EE411 – Fall 2009, Test 2

Please, show all your work on the test sheets. A correct answer without supporting work gets no credit. One sheet of notes is permitted. Write your name in all pages. Do not un-staple. You have 60 minutes to complete the test.

Problem 1 (30 points)
Consider a series RLC circuit excited by a sinusoidal voltage source with an amplitude of 10 V. Consider also that R = 5 Ω, L = 2 mH, and C = 8 mF. Please answer the following questions:

a) Please write down the expression without numbers for the current phasor **I** in terms of the input voltage phasor $\mathbf{V}_s$.
b) What is the (resonant) <u>frequency</u> of the voltage source that yields the maximum magnitude for **I**? Write down the expression without numbers for **I** at this specific condition.
c) What is the maximum magnitude of **I**? (with numbers)

a) $\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{\mathbf{V}_s}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$

b) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{16 \cdot 10^{-6}}} = 0.25 \cdot 10^3 = 250$

$f_0 = \frac{\omega_0}{2\pi} = \frac{250}{2\pi} = 39.79\ Hz$

$\mathbf{I} = \frac{\mathbf{V}_s}{R}$

c) $\mathbf{I} = \frac{10}{5} = 2A$

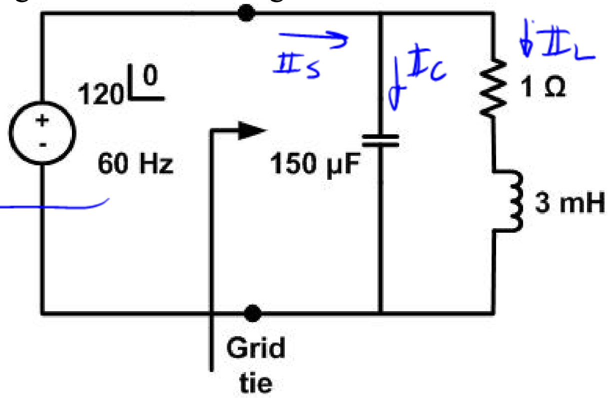
EE411 – Fall 2009, Test 2

Problem 2 (30 points)

The figure shows the equivalent circuit of a typical industrial load with capacitors compensating the power factor. Unfortunately, the person that calculated the capacitors didn't take EE411 and the capacitance does not provide enough compensation.

a) What is the active and reactive power measured at the grid tie?
b) What is power factor?
c) How much capacitance should be <u>added</u> to reach a power factor of at least 0.9?
d) How much capacitance should be added to reach a power factor of at least 0.9 if you were in Europe where the line frequency is 50 Hz?

Consider that all voltages and currents magnitudes are rms values.



$\omega = 2\pi f = 2\pi 60 = 377$ rad/sec

a) $P_L = I_L^2 R = \dfrac{120^2}{\left(\sqrt{1+(3\cdot10^{-3}\cdot377)^2}\right)^2} \, 1 \cong 6.32$ kW

$Z_L = 1 + j\,377\cdot3\cdot10^{-3} = 1.51 \angle 48.52$

$Q_L = I_L^2 X_L = (79.47)^2\, 3\cdot10^{-3}\, 377 \cong 7.14$ kVAr

$\hookrightarrow I_L = \dfrac{120}{1.51} = 79.47$

$Q_C = \dfrac{V_S^2}{X_C} = 120^2\, 377\, 150\cdot10^{-6} = 814.32$ VAr

At the grid tie $\rightarrow P \simeq 6.32$ kW

$\hookrightarrow Q \simeq 7.14 - 0.814 \cong 6.33$ kVAr

b) $pf = \dfrac{P}{S} = \dfrac{6.32}{\sqrt{6.32^2 + 6.33^2}} = 0.706 \rightarrow \varphi_0 = \cos^{-1} 0.706 \cong 45°$

c) For $\cos\varphi_n = 0.9 \longrightarrow \varphi_n = 25.84°$

$$\text{New } C = \left| \frac{P(\tan\varphi_n - \tan\varphi_0)}{\omega V_s^2} \right| = \left| \frac{6320\,(\tan 25.84 - \tan 45)}{377 \cdot 120^2} \right| =$$

