Baseband Equivalent Channel

Learning Objective:

Use the complex envelope to represent a passband channel at baseband.
Lecture #6

- Broadband equivalent channel
  \[ x_p(t) \xrightarrow{h(t)} y_p(t) \]

- Linear-time invariant channel
  - Good model linear-time varying system
    - Linearity from propagation medium
      - Time-varying SIC mobility
  - Design parameters of wireless system so LTI assumption is reasonable in short intervals
\[ y_p(t) = \int_{-\infty}^{\infty} h(t - \tau) x_p(\tau) d\tau \]

- Passband
- Not-passband
- Passband

- Find a baseband equivalent channel \( h_e(t) \)

\[ y_e(t) = \int_{-\infty}^{\infty} h_e(t - \tau) x_b(\tau) d\tau \]
\[ y_p(t) = \text{Re} \left\{ (h(t) + x_b(t)) e^{j2\pi fc t} \right\} \]
\[ = \int_{-w}^{w} h_p(t) x_b(t+\tau) \, d\tau \]

Derive \( h_p(t) \) in frequency domain:

\[ Y_p(f) = H(f) X_p(f) \]
\[ = H_p(f) X_p(f) \]

\[ H_p(f) = P(f) H(f) \]

\[ P(f) = \text{rect} \left( \frac{f - \frac{fe}{W}}{\frac{1}{W}} \right) + \text{rect} \left( -\left( \frac{f + \frac{fe}{W}}{\frac{1}{W}} \right) \right) \]
\[ = \text{rect} \left( \frac{f - \frac{fe}{W}}{\frac{1}{W}} \right) + \text{rect} \left( \frac{f + \frac{fe}{W}}{\frac{1}{W}} \right) \]

\[ P(f) \]

\[ w \quad \frac{fe}{W} \]

\[ f \]

\[ 1 \]
\[ y_p(t) = \frac{1}{2} H_e(t-t_0) x_0(t-t_0) + \frac{1}{2} H_e(t-t_0) \tilde{x}_0(t-t_0) \]

\[ = H_p(t) \tilde{x}_p(t) \]

\[ = \left( \frac{1}{2} H_b(t-t_0) + \frac{1}{2} H^*_e(t-t_0) \right) \]

\[ - \left( \frac{1}{2} \tilde{x}_b(t-t_0) + \frac{1}{2} \tilde{x}_e(t-t_0) \right) \]

\[ = x_q H_b(t-t_0) \tilde{x}_b(t-t_0) + y_q H_b(t-t_0) \tilde{x}_b(t-t_0) \]

Let \( H_e(t) = \frac{1}{2} H_b(t) \)

\[ \Rightarrow \frac{1}{2} H_e(t-t_0) \tilde{x}_b(t-t_0) + \frac{1}{2} H_e(t-t_0) \tilde{x}_b(t-t_0) \]
In time domain

\[ h_\text{e}(t) = \frac{1}{2} \cdot 2L_\text{w}(t) \ast \left( (h(t) \ast p(t)) e^{-j2\pi ft} \right) \]

\[ p(t) = 2W \frac{\sin(\pi ft)}{\pi ft} \cos(2\pi ft) \]

- Discrete-time

\[ h[n] = T h_\text{e}(nT) \]

- No bandlimiting \( h_\text{e}(t) \) is required \( \forall c \in \mathbb{L}_\text{w}(t) \)

- Example

100m

attenuates by 0.1
Baseband BW is 5MHz
Carrier is \( f_c = 2G\text{Hz} \)
8/ ① Compute delay

\[ c = 3 \times 10^9 \text{m/s} \]

\[ T_d = \frac{100 \text{m}}{3 \times 10^8 \text{m/s}} = \frac{1}{3} \times 10^{-6} = \frac{1}{3} \mu \text{s} \]

\[ h(t) = 0.1 \delta(t - \frac{1}{3} \times 10^{-6}) \]

② \( h_p(t) = h(t) * p(t) \)

\[ = \int h(t - \tau) p(\tau) d\tau \]

\[ = 0.1 p(t - \frac{1}{3} \times 10^{-6}) \]

\[ = 2 \times 0.1 \times 10^7 \text{sinc} \left( \pi 10^7 (t - \frac{1}{3} \times 10^{-6}) \right) \]

\[ W = 2 \times 5 \text{MHz} = 10 \times 10^6 = 10^7 \text{Hz} \]

\[ h_p(t) = \text{Re} \left\{ 2 \cdot h(t) e^{-j2\pi f_c t} \right\} \]

\[ \Rightarrow h(t) = 0.1 \times 10^7 \text{sinc} \left( \pi 10^7 (t - \frac{1}{3} \times 10^{-6}) \right) e^{-j2\pi f_c t} \]

\[ f_c = \frac{2}{3} \times 10^9 \times \frac{1}{3} \times 10^{-6} = \frac{2}{3} \times 10^3 \]
\[ h\{n\} = T \cdot h_x(nT) \]

Nyquist: \[ T = \frac{1}{\nu} = \frac{1}{\text{bandwidth}} = 10^{-7} \]

\[ h\{n\} = 0.1 \times 10^{-7} \times 10^7 \times \text{sinc}\left(\frac{\pi}{10^7}\left(10^{-7} - \frac{1}{2} \times 10^{-9}\right)\right) e^{j2\pi \frac{2}{3} \times 10^{-9}} \]

\[ = 0.1 \text{sinc}(\pi n - 10^{-3} \pi) e^{-j2\pi \frac{2}{3} \times 10^{-9}} \]
Intermission

What are the killer applications for device-to-device communication?
Linear Complex Amplitude Modulation Techniques

Learning Objectives:

Explain the concept of linear complex pulse amplitude modulation and relevant notation.
Linear complex pulse amplitude modulation

\[ x(t) = \sqrt{E_x} \sum_{n=-\infty}^{\infty} s(n) g_{tx}(t-nT) \]

*Baseband signal*

\( E_x \) is transmit energy

\( T \) is symbol period

\( Y T \) is symbol rate

\( s(n) \) is complex symbol at time \( n \)

\( s(n) \in \mathcal{C} = \{ s_0, \ldots, s_{M-1} \} \quad |E| = M \)

*Constellation* \( \uparrow \) *Symbols*

\( g_{tx}(t) \) is pulse shape \( @ T X \)

\( M \) symbols \( M = 2^6 \leftarrow \text{bits per symbol} \)

![Diagram showing the process from bits to signal](image)
Constellations

Learning Objectives:

Draw a constellation diagram including bit labels.

Calculate the energy of a constellation and normalize it.
Constellations

- $\mathcal{C} = \{ s_0, s_1, \ldots, s_{M-1} \}$

- BPSK: $\mathcal{C} = \{ 1, -1 \}$, $M = 2$

- 4-QAM, 8-PSK: $\mathcal{C} = \{ 1, i, -1, -i \}$

- $M$-PAM: $M$ is power of 2
  - 4-PAM
  - 3-PAM

- $M$-QAM: $M$ is power of 4
  - 4, 16, 64, $\left( \frac{M}{2} \text{-PAM} \right) \times \left( \frac{M}{2} \text{-PAM} \right)$

- $\mathbb{C}$: Real $\rightarrow$ Imag
  - $3 + 3i$
Figure 17: Several different constellations with generally accepted bit to symbol mappings based on Gray labeling.

causes problems with transmission and reception.