Wireless Communications Lab

Lecture #9

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Pulse Shaping at the Transmitter

- **Bits** → Symbol Mapping → Create Pulse Train → \( g_{TX}(t) \) → (Gain) → \( x(t) \)
- Pulse Shaping Filter (to create bandlimited signal)

\[ g_{TX}(n) = g(nT_x/L) \]

**Diagram 1:**
- \( s[n] \) → \( \uparrow L \) → \( g_{TX}[n] \) → D/C → \( \sqrt{E_x} \) → \( x(t) \)

**Diagram 2:**
- \( s[n] \) → \( g_{TX}^{(0)}[n] \) → \( \uparrow L \) → D/C → \( \sqrt{E_x} \) → \( x(t) \)
- \( g_{TX}^{(i-1)}[n] \) → \( \uparrow L \) → D/C → \( \sqrt{E_x} \) → \( x(t) \)
Pulse Shaping at the Receiver

Learning Objectives:

Implement discrete-time pulse shaping at the receiver using oversampling combined with downsampling.
Pulse shaping at RX

- Implement discrete-time processing of a CT signal

\[
\begin{align*}
Z(t) &\rightarrow g_{md}(t) \rightarrow C/T \rightarrow \text{Detection} \rightarrow (\text{sym})^{-1} \rightarrow \hat{b}/b
\end{align*}
\]

- \( Z(t) \) is bandlimited
- \( g_{rx}[n] = T_{z} g_{rx}(nT_{z}) = g_{rx}(nT_{z}) \)
- Select \( T_{z} \) s.t. Nyquist is satisfied
- Take \( T_{z} = T/M \) \( M \in \mathbb{Z}^+ \) s.t. Nyquist satisfied
  \[ \frac{T}{T_{z}} > \text{Nyquist}, \text{ called oversampling} \]
\[ y[n] \rightarrow D/L \rightarrow \lfloor \frac{n}{M} \rfloor \rightarrow y[nM] \]

\[ T_2 \quad T = MT_2 \]

\[ y[n] \rightarrow \lfloor nM \rfloor \rightarrow y[nM] \quad \text{downsampling} \]

\[ Y(e^{j\omega}) = \frac{1}{M} \sum_{m=-\infty}^{\infty} Z(e^{j\omega M - j2\pi M/2}) \]

\[ z[n] \rightarrow D/L \rightarrow \lfloor g_x[n] \rfloor \rightarrow \lfloor n \rfloor \rightarrow y[n] \]

\[ y[n] = \sum \delta[nM - e] \]

\[ g_x[n] = \sum \delta[nM - e] \]
Intermission

Exam Related Questions?
Maximum Likelihood Detection

Learning Objectives:

Derive the ML detector for the AWGN channel.

Implement the ML detection algorithm.
ML Detector

- Use matched filter + composite pulse shape satisfies Nyquist
  \[ y[n] = \sqrt{E_x} \sum_{m} s[m] g((n-m)T) + \eta[n] \]
  \[ = \sqrt{E_x} s[n] + \eta[n] \]
  \( \eta[n] \) i.i.d. \( \mathcal{N}(0, \sigma^2) \)

- Detection problem:
  Given an observation of \( y[n] \)
  find "best" guess of \( s[n] \) (call \( \hat{s}[n] \))
Maximum likelihood detection rule that maximizes the likelihood of the observation under hypothesis a given symbol was transmitted.

