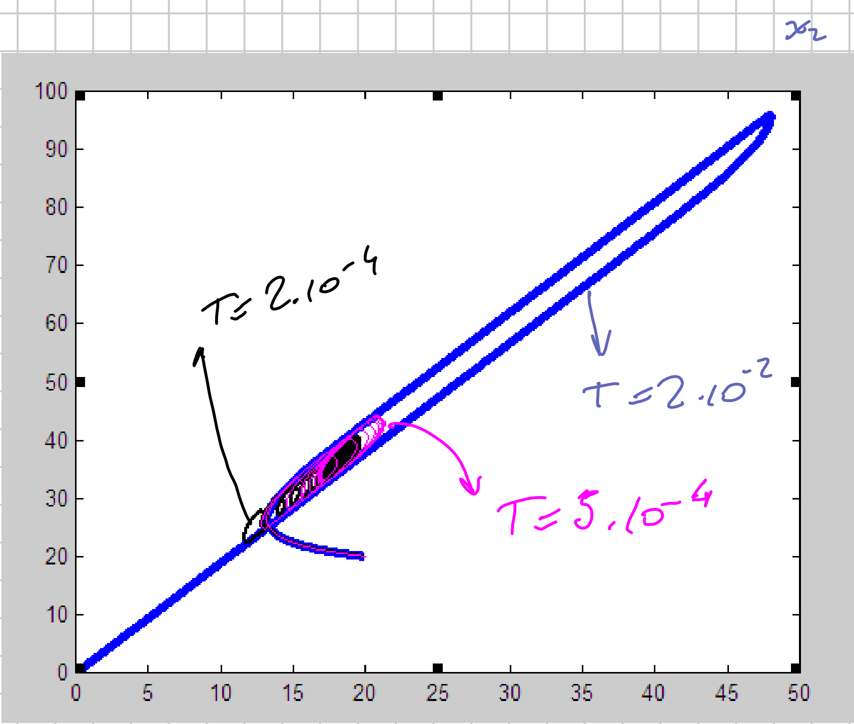
$$\begin{cases} L\dot{x}_1 = f(t)E - x_2 \\ C\dot{x}_2 = x_1 - \frac{x_2}{R} \end{cases} \qquad x(0) = x_0$$

