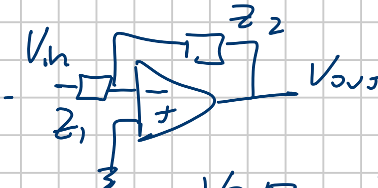
Note Title 11/22/2009

Some important concepts for test #2

- **Operational amplifiers:** → Only Some not All
  - Analysis of circuits with ideal op. amp. → $Z_{in} = \infty$ → $V^+ = V^-$
    - → $Z_{out} = 0$
  - Circuits with op. amps. → e.g.



$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1}$$

we can make integrators and differentiators with one R and one C

- **First order circuits**

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$

$$\delta(t) = \frac{du}{dt} = \begin{cases} 0 & t<0 \\ \text{undefined} & t=0 \\ 0 & t>0 \end{cases}$$

$$\int_{t=0^-}^{t=0^+} \delta(t)\,dt = 1$$

$$r(t) = \int_{-\infty}^{t} u(t)\,dt = \begin{cases} 0 & t\leq 0 \\ t & t>0 \end{cases}$$

$$\int_a^b f(t)\,\delta(t-t_0) = f(t_0)$$

↳ $a \leq t_0 \leq b$

Complete response = natural response (stored energy) + forced response (source)

natural response → time constant → depends on circuit parameters

Complete response = transient response + steady state response

(transient response → temporary → evolution from initial energy conditions t_0 → Final energy condition)

(steady state response → behavior as $t \to \infty$)

$$v(t) = v_t(t) + v_{ss}(t)$$

$$f(t) = f(t\to\infty) + \left[f(0) - f(t\to\infty)\right] e^{-\frac{t}{\tau}}$$

→ some signal usually → voltage for capacitors / current for inductors

- Second order circuits
  - No external source
    → First write down characteristic equation:

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

↓ solution leads to 3 cases

- case a) $\alpha > \omega_0$
  → $f(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$
  → solution
  → A_1 and A_2 are obtained from initial conditions

  $s_1 \neq s_2$ → Both s_1 & s_2 are real numbers

- case b) $\alpha = \omega_0$ → $s_1 = s_2 = s$ → Real number

$$f(t) = (A_1 + A_2 t) e^{-st}$$

→ From initial conditions

→ Case c) $\alpha < \omega_0$

$$i_L(t) = e^{-\alpha t}\left(B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right)$$

From initial conditions

↳ S_1 and S_2 are complex conjugates

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

- With external source:

$$v(t) = v_n(t) + v_f(t)$$

$v_n(t)$ ↓ Natural; $v_f(t)$ ↓ forced

Where → $v_f(t)$ → depends on $v_s(t)$

↳ $v_f(t)$ has the same form than $v_s(t)$

So if $v_s(t) = V_s$

then $v_f(t) = V_{f_0}$

or if $v_s(t) = V_s \cos \omega_s t$

then $v_f(t) = V_{f_1} \cos(\omega_f t) + V_{f_2} \sin(\omega_f t)$

In all these cases V_{f_i} is unknown.

(we'll see in a short while how to obtain it)

Also we know that $v_n(t)$ → depends on → circuit

↳ initial conditions

(*) Overdamped circuit $\rightarrow V_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Critically damped circuit $\rightarrow V_n(t) = (A_1 + A_2 t) e^{-\alpha t}$

underdamped circuit $\rightarrow V_n(t) = e^{-\alpha t}(A_1 \cos w_0 t + A_2 \sin w_0 t)$

So here it is the recipe:

1) Solve characteristic eq

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

↓ obtain $(s_{1,2})$ or α or (α, w_0) depending which of the cases (*) is present in this problem

2) Look at $V_s(t)$ and suggest $V_f(t)$ = "same form than $V_s(t)$"

if $V_s(t) = V_s e^{kt} \longrightarrow V_f(t) = V_s e^{kt}$

if $V_s(t) = V_s \longrightarrow V_f(t) = V_{f_0}$

if $V_s(t) = M\cos(W_s t) \longrightarrow i_f(t) = V_{f_1}\cos(W_s t) + V_{f_2}\sin(W_s t)$