$$= \left| \frac{6320\,(0.4842 - 1)}{5428800} \right| \cong 600\,\mu F$$

d) In Europe

$$\omega = 2\pi 50 = 314.16$$

$$I_L = \frac{120}{\sqrt{1 + (3 \cdot 10^{-3}\; 314.16)^2}} = \frac{120}{1.374} = 87.32$$

$$P_L = I_L^2 R = (87.32)^2\; 1 \cong 7.63\text{ kW}$$

$$Q_L = I_L^2 X_L = (87.32)^2\; 314.16\; 3 \cdot 10^{-3} = 7.187\text{ kVAr}$$

$$Q_C = \frac{V_s^2}{X_C} = 120^2\; 314.16\; 150 \cdot 10^{-6} \cong 678.6\text{ kVAr}$$

$$P_L = 7.63\text{ kW}$$

$$Q = 7.187 - 0.678 = 6.508\text{ kVAr}$$

$$pf = \frac{7.63}{\sqrt{7.63^2 + 6.51^2}} = 0.76 \longrightarrow \varphi_0 = 40.47$$

$$\text{New } C = \left| \frac{7630\,(\tan 25.84 - \tan 40.47)}{314.16\; 120^2} \right| = 622\,\mu F$$

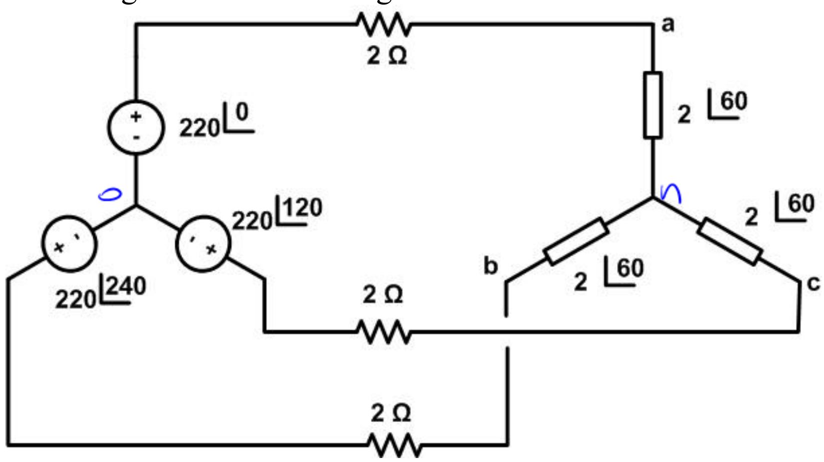
EE411 – Fall 2009, Test 2

Problem 3 (40 points)

For the 3-phase circuit in the figure calculate:

a) Load's phase and line voltage phasors (all 6 of them).
b) Load's phase and line current phasors (all 6 of them).
c) Load's total active and reactive power.
d) Load's complex power.
e) Source's total reactive power.

Consider that all voltages and currents magnitudes are rms values.



b) Load's phase and line currents are the same

$Z_T = 2\angle 0 + 2\angle 60 = 3.46\angle 30$

$I_a = \frac{V_{ao}}{Z_T} = \frac{220\angle 0}{3.46\angle 30} = 63.51\angle -30$

$I_b = I_a \angle -120 = 63.51\angle -150$

$I_c = I_a \angle 120 = 63.51\angle 90$

a) $V_{an} = I_a Z_L = 63.51\angle -30 \; 2\angle 60 = 127.02\angle 30$

$V_{bn} = V_{an}\angle -120 = 127.02\angle -90$

$V_{cn} = V_{an}\angle 120 = 127.02\angle 150$

Line voltages

$V_{ab} = V_{an}\sqrt{3}\angle 30 = 220\angle 0$

$V_{bc} = V_{ab}\angle -120 = 220\angle -120$

$V_{ca} = V_{ab}\angle 120 = 220\angle 120$

c) $P_L = \sqrt{3}\, V_L I_L \cos\varphi = \sqrt{3}\; 63.51 \; 220 \cos(60) = 12.1\,kW$

$Q_L = \sqrt{3} V_L I_L \sin\varphi \cong 21\,kVAr$

d) $S = P_L + jQ_L = 12.1 + j21$

$\hookrightarrow S_L = \sqrt{P_L^2 + Q_L^2} = 24.2$ kVA

e) From conservation of energy $Q_S = Q_L = 21$ kVAr