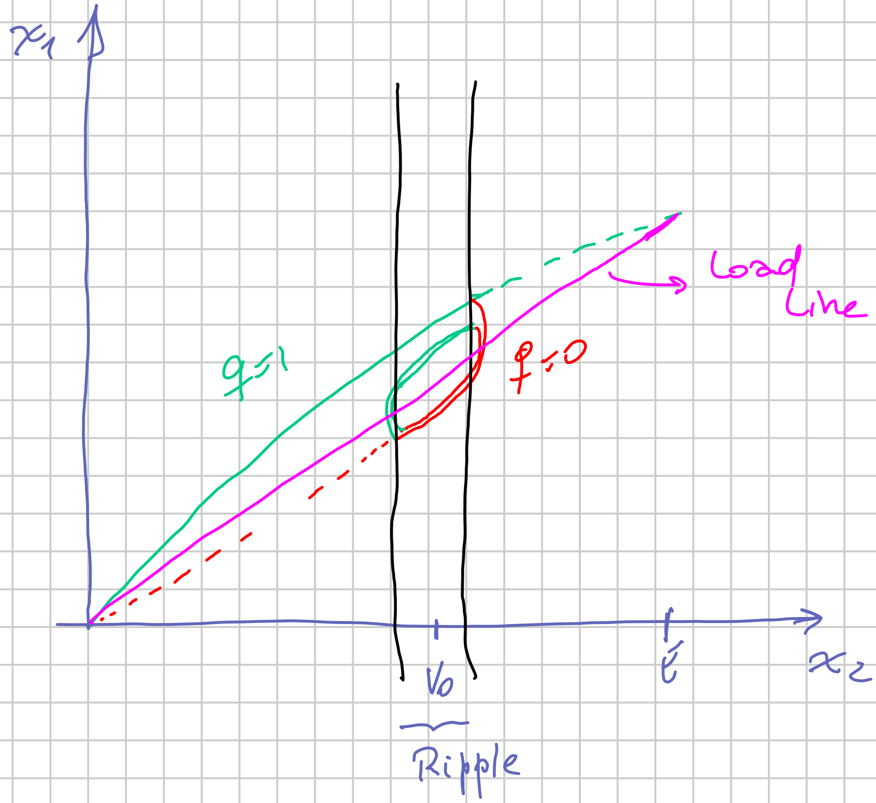
equilibrium

For $f(t) = 0 \longrightarrow x_2 = 0,\ x_1 = 0$

For $f(t) = 1 \longrightarrow x_2 = E,\ x_1 = \frac{E}{R}$

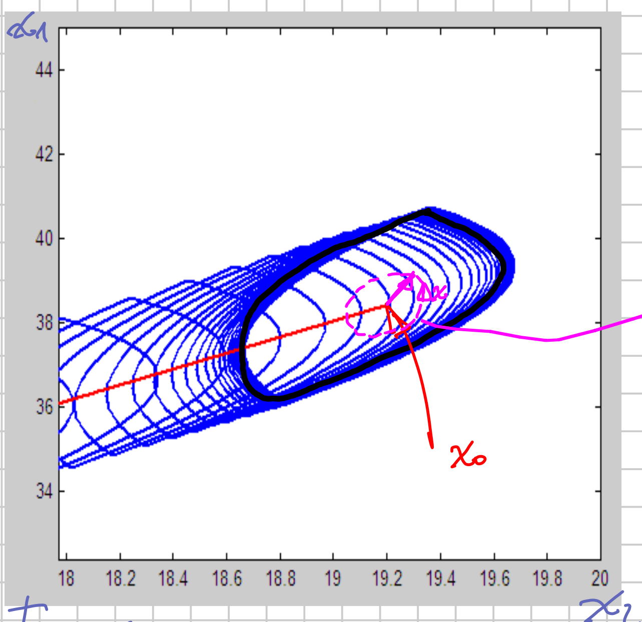






Small signal model

Let's consider once again the buck converter



Let's represent the converter behavior in a small region close around the equilibrium point x_0

From (2)

$$\begin{cases} L\dot{\bar{x}}_1 = \bar{d}(t)E - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \dfrac{\bar{x}_2}{R} \end{cases}$$

Consider the linear operator Δ that is defined as

$$\Delta(f) = f - f_0$$ → so it just calculates the difference with respect to a point f_0.

So $\delta_{\bar{x}_1} = \Delta(\bar{x}_1) = \bar{x}_1 - \bar{x}_{10}$ → $(\dot{\delta x_1}) = \dot{\bar{x}}_1$

→ coordinate in x_1 of the equilibrium point.

Let $r_1 = x_1$, $r_2 = x_2$ then

$$L\,\delta\dot{\bar{x}}_1 = E\,\delta_{\bar{d}} - \delta_{\bar{x}_2}$$

In the same way $C\,\delta\dot{\bar{x}}_2 = \delta_{\bar{x}_1} - \frac{\delta \bar{x}_2}{R}$

Thus,

$$\boxed{\begin{cases} L\,\delta\dot{\bar{x}}_1 = E\,\delta_{\bar{d}} - \delta\bar{x}_2 \\ C\,\delta\dot{\bar{x}}_2 = \delta\bar{x}_1 - \dfrac{\delta\bar{x}_2}{R} \end{cases}}$$ → Model valid in a small neighborhood around the equilibrium point x_0

If the input voltage E is allowed to vary then:

$$\begin{cases} L\,\delta\dot{\bar{x}}_1 = \underbrace{E\,\delta_{\bar{d}} + \delta_E D_0}_{} - \delta\bar{x}_2 \\ C\,\delta\dot{\bar{x}}_2 = \delta\,\bar{x}_1 - \dfrac{\delta\bar{x}_2}{R} \end{cases}$$

→ the product of two variables (d and E) becomes two terms

So I can now do standard linear analysis. For example I can calculate the transfer functions from

$$\begin{pmatrix} \dot{\tilde{\delta x_1}} \\ \dot{\tilde{\delta x_2}} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1/L \\ 1/C & -1/RC \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} \tilde{\delta x_1} \\ \tilde{\delta x_2} \end{pmatrix}}_{\tilde{\delta x}} + \underbrace{\begin{pmatrix} E/L & D_0/L \\ 0 & 0 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \delta d \\ \delta_E \end{pmatrix}}_{\tilde{\delta u}}$$

Notice that δx_2 is controlled through δx_1

$$\tilde{\delta y} = \tilde{\delta x_2}$$

$C = 1$

Using Laplace

$$\mathcal{L}(\dot{\tilde{\delta x_1}}) = s\,\Delta x_1(s) \;- - - -$$

$$\begin{cases} L s\,\Delta x_1(s) = -\Delta x_2(s) + E\,\Delta_D(s) + D_0\,\Delta_E(s) \\ C s\,\Delta x_2(s) = \Delta x_1(s) - \dfrac{\Delta x_2(s)}{R} \end{cases}$$

$$G(s) = \frac{\Delta x_2(s)}{\Delta_D(s)} \longrightarrow \frac{\text{output}}{\text{control input}}$$