Important: see special cases below

3) Calculate $\frac{dV_f}{dt}$ and $\frac{dV_f^2}{dt^2}$ and replace them in the circuit equation, i.e.

$$\frac{d^2 V_f}{dt^2} + \frac{R}{L}\frac{dV_f}{dt} + \frac{1}{LC}V_f = \frac{V_s}{LC}$$

↳ Solve the equation to obtain V_{f0} or (V_{f_1}, V_{f_2})

4) Now, the solution is

$$v_c(t) = v_n(t) + v_f(t)$$

5) Make $v_c(t=0) = V_0 = v_n(t=0) + v_f(t=0)$

make $\frac{dv_c}{dt}(t=0) = \frac{dv_n}{dt}(t=0) + \frac{dv_f}{dt}(t=0)$

→ Solve to obtain A_1 and A_2.

So initial conditions are applied in the last step!!

Special case

- Suppose that from the characteristic equation:

$$s_{1,2} = \pm j\omega_0$$

And $v_s(t) = V_s \cos\omega_0 t$

then $v_n(t) = A_1 \cos\omega_0 t + A_2 \sin\omega_0 t$

If we do as above then

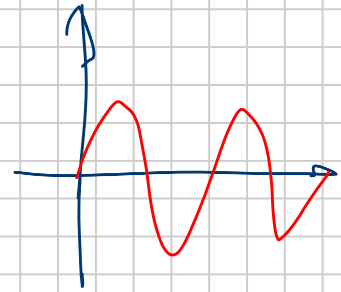
$$v_f(t) = V_{f1}\cos(\omega_0 t) + V_{f2}\sin(\omega_0 t)$$

Since they are the same it won't work

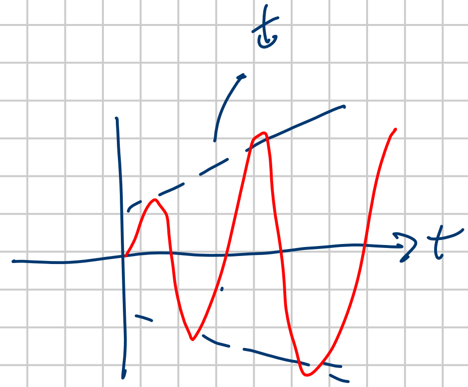
Instead, propose

$$v_f(t) = t\left[V_{f1}\cos(\omega_0 t) + V_{f2}\sin(\omega_0 t)\right]$$

so instead of having a solution like →



I have →



This is just a special case for when

$$v_s(t) = V_{s_1} e^{k_1 t} + V_{s_2} e^{k_2 t}$$

and $k_1 = s_1$ and/or $k_2 = s_2$

- Sinusoids and phasors/Sinusoidal steady state analysis

$$v(t) = V\cos(\omega t + \phi) = \Re(\underbrace{V e^{j\phi}} e^{j\omega t})$$

↳ $\mathbf{V} = V\angle\phi = V(\cos\phi + j\sin\phi)$

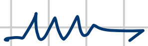
↳ complex number → requires 2 inputs → V, ϕ

$$\mathbf{V}_1\mathbf{V}_2 = V_1 V_2 \angle \phi_1 + \phi_2$$

time domain		frequency domain
$\frac{dv(t)}{dt}$	⟷	$j\omega\mathbf{V}$
$\int v(t)dt$	⟷	$\frac{\mathbf{V}}{j\omega}$

Ohm's law in phasor's domain

↳ $\mathbb{V} = \mathbb{I}\,\mathbb{Z}$

Element	$\mathbb{Z}$	$\mathbb{Z}$ (polar)	Resistance	Reactance
	R	$R\angle 0$	R	0
	$j\underbrace{\omega L}_{X_L}$	$\omega L \angle 90$	0	ωL
	$\frac{1}{j\omega C}$ ↳ $-X_C$	$\frac{1}{\omega C} \angle -90$	0	$-\frac{1}{\omega C}$

