1. (35 pts) DTFT problems. Solve the following using the properties.
   
   (a) $\mathcal{F}\{u[n - 1] - u[n - 5]\}$
   
   (b) $\mathcal{F}^{-1}\{\sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \pi k/3)\}$
   
   (c) $\mathcal{F}\{3^n \sin(\pi n/4)u[-n]\}$
   
   (d) $\mathcal{F}\{\sin(\pi n/2) + \cos(7\pi n/3)\}$
   
   (e) $\mathcal{F}\{\sum_{k=-\infty}^{\infty} \delta[n - 3k - 1]\}$
   
   (f) $\mathcal{F}^{-1}\left\{\frac{e^{-j\omega} - \frac{1}{2}}{1 - \frac{1}{4} e^{-j\omega}}\right\}$
   
   (g) $\mathcal{F}^{-1}\left\{\frac{1 - (\frac{1}{3})^6 e^{-j6\omega}}{1 - \frac{1}{4} e^{-j\omega}}\right\}$

2. (20 pts) O & W 5.30 (a), (b)(i) and (c)(i)

3. (25 pts) DFT calculations. Compute the DFT of the following signals by hand and plot $X[k]$. Label the low and high frequencies. Feel free to check your results on the computer. For the following problems, we use vector notation to represent finite length signals. For example $[-1, 2, 3]$ means a signal with $x[0] = -1$, $x[1] = 2$, and $x[2] = 3$. Label the low frequency and the high frequency coefficients.
   
   (a) DFT of $[1, 1, -1, -1]$
   
   (b) DFT of $[1, 0, 1, 0]$
   
   (c) Inverse DFT of $[0, 0, 1, 0]$

4. (20 pts) Computer Experiment. Consider a discrete-time signal

   $$x[n] = \cos(\omega_1 n) + \sin(\omega_2 n)$$

   where $\omega_1 = 2\pi/7$ and $\omega_2 = 2\pi/9$. Suppose that you create a length $L$ signal by taking $x[0], x[1], \ldots, x[L]$. Use the DFT with zero-padding as described in Lecture 20 to plot one period of the DTFT from $[-\pi, \pi]$ (hint: you need to use FFT shift) using a sufficiently long FFT (at least $N = 512$). Plot $|X_L(e^{j\omega})|$ (the DTFT of your finite length signal) for different values of $L$. Turn in your plots and MATLAB code.
   
   (a) Determine the value of $L$ where you can clearly distinguish the two sinusoids. Generate a plot for this value of $L$ and for $L-10$ to show the difference. Explain this result and connect it to DTFT properties.
(b) For both plots, try different values of $N$. Can you increase resolution with larger values of $N$ to overcome this problem? Explain the result intuitively.

(c) Assuming again a large value of $N$, what happens to your optimum value of $L$ if $\omega_2 = 2\pi/9$?

(d) Suggest through experiments a relationship between $\omega_2 - \omega_1$ and $L$.

Extra Problems (NOT GRADED) Note from Prof. Heath: You will be given solutions to these problems. You should know how to work them for exams.

5. O & W 5.34 (LTI system response)
6. O & W 5.42 (frequency shift property proof)
7. O & W 5.53 (DFT)
8. O & W 5.54 (FFT)