$\to$ transfer function

$$L s\,\Delta x_1(s) = L s\left(C s\,\Delta x_2(s) + \frac{\Delta x_2(s)}{R}\right) = -\Delta x_2(s) + E\,\Delta_D(s) + D_0\,\Delta_E(s)$$

$$\Delta x_2(s)\left(LCs^2 + \frac{L}{R}s + 1\right) = E\,\Delta_D(s)$$

When calculating the transfer function with respect to the control input, it is considered that the power input is fixed $\Rightarrow \Delta_E(s) = 0$

$$\boxed{G(s) = \frac{ER}{LCRs^2 + Ls + R}}$$

For the boost converter:

Linearization:

From $\Delta f \simeq \left.\frac{\partial f}{\partial r_1}\right|_{r_0} \Delta r_1 + \left.\frac{\partial f}{\partial r_2}\right|_{r_0} \Delta r_2$

If $f = \bar{d}\,\bar{x}_i$

then $\delta_f = X_{i0}\,\delta_d + D_0\,\delta\bar{x}_i$

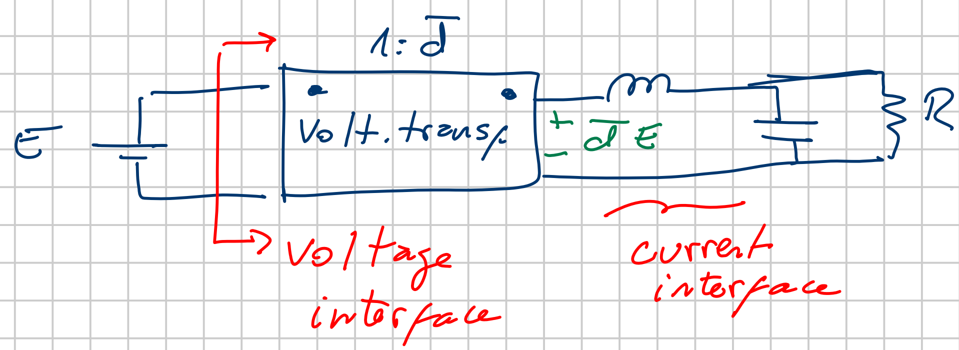
thus,

$$\boxed{\begin{cases} L\,\delta\dot{\bar{x}}_1 = X_{20}\,\delta_d - (1-D_0)\,\delta x_2 \\ C\,\delta\dot{x}_2 = -X_{10}\,\delta_d + (1-D_0)\,\delta x_1 - \frac{\delta x_2}{R} \end{cases}}$$

Equivalent circuits based on the fast average model

– Buck converter

$$\begin{cases} L\dot{\bar{x}}_1 = \bar{d}(t)E - \bar{x}_2 \\ C\dot{\bar{x}}_2 = \bar{x}_1 - \frac{\bar{x}_2}{R} \end{cases}$$



– Boost converter

$$\bar d' = 1 - \bar d$$

$$\begin{cases} L\,\dot{\bar x}_1 = E - \bar d'(t)\,\bar x_2 \\ C\,\dot{\bar x}_2 = \bar d'\,\bar x_1 - \dfrac{\bar x_2}{R} \end{cases}$$

– Fly Back

$$\begin{cases} \dot{\bar\phi} = \dfrac{\bar d\,E}{N_1} - \dfrac{\bar d'}{N_2}\,\bar x_2 \\ C\,\dot{\bar x}_2 = \dfrac{\bar d'\,\bar x_1}{\Delta_L N_2} - \dfrac{x_2}{R} \end{cases} \rightarrow \begin{cases} L\dfrac{d i_1}{dt} = \bar d E - \dfrac{N_1}{N_2}\bar d' v_c \\ C\dfrac{d v_c}{dt} = \bar d' \dfrac{N_1}{N_2^2}\,\bar i_1 - \dfrac{v_c}{R} \end{cases}$$

$$\frac{\bar\phi}{\Delta_L N_2} = \frac{\bar\phi\,\mathcal{R}}{N_2} = \phi\,\frac{N_2}{L_2} = \frac{N_1}{N_2^2}\,\bar i_1$$

Some good papers for reference:

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Richard M. Bass[1] and Philip T. Krein[2]

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