KVL → $\sum_{i=1}^{n} \mathbb{V}_i = 0$

KCL → $\sum_{i=1}^{N} \mathbb{I}_i = 0$

$\mathbb{Z}_{eq_S} = \sum_{i=1}^{N} \mathbb{Z}_i$

$\mathbb{Z}_{eq_{\parallel}} = \dfrac{1}{\sum_{i=1}^{N} \frac{1}{\mathbb{Z}_i}}$

At resonance $|X_L| = |X_C|$

- **AC Power**

Instantaneous power

↳ $p(t) = v(t)\,i(t) = \frac{V_m I_m}{2}\left[\underbrace{\cos(\varphi_v - \varphi_i)}_{\text{constant}} + \underbrace{\cos(2\omega t + \varphi_v + \varphi_i)}_{\text{time varying}}\right]$

Average power → $P(t) = \frac{1}{T}\int_0^T p(t)\,dt = \frac{V_m I_m}{2}\cos(\varphi_v - \varphi_i)$

rms value → $x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$

Complex power $\longrightarrow \mathbb{S} = \mathbb{V}\,\mathbb{I}^* = P + j\,Q \longrightarrow S = V_{rms} I_{rms}$

$\mathbb{S} = \mathbb{I}_{rms}^2\, \mathbb{Z}$ $\longrightarrow$ power factor

$P = V_{rms} I_{rms} \cos(\varphi_v - \varphi_i)$

$p.f. = \frac{P}{S}$

$Q = V_{rms} I_{rms} \sin(\varphi_v - \varphi_i)$

$p.f. = \cos(\arg(\mathbb{Z}))$

Load	P	Q
R	$I_{rms}^2 R$	0
L	0	$I_{rms}^2 X_L = I_{rms}^2 \omega L$
C	0	$I_{rms}^2 X_C = -\frac{I_{rms}^2}{\omega C}$

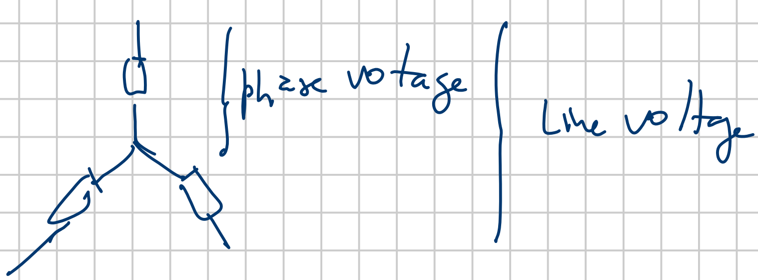
Power factor correction

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

- 3-phase circuits

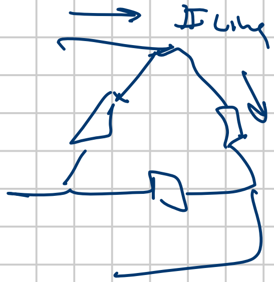
wye

$\mathbb{V}_L = \sqrt{3}\,\mathbb{V}_{ph} \angle 30$



$\mathbb{I}_{ph} = \mathbb{I}_{Line}$

Delta



$$I_{Line} = \sqrt{3}\, I_{phase} \angle -30$$

$$V_{ph} = V_{Line}$$

$$p(t) = 3 V_{rms_{ph}} I_{rms_{ph}} \cos\varphi$$

$$P_{ph} = V_{rms_{ph}} I_{rms_{ph}}$$

$$P = \sqrt{3}\, V_{rms_L} I_{rms_L} \cos\varphi = 3 V_{rms_{ph}} I_{rms_{ph}} \cos\varphi$$

$$Q = \sqrt{3}\, V_{rms_L} I_{rms_L} \sin\varphi = 3 V_{rms_{ph}} I_{rms_{ph}} \sin\varphi$$

$$S = \sqrt{3}\, V_{rms_L} I^*_{rms_L}$$