EE313 - Signals and Systems

Homework #7
Due: March 23 at the beginning of class

Remember, discussion of homework questions is encouraged. Please be absolutely sure to submit your own independent homework solution.

1. (10 points) Consider a signal $x(t)$ that is periodic with a fundamental period of $T$ and Fourier series coefficients denoted by $a_k$. Consider a signal $y(t)$ that is periodic with fundamental period $TM$ where $M$ is a positive integer. Denote the Fourier series coefficients of $y(t)$ as $b_k$. Let $z(t) = x(t)y(t)$.

(a) Show that $z(t)$ is periodic with period $TM$.
(b) Determine the Fourier series coefficients of $z(t)$ in terms of the Fourier series coefficients of $x(t)$ and $y(t)$.
(c) Determine the Fourier transform of $z(t)$.

2. (30 pts) Consider a standard RLC circuit with a resistor $R$, inductor $L$ and capacitor $C$ all in series driven by a voltage source $v(t)$. The voltage source gives pulses of 5 volts that last 2 msec, every 10 msec and we are interested in the output $y(t)$ which is the current flowing through the circuit at time $t$.

(a) Find a general expression for the frequency response $H(j\omega)$ of this system.
(b) Let $R = 5$, $C = 0.1$, $L = 0.2$. Plot by hand using straight-line approximations the gain $|H(j\omega)|$ in dB vs. $\omega$ using a log scale for the x-axis. Use a range of $\omega \in (10^{-2}, 10^4)$.
(c) Now, plot the gain in Matlab over the same range. Some functions you might find useful for this are: logspace, abs, and semilogx. Print out your plot and turn it in. Comment on how close it comes to your hand drawn plot above.
(d) Find the Fourier series representation of the output current $y(t)$.
(e) Using this representation, plot $y(t)$ in Matlab for the first 20 harmonics. What is the effect of this filter?

3. (30 points) Compute Fourier transforms for the following signals.

(a) $[e^{-\alpha t} \cos \omega_0 t] u(t)$ (also determine the values of $\alpha$ where the Fourier transform exists)
(b) $[e^{-\alpha t} \cos \omega_0 t] \text{rect}(t)$ (also determine the values of $\alpha$ where the Fourier transform exists)
(c) $\text{sinc}(t) \text{sinc}(2t - 1)$

4. (30 points) Compute the inverse Fourier transform for the following signals.

(a) $\delta(\omega + 1) + \delta(\omega - 1) + j\delta(\omega + 3) - j\delta(\omega - 3)$
(b) $\cos(4\omega + \pi/3)$
(c) sinc(2ω − 1)

Extra Problems (NOT GRADED) Note from Prof. Heath: You will be given solutions to these problems. You should known how to work them for exams.

5. Compute the Fourier transform of each of the following signals:

(a) $x(t)$ as shown below

(b) $x(t)$ as shown below in Figure 2

3. (10 pts) Oppenheim & Willsky: Problem 4.22(a)
4. (10 pts) Oppenheim & Willsky: Problem 3.34
5. (25 pts) Oppenheim & Willsky: Problem 3.65(a) - (d)