Wireless Communications Lab

Lecture #15

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Review

SC-FDE Equalization

OFDM Equalization
Introduction to Frequency Offset

Learning Objectives:

Explain origin of frequency offset and discuss nature of the impairment.
Frequency offset

- Consider a passband signal
  \[ y_p(t) = \text{Re} \{ y(t) e^{j \omega_c t} \} \]

- Suppose have \( \omega_c \) at receiver?
  - Creates carrier frequency offset (CFO)
  - Carrier synchronization

- Assuming offset is small and local and bandwidth is wide enough

\[ y_p(t) \]

\[ \cos(\omega_c t) \]

\[ \sin(\omega_c t) \]

\[ \text{LPF} \]

\[ \text{LPF} \]

\[ \text{Re}\{y(t)\} \]

\[ \text{Im}\{y(t)\} \]

\[ \text{not } \omega_c ! \]
2/ \[ y[n] = \sum_{n=-\infty}^{\infty} h[n] e^{j2\pi f T n} + \text{noise} \]

\[ \begin{align*}
\varepsilon &= \delta_e T \\
\delta_e &= \delta_c - \delta_e \\
\text{Rotation occurs after convolution} \\
\text{Channel estimation & EQ impacted}
\end{align*} \]

- Flat fading

\[ y[n] = e^{j2\pi f_c n} h[n] + \text{noise} \]

\[ e^{j2\pi f_c n} \]

\[ \text{Carrier}
\begin{align*}
\text{Freq.} & \\
\text{Offset} & \\
\text{Estn}
\end{align*} \]
Frequency Offset Estimation Using Periodic Training

Learning Objectives:

Use the Moose method to perform joint frequency offset estimation and frame synchronization.
Frequency offset estimation using periodic training


Consider

\[ y[n] = e^{j\frac{2\pi}{e} n} \sum_{e=0}^{L} h[e] s[n-e] + u[n] \]

\[ s[n] = t[n] \quad n = 0, \ldots, N_b-1 \]

\[ s[n+N_b] = t[n] \quad n = 0, \ldots, N_b-1 \]

\[ y[n] = e^{j\frac{2\pi}{e} n} \sum_{e=0}^{L} h[e] s[n+N_b-e] + u[n] \quad n = 0, \ldots, N_b-1 \]

\[ y[n+N_b] = e^{j\frac{2\pi}{e} (n+N_b)} \sum_{e=0}^{L} h[e] s[n+N_b-e] + u[n+N_b] \]

\[ s[n] = e^{j\frac{2\pi}{e} N_b} y[n] \]
4/1. Relaxed L.S. problem

\[ J(a) = \sum_{e=2}^{N_b-1} (y(n+ne) - ay(n))^2 \]

relax \( e \) to \( a \)

\[ a = \frac{\sum_{e=2}^{N_b-1} y^* (n+ne) y(n)}{\sum_{e=2}^{N_b-1} |y(n+ne)|^2} \]

\[ \hat{a} = \arg \left\{ \sum_{e=2}^{N_b-1} y^* (n+ne) y(n) \right\} \]

or \( \hat{a} = \arg \left\{ \frac{\sum_{e=2}^{N_b-1} y^* (n+ne) y(n)}{2\pi N_b} \right\} \)

- Maximum offset \( |e| N_b \leq \frac{1}{2} \)

\[ \Rightarrow |e| N_b \leq \frac{1}{2} \]

\[ |e| \leq \frac{1}{2 N_b} \]

\[ |\hat{e}| \leq \frac{1}{2 N_b T} \]

\( N_t \) correction range decreases, but better noise averaging
5/  * Example

\( S_c = 20 \text{ khz} \)  1M symbol/s QAM,  \( N_\varepsilon = 10 \)

\[ \max |S_\varepsilon | \leq \frac{1}{2 \cdot 10^{-6}} = \frac{1}{2} \times 10^5 = 50 \, \text{kHz} \]

\[ \max |13| \leq \frac{1}{2} \times 10 = 50 \]

* Self-reference frame sync

\[ d = \arg \max \left( \sum_{e=2}^{N_\varepsilon-1} \frac{\left| \sum_{e=2}^{N_\varepsilon-1} y^* (n+e+d) y (n+e) \right|^2}{\sum_{e=2}^{N_\varepsilon-1} |y^* (n+e+d)|^2} \right) \]

\[ \sum_{e=2}^{N_\varepsilon-1} |y^* (n+e+d)|^2 \]

and variations ...
Frequency Offset Estimation Using Periodic Training

Learning Objectives:

Describe and implement carrier frequency offset estimation for an OFDM system.
Frequency offset in OFDM

- OFDM is very sensitive to frequency offset
- Coax + Schmidt modification of Moore
- Consider turn off odd subcarrier

\[ W[n] = \frac{1}{N} \sum_{m=0}^{N-1} s[m] e^{j \frac{2\pi m(n-\ell_c)}{N}} \quad n = 0, \ldots, N+\ell_c-1 \]

\[ = \frac{1}{N} \sum_{m=0}^{\ell_c-1} s[m] e^{j \frac{2\pi m(n-\ell_c)}{N}} \]

\[ = \frac{1}{N} \sum_{m=0}^{\ell_c-1} s[m] e^{j \frac{2\pi m(n+\ell_c)}{N/2}} \]

\[ W[n+\ell_c] = \frac{1}{N} \sum_{m=0}^{\ell_c-1} s[m] e^{j \frac{2\pi mn}{N/2}} \quad n = 0, \ldots, \ell_c-1 \]

\[ W[n+\ell_c+N/2] = \frac{1}{N} \sum_{m=0}^{\ell_c-1} s[m] e^{j \frac{2\pi mn}{N/2}} \quad m = 0, \ldots, \ell_c-1 \]

\[ = \frac{1}{N} \sum_{m=0}^{\ell_c-1} s[m] e^{j \frac{\pi m}{N/2}} \]

\[ = W[n+\ell_c] \quad n = 0, \ldots, \ell_c-1 \]
Turn off subcarriers to create periodicity in one OFDM symbol.

- Extends to turn off $k-1$ subcarriers

$\Rightarrow$ periodicity of $n/k$
Apply to OROF \( k = 2 \)

\[ y[n] = e^{j\omega_n} \sum_{k=0}^{\infty} h(k) e^{-j\omega_k n} + w[n] \]

\[ y[n + \frac{N}{2}] = e^{j\omega_n} y[n + \frac{N}{2}] \]

\[ \theta = \arg \left\{ \sum_{n=0}^{\frac{N}{2}-1} y[n + \frac{N}{2}] y^*\left[n + \frac{N}{2} + \frac{N}{2}\right] \right\} \]

\[ \frac{1}{N} \sum_{n=0}^{N/2-1} y[n + \frac{N}{2}] y^*\left[n + \frac{N}{2} + \frac{N}{2}\right] \]

- Maximum offset

\[ |13| \leq \frac{1}{2 \cdot N^{1/2}} = \frac{1}{\sqrt{N}} \]

- Note: \( N \) large better resistance to noise but smaller range

\( K > 2 \) more periods to get better estimate and more range
9/ How to correct larger offsets?

Send two OFDM symbols with different contents (still periodic).