- Likelihood function

$$f_{ys} (y|s|/s_n = s)$$

$$y_n = \sqrt{\xi + s_n} + v \frac{s}{n}$$

$$f_{ys} (y|s) = \frac{1}{\sqrt{\pi} \sigma_0} e^{-\frac{(y - \sqrt{\xi} s)^2}{\sigma_0^2}}$$

b/c $V(s)$ is iid complex Gaussian
ML detector solve optimization

\[ s_{\text{ML}} = \arg \max_{s} \mathbf{P}_{\text{sys}} (y(n) | s(n) = s) \]
\[ = \arg \max_{s} \frac{1}{n \nu_0} e^{-\frac{|y(n) - \mathbf{V}_\text{ex} s|^2}{\nu_0}} \]
\[ = \arg \max_{s} e^{-\frac{|y(n) - \mathbf{V}_\text{ex} s|^2}{\nu_0}} \]
\[ = \arg \min_{s} |y(n) - \mathbf{V}_\text{ex} s|^2 \]

Minimum Euclidean distance

\( \sqrt{\mathbf{E}_{\text{H}}(\mathbf{y}, \mathbf{y})} \)  \( \sqrt{\mathbf{E}_{\text{H}}(\mathbf{y}, \mathbf{y})} \)

Scaled constellation
Voronoi region:

\[ V_s = \{ y : |y - \text{Vexs}|^2 < |y - \text{Vexc}|^2 \} \]

1. Voronoi used for UQAM
2. Symbol error rate
3. Simplify detection algorithm
Probability of Symbol Error Analysis

Learning Objectives:

Derive the union bound on the probability of symbol error for the ML detector in an AWGN channel.

Calculate the probability of symbol error for QAM using both the union bound and the exact formula.
Probability of symbol error

- Probability of error
  - Closed form in some cases, not others
- Equally likely symbols for AWGN

\[
P_e\left(\frac{E_b}{W_0}\right) = \frac{1}{M} \sum_{m=0}^{M-1} P_e(\frac{E_b}{W_0})
\]

\[
P_e(\frac{E_b}{W_0}) = P_r\{S_m \text{ detected incorrectly } | S_m \text{ was transmitted}\}
\]

\[
= \sum_{e=0}^{M-1} P_r\{S_m \text{ decided as } S_e | S_m \text{ was transmitted}\}
\]
Use pairwise error probability (PEP)

\[ \Pr \{ S_m \rightarrow S_e \} \]

- Probability \( S_m \) is decoded as \( S_e \) assuming \( C = \{ S_m, S_e \} \)

\[ \Pr \{ S_m \rightarrow S_e \} = \min \left( \sqrt{\frac{E_b}{N_0}} \frac{11 \cdot S_m - S_e}{2} \right) \]
\[
\Phi_{\text{elsm}}(\frac{E_x}{\text{No}}) \leq \frac{M-1}{e^{2\gamma}} \text{ for } e \leq m \\
\Phi_{\text{el}}(\frac{E_x}{\text{No}}) \leq \frac{1}{M} \sum_{m=2}^{M-1} \frac{M-1}{e^{2\gamma}} \sqrt{1 - \left(\frac{E_x}{\text{No}}\right)^2} \\
\text{Define } d_{\text{min}} = \min_{m \neq n} 1 - s_{m-n}^2 \\
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} dt \\
\Phi(\frac{E_x}{\text{No}}) \leq \frac{1}{M} \sum_{m=2}^{M-1} (M-1) \Phi\left(\sqrt{1 - \left(\frac{E_x}{\text{No}}\right)^2}\right) \\
= (M-1) \Phi\left(\sqrt{\frac{E_x}{\text{No}}}\right) \frac{d_{\text{min}}}{\text{No}^2} \\
\text{Union bound (close at low SNR)}
\[ \dimin^2 = \frac{6}{M-1} \]
\[ P_e (\frac{\varepsilon}{N_0}) \leq (M-1) \alpha (\sqrt{\frac{\varepsilon}{N_0}} \frac{3}{M-1}) \]

Exact
\[ P_e (\frac{\varepsilon}{N_0}) = 4 (1 - \frac{1}{\sqrt{M}}) \alpha (\sqrt{\frac{\varepsilon}{N_0}} \frac{3}{M-1}) \]
\[- \quad 4 (1 - \frac{1}{\sqrt{M}})^2 (\alpha \sqrt{\frac{\varepsilon}{N_0}} \frac{3}{M-1})^2 \]
\[ o (1) \leq \frac{1}{2} e^{-x^{2/2}} \]