Wireless Communications Lab

Lecture #5

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Discrete-time Processing of Bandlimited Continuous-Time Signals

Learning Objective:

Determine equivalent combination of sampling, digital filtering, and reconstruction to process a bandlimited continuous-time signal with discrete-time signal processing.
II. Discrete-time processing of bandlimited continuous-time signal

Let \( x_c(t) \) be bandlimited with \( f_N \) and let

\[
y_c(t) = \int_{-\infty}^{\infty} h(\tau) x_c(t-\tau) \, d\tau
\]

Observe:

1. \( x_c(t) \) can be represented by \( x[n] \)
2. \( y_c(t) \) is also bandlimited!
3. \( y_c(t) \) can be represented by \( y[n] \)

Think:

\( x_c(t) \) is generated
\( y_c(t) \) is processed
2/ $X_c(t) \rightarrow h(t) \rightarrow y_c(t)$

$x[n] \rightarrow D/L \rightarrow h(t) \rightarrow C/O \rightarrow y[n]$  

CT channel

b/c bandlimited

$b/c$ bandlimited

$x[n] \rightarrow h[n] \rightarrow y[n]$  

DT Equivalent channel

$X_c(s) \rightarrow H(s) \rightarrow Y(s)$  

$X(e^{j2\pi f}) = \frac{1}{2}X(s)$

$H(e^{j2\pi f}) = -$  

$Y(e^{j2\pi f}) = 4y_c(s)$
If $x_c(t)$ is BICW, $y_c(t)$ is

$$y_c(t) = \sum_{n} y[n] \frac{\sin(\pi(t - nT)/T)}{\pi(t - nT)/T}$$

Then

$$y_c(t) = \sum_{n} y[n] \frac{\sin(\pi(t - nT)/T)}{\pi(t - nT)/T}$$

where

$$y[n] = x(n) \ast h[n]$$

where $h[n]$ is equivalent channel

$$H(\omega) = H_c(\frac{\omega}{T}) \frac{1}{T} L_{SIN}(\frac{\pi}{T})$$

Ideal low-pass filter w/ gain of 1 cutoff f/T

$$L_{SIN}(\frac{\pi}{T}) = rect(\frac{\pi}{T}) \leftrightarrow T \text{sinc}(\frac{\pi}{T})$$

$$h_c(t) = T \left( \frac{T}{T} \right) \text{sinc}(\frac{\pi}{T}) \ast h(t)$$

$$h[n] = h_c(nT) = h(t) \ast \text{sinc}(\frac{\pi}{T}) \bigg|_{t=nT}$$
Defining the Frequency Response of a Random Signal

Learning Objectives:

Define and calculate the power spectrum of a wide sense stationary stochastic process.
Defining frequency of a random signal

- Conventional random signals have a random Fourier transform
  \[ X_c(t) \rightarrow X(\xi) \] _R.P._ may not be well defined

- Autocovariance provides a definition of spectrum for WSS R.P.

- Power spectrum

  \[ P_x(\xi) = \int_{-\infty}^{\infty} C_x(t, t) e^{-j2\pi\xi t} \, dt \]

  \[ P_x(\xi) = \sum_{\nu=-\infty}^{\infty} C_x(\nu) e^{-j2\pi\nu} \]

  Use power spectrum to define B/W of a random signal
Intermission

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Bandwidth

Learning Objectives:

Define and calculate various real measures of bandwidth.
Bandwidth

- Define an operational notion of bandwidth for non-ideal time-limited signals
- Half-power at passband

\[ \frac{P_x(f_1)}{P_x(f_{max})} = \frac{1}{2} \quad \text{and} \quad \frac{P_x(f_2)}{P_x(f_{max})} = \frac{1}{2} \]

\( f_{max} \) is the frequency that maximizes \( P_x(f) \)

\[ \frac{P_x(f)}{P_x(f_{max})} < \frac{1}{2} \quad f > f_1 \quad \frac{P_x(f)}{P_x(f_{max})} < \frac{1}{2} \quad f < f_2 \]

- XdB bandwidth use XdB instead of \( \frac{1}{2} \) dB
- Fractional containment \( X \) is the containment

\[ \frac{\int_{f_1}^{f_2} P_x(f) \, df}{\int_{-\infty}^{\infty} P_x(f) \, df} = \frac{1-x}{2} \]
Complex Envelope Representation of Wireless Passband Signals

Learning Objectives:

Use the complex envelope to represent a passband signal at baseband.
Complex envelope representation

- All wireless signals are real pass band
- Most passband signals in wireless are narrowband $B \ll f_c$

Bandwidth is much smaller than the carrier.
Passband signal with carrier $S_c$

$$X_p(t) = A(t) \cos (2\pi f_c t + \phi(t))$$

$$= A(t) \cos (\phi(t)) \cos (2\pi f_c t)$$

$$- A(t) \sin (\phi(t)) \sin (2\pi f_c t)$$

$$\sim \text{inphase} \quad \quad \quad \quad \quad \quad \quad \quad \quad \sim \text{quadrature}$$

Let $X(t) = x_i(t) + j x_q(t)$ \underline{complex envelope}

or $X_p(t)$ \underline{complex baseband}

$$X_p(t) = \text{Re} \{ X(t) e^{j 2\pi f_c t} \}$$

$$= x_i(t) \cos (2\pi f_c t) - x_q(t) \sin (2\pi f_c t)$$
Upconversion

baseband $\rightarrow$ passband

$X_p(t) = x_i(t) \cos(2\pi f_c t) - x_q(t) \sin(2\pi f_c t)$

$X_p(f) = \frac{1}{2} \left( X(f-f_c) + X^*(f+f_c) \right)$

$\Re \{ x(t) e^{j2\pi f_c t} \} \quad \uparrow$

![Diagram]

$|X(f)|$

$|X_p(f)|$
10/1 Downconversion

passband \rightarrow\ baseband

y_p(t) \rightarrow y(t)

y_p(t) = y_i(t) \cos(2\pi f_c t)
- \frac{1}{2} y_q(t) \cos(2\pi f_c t)
- \frac{1}{2} y_q(t) \sin(2\pi f_c t)

\text{carrier @ } f_c

y_p(t) \cos(2\pi f_c t) = \frac{1}{2} y_i(t) - \frac{1}{2} y_q(t) \cos(4\pi f_c t)
- \frac{1}{2} y_i(t) \sin(4\pi f_c t)

\text{carries @ } f_c

y_p(t) \sin(2\pi f_c t) = -\frac{1}{2} y_q(t) - \frac{1}{2} y_q(t) \cos(4\pi f_c t)
- \frac{1}{2} y_i(t) \sin(4\pi f_c t)

\text{carries @ } f_c

\times 2
Downconversion

\[ y(t) \]

\[ y_p(t) \]

\[ \cos(\omega + \Delta \omega t) \]

\[ \sin(\Delta \omega t) \]

\[ y_q(t) \]