Announcements

Please return exam regrade requests by Friday, 5pm.
Review of Receive Diversity

\[ y_k(n) = h_k s(n) + v_k n \]

different channel

different noise
Transmit Diversity

Learning Objectives:

Explain why exploiting transmit diversity is hard.

Derive the maximum ratio transmit diversity solution.
Transmit diversity

- Naïve approach
  - Send same signal from all TX antennas

\[ \sqrt{\frac{\mathbb{E} x}{N_t}} s \]

- Received signal

\[ y[n] = h_1 \sqrt{\frac{\mathbb{E} x}{N_t}} s[n] + h_2 \sqrt{\frac{\mathbb{E} x}{N_t}} s[n] + \ldots \]
\[ h_{N_t} \sqrt{\frac{\mathbb{E} x}{N_t}} s[n] + \mathcal{V}[n] \]
\[ = \sqrt{\frac{\mathbb{E} x}{N_t}} s[n] \left( h_1 \mathcal{H} r_2 + \ldots + h_{N_t} \right) + \mathcal{V}[n] \]
\[ z = \sqrt{\lambda} \, s[y] \left( \frac{1}{\sqrt{n}} \left( h_1 + \cdots + h_{\lambda n} \right) + u \right) \]

Assume \( h_k \sim N(0, 1) \) i.i.d.

\( \frac{1}{\sqrt{n}} \left( h_1 + \cdots + h_{\lambda n} \right) \) is Gaussian

If \( \frac{1}{\sqrt{n}} \left( h_1 + \cdots + h_{\lambda n} \right) = 0 \)

\[ \text{VAR} \left( \frac{1}{\sqrt{n}} \left( h_1 + \cdots + h_{\lambda n} \right) \right) \]

\[ = \frac{1}{n} \text{VAR}(h_1) + \frac{1}{n} \text{VAR}(h_2) + \cdots \]

\[ = 1 \]

\( \frac{1}{\sqrt{n}} \left( h_1 + \cdots + h_{\lambda n} \right) \sim N(0, 1) \)
3/ \quad y[n] \overset{\text{d}}{=} \sqrt{\text{E} g \cdot s[n]} + v[n]

\quad g \sim \mathcal{N}(0,1)

\quad \text{Pe}(\{s[n]\}) \approx (M-1)

\quad \frac{\text{E} \text{median}^2}{\frac{\text{E}}{\text{N}_0}} + 1

No diversity advantage
Maximum ratio transmission (MRT)

Define \( h = \begin{bmatrix} h_1 \\ \vdots \\ h_{N_t} \end{bmatrix} \), \( w = \begin{bmatrix} w_1 \\ \vdots \\ w_{N_t} \end{bmatrix} \)

\[ y = \sqrt{\mathbf{E}_x} \; w^* h \; s + v \]

\( w^* w = 1 \) to satisfy power constraint
Optimum \( \omega ? \)

Maximize \( \frac{|w^*h|^2 \varepsilon_r}{N_0} \)

Subject to \( ||w||^2 = 1 \)

\( \frac{|w^*h|^2 \varepsilon_r}{N_0} \leq \frac{\lambda^2 |h^*h|^2 \varepsilon_r}{N_0} \)

\( \omega = \lambda h \)

\( w^*w = |\lambda|^2 h^*h = 1 \)

\( \Rightarrow |\lambda|^2 = \frac{1}{||h||^2} \)

Take \( \lambda = \frac{1}{||h||} \)
6. Optimum \( w = \frac{h}{\|h\|} \)

\[
W_k^* h_k = \frac{h_k^* h_k}{\|h\|}
\]

\[
\text{SNR} = \frac{|W_k^* h_k|^2 \varepsilon_+}{N_0} = \frac{|h_k^* h_k|^2 \varepsilon_+}{\|h\|^2 N_0} = \frac{\|h\|^2 \varepsilon_+}{N_0}
\]

\[
Pr(\varepsilon_+|N_0) = \frac{1}{(\frac{\varepsilon_+ d_n^2}{4N_0} + 1)^{N_0}}
\]
Note: Requires $h \circ \mathcal{Q}$ at $tx$.

Limited feedback
Alamouti Code

Learning Objectives:

Derive the Alamouti space-time code.

Show that the Alamouti code has a second order diversity gain.
Alamouti code

- Space-time code way of mapping symbols to antennas for diversity gain
- \( N_t = 2 \)
- Part of WCDMA 3GPP

\[
\sqrt{\frac{E_s}{2}} \begin{bmatrix}
  s_1 & -s_2^* \\
  s_2 & s_1^*
\end{bmatrix}
\begin{array}{c}
\rightarrow h_1 \\
\rightarrow h_2
\end{array}
\]

first time  second
\[ y_1 = \sqrt{\frac{\mu}{2}} (h_1 s_1 + h_2 s_2) + v_1 \]
\[ y_2 = \sqrt{\frac{\mu}{2}} (-h_1 s_2^* + h_2 s_1^*) + v_2 \]
\[ y_2^* = \sqrt{\frac{\mu}{2}} (-h_1^* s_2 + h_2^* s_1) + v_2^* \]

\[
\begin{bmatrix}
  y_1 \\
  y_2^*
\end{bmatrix}
= \sqrt{\frac{\mu}{2}}
\begin{bmatrix}
  h_1 & h_2 \\
  h_2^* & -h_1^*
\end{bmatrix}
\begin{bmatrix}
  s_1 \\
  s_2
\end{bmatrix}
+ \begin{bmatrix}
  v_1 \\
  v_2^*
\end{bmatrix}
\]

