EE313 - Signals and Systems

Homework #4
Due: February 16 at the beginning of class

Remember, discussion of homework questions is encouraged. Please be absolutely sure to submit your own independent homework solution.

1. (20 points) For \(x(t), h(t)\) defined below, use the convolution integral to find the response \(y(t)\) of the LTI system characterized by impulse response \(h(t)\) when the input is \(x(t)\). Solve the problem both analytically and graphically (i.e., as a sequence of plot sketches).

\[
x(t) = e^{-\alpha t}u(t)
\]
\[
h(t) = e^{-\beta t}u(t)
\]

Be sure to consider both special cases: \(\alpha \neq \beta\) and \(\alpha = \beta\).

2. (20 points)
   (a) Consider an LTI system with input and output related through the equation

   \[
y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau - 3)d\tau.
   \]

   What is the impulse response \(h(t)\) for this system?

   (b) Determine the output of the system when the input is given by

   \[
x(t) = \text{rect} \left( \frac{t}{2} - \frac{1}{3} \right).
   \]

   Recall that \(\text{rect}(t) = 1\) for \(t \in (-0.5, 0.5)\) and is zero elsewhere.

3. (20 points) Determine \(y(t) = \text{rect}(t) \ast \text{rect}(t)\).

4. (25 points) Determine whether each system is causal, memoryless and/or stable. Justify your answers.
   (a) \(h[n] = (-\frac{1}{2})^n u[n] + (1.02)^n u[n - 1]\)
   (b) \(h(t) = (t + 1)e^{-(t+1)}u(t)\)
   (c) \(h[n] = 4^n u[2 - n]\)
   (d) \(h(t) = e^{-2t}u(t - 2)\)
   (e) \(h(t) = e^{-2t}u(2 - t)\)

5. (15 points) Determine if each of the following systems is invertible. If it is, construct the inverse system. If not, find two input signals to the system that have the same output.
(a) \( y(t) = \frac{dx(t)}{dt} \)

(b) \( y[n] = \sum_{k=-\infty}^{n} \left( \frac{1}{2} \right)^{n-k} x[k] \)

**Extra Problems (NOT GRADED)** Note from Prof. Heath: You will be given solutions to these problems. You should known how to work them for exams.

6. Prove the equality
\[
[x(t) * h(t)] * g(t) = x(t) * [h(t) * g(t)]
\]
by showing that both sides of the above equation equal
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) h(\sigma) g(t - \tau - \sigma) d\tau d\sigma
\]

7. Consider a discrete-time system whose input \( x[n] \) and output \( y[n] \) are related by
\[
y[n] = \left( \frac{1}{2} \right) y[n-1] + x[n].
\]

(a) If the system satisfies the condition “if \( x[n] = 0 \) for \( n < n_0 \), then \( y[n] = 0 \) for \( n < n_0 \)”, then is it linear and time invariant?

(b) If the system does not satisfy the above condition, but instead satisfies \( y[0] = 0 \), is it causal?

8. A linear system has the relationship
\[
y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-2k]
\]
between its input \( x[n] \) and its output \( y[n] \), where \( h[n] = u[n] - u[n-3] \).

(a) Determine \( y[n] \) when \( x[n] = \delta[n-2] \).

(b) Is the system LTI?

(c) Determine \( y[n] \) when \( x[n] = u[n] \).

9. O&\text{W} 2.48 – Properties of LTI systems