\[ y = \sqrt{\frac{\mu}{2}} H s + v \]
\[ H^s = \sqrt{\frac{\mu}{2}} H^s H s + H^s v \]
\[
\begin{bmatrix}
h_1^* & h_2 \\
h_2^* & -h_1
\end{bmatrix}
\begin{bmatrix}
h_1 \\
h_2
\end{bmatrix}
\]

\[
H^* = \begin{bmatrix}
h_1^2 + h_2^2 & h_1 h_2 \\
h_2 h_1 - h_1 h_2^* & h_1^2 + h_2^2
\end{bmatrix}
\]

\[
= \begin{bmatrix}
h_1^2 + h_2^2 & 0 \\
0 & |h_1|^2 + |h_2|^2
\end{bmatrix}
\]
\[ H_0^y = \sqrt{\frac{3}{2}} \begin{bmatrix} (\text{Re}^2 + \text{Im}^2) & 0 \\ 0 & (\text{Re}^2 + \text{Im}^2) \end{bmatrix} v \]

\[ = \sqrt{\frac{3}{2}} (\text{Re}^2 + \text{Im}^2) \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \frac{H^o}{L} \]

\[ \Rightarrow \text{Decoupled symbols} \]

\[ H_0^v \text{ given } H \]

\[ v \sim N_c (0, N_0 I) \]

\[ \mathbb{E} H_0^v = H^o \mathbb{E}v = 0 \]

\[ \mathbb{E} H^o v^* H = H^o \mathbb{E} v^* v H = N_0 \frac{H^o H}{\text{Re}^2 + \text{Im}^2} \]
$H^0 \nu$ has uncorrelated entries!

$W$'s variance $\sim N_0 (|h_1|^2 + |h_2|^2)$

$$y_k = \sqrt{\frac{\xi_k}{2}} (|h_1|^2 + |h_2|^2) s_k + \tilde{v}_k$$

$$SNR = \frac{\xi_k (|h_1|^2 + |h_2|^2)^2}{2 N_0 (|h_1|^2 + |h_2|^2)}$$

$$= \frac{\xi_k}{2 N_0} (|h_1|^2 + |h_2|^2)$$

Looks like MRC
\[ P_e (\text{Ex/N0}) \leq (M-1) \frac{1}{(\frac{E_x}{\text{SNR}} + 1)^2} \]

factor of 2 penalty

3dB gap due to 2
1. Alamouti achieves second order diversity
2. Alamouti code loses 3dB relative to MRT w/ $M_t=2$
3. Does not require $h_1$ or $h_2$ at TX
4. Decoding complexity is low
Introduction to MIMO Communication

Learning Objectives:

Explain the concept of spatial multiplexing.
MIMO (Multiple-input, Multiple-output)

\[ \{ h_{km}(t) \}_{k=1}^{Nt}, m=1, \ldots, Nt \]

- Focus on spatial multiplexing
- Consider flat fading

Spatial multiplexing

\[ S_1, S_2, \ldots \rightarrow \text{Spatial Multiplexer} \rightarrow S_1, S_{Nt+1}, \ldots, S_{Nt} \]

Spatial multiplexing (SM)
Also called V-BLAST
Vertical Bell Labs Space-Time Architecture
- Spatial division multiplexing (SDM)

Part of 802.11n, 802.11ac, 3GPP R7+, LTE, LTE-A, uCPE, etc.
Intermission
FCC sets workshop date for 3.5 GHz band spectrum use

Posted on 19 November 2013 by Dan Meyer. Tags: FCC, federal communications commission, small cells, Spectrum, white spaces, Wi-Fi

The Federal Communications Commission is moving forward with plans to free up spectrum in the 3550-3650 MHz band for use by small cell network deployments and spectrum sharing, announcing it will host a technical workshop focused on the band on Jan. 14.

As part of the workshop, the FCC’s Wireless Telecommunications Bureau and Office of Engineering and Technology will look at “technical requirements, architecture, and operational parameters of the proposed spectrum access system (SAS) for the 3550-3650 MHz band (3.5 GHz band).” The SAS would operate similar to the TV White Spaces database in governing use of the 3.5 GHz band.

The workshop is set to focus on four aspects of SAS implementation:

1. General responsibilities and composition of SAS.
2. Key SAS functional requirements.
3. SAS monitoring and management of spectrum use.
4. Issues related to initial launch and evolution of SAS band planning.

Both bureaus have asked that interested parties submit papers discussing aspects of the SAS, which it had previously proposed would manage three service tiers described as incumbent access, priority access and general authorized access. The incumbent access users would include authorized federal and “grandfathered fixed satellite service” users currently operating in the 3.5 GHz band and would receive “protection from harmful interference from all other users in the 3.5 GHz band.”

Priority access users would include “critical quality-of-service needs,” including hospitals, utilities and public safety, and would operate with “some” interference protection. The general authorized access users would be allowed to operate “opportunistically” in certain areas and would have to deal with potential interference issues from other users in that space.

The FCC late last year released a notice of proposed rulemaking in relation to the 3.5 GHz band for use in the deployment of small cells and “spectrum sharing.” The NPRM was set up to look at whether it would be feasible to open up approximately 100 megahertz of spectrum in the 3550-3650 MHz bands for small cell technologies, possibly on an unlicensed basis. Currently, the most prolific unlicensed spectrum used for wireless services resides in the 2.4 GHz band that is used for Wi-Fi services.

The 3.5 GHz band is now in the hands of the Department of Defense for use in certain radar installations, as well as by “non-federal fixed satellite service earth stations for receive-only, space-to-earth operations and feeder links.” The somewhat limited propagation characteristics of the 3.5 GHz band are thought to be a good fit for the dense deployment plans for small cells and would likely limit interference with current users.

The FCC is also looking at potentially extending the spectrum allocation an additional 50 megahertz up to the 3700 MHz band. That spectrum band is currently used by the federal government in just a few locations.