Lecture #14

EE 471C / EE 381K-17 Wireless Communication Lab

Professor Robert W. Heath Jr.
From previous lectures

- Frequency selective channel model

- The channel includes all **frequency selective effects**
  - Pulse-shaping filter
  - Transmit signal scaling
  - Propagation effects
  - Receive filter

- **Previous lectures**: channel estimation and equalization
- **This lecture**: frequency offset correction and frame synch
Preview of today’s lecture

- Carrier frequency offset correction in frequency-selective channels
  - Model for frequency offset in frequency-selective channels
  - Application of single frequency estimation

- Frequency offset estimation and frame synchronization using periodic training for single carrier systems

- Frequency offset estimation and frame synchronization using periodic training for OFDM
Including carrier frequency offset

- Carrier frequency offset multiplies the matched filtered received signal by $e^{j2\pi f_e t}$

$$y(t) = e^{j2\pi f_e t} \int r(t - \tau) g_{rx}(\tau) \, d\tau.$$  

- Sampling at the symbol rate and using the FIR channel model

$$\epsilon = f_e T$$

$$y[n] = e^{j2\pi \epsilon n} \sum_{\ell=0}^{L} h[\ell] s[n - \ell] + v[n]$$

- Correction of frequency offset
  - Estimate $\epsilon$ and derotate the received signals

$$e^{-j2\pi \epsilon n} y[n]$$
Frequency offset estimation using a sinusoid

- Suppose that frame synchronization has been solved
- Choose as a training sequence a complex sinusoid

\[ t[n] = \exp(j2\pi f_t n) \text{ for } n = 0, 1, \ldots, N_{tr} - 1 \]

- The received signal during training is

\[ y[n] = e^{j2\pi \epsilon n} e^{j2\pi f_t n} h + v[n] \]

- Removing the known offset

\[ e^{-j2\pi f_t n} y[n] = e^{j2\pi \epsilon n} h + e^{-j2\pi f_t n} v[n] \]

An example that is relevant also for frequency selective channels since sinusoids are eigenfunctions of LTI systems

This becomes a classic problem of estimating a complex sinusoid with unknown amplitude, frequency and phase (amplitude and phase from the channel)
How does this work for frequency selective channels?

- Now the received signal is

\[ y[n] = e^{j2\pi \epsilon n} \sum_{\ell=0}^{L} h[\ell] t[n - \ell] + v[n] \quad n = L, L + 1, \ldots, N_{tr} - 1 \]

- For the exponential training signal \( t[n] = \exp(j2\pi f_t n) \)

\[ y[n] = e^{j2\pi \epsilon n} \sum_{\ell=0}^{L} h[\ell] e^{j2\pi f_t (n - \ell)} + v[n] \]

\[ = e^{j2\pi \epsilon n} e^{j2\pi f_t n} \sum_{\ell=0}^{L} h[\ell] e^{-j2\pi f_t \ell} + v[n] \]

\[ = e^{j2\pi \epsilon n} e^{j2\pi f_t n} h(e^{j2\pi f_t}) + v[n] \]

Discarding samples that do not depend on the training (remove edge effects)

Rotation by the known freq. of the training sinusoid
Simple estimator for frequency selective channels

- Derotate by the frequency of the training signal

\[ e^{-j2\pi f_t n} y[n] = e^{j2\pi \epsilon n} h(e^{j2\pi f_t}) + e^{-j2\pi f_t} v[n] \]

Same form as in the flat channel case!

- The same estimator can be applied to frequency selective channels!
- This approach was taken in the GSM standard

Transmission of a sinusoid makes frequency offset estimation easy, exploiting the eigenfunction property of the channel
Revisiting single frequency estimation

Learning objectives

- Extend simple frequency offset estimator for flat channels to frequency selective channels
Frequency offset estimation using periodic training

Learning objectives

- Use the Moose method to perform joint frequency offset estimation and frame synchronization
Frequency offset estimation using periodic training

◆ Focus on a tricky approach for estimating carrier frequency offset using periodic structure in the transmitted signal


◆ This approach was described for OFDM but applies more generally to single carrier as well

◆ Apply a more sophisticated variation to OFDM later
Periodic training structure

\[ L \leq n \leq N_t - 1 \]

- The Moose algorithm exploits periodicity in the training sequence.
- Let training start at \( n=0 \)
  \[
s[n] = s[n + N_{tr}] = t[n] \quad n = 0, 1, \ldots, N_{tr} - 1
\]
- For \( L \leq n \leq N_{tr}-1 \)
  \[
y[n] = e^{j2\pi \epsilon n} \sum_{\ell=0}^{L} h[\ell] s[n - \ell] + v[n]
\]
  \[
y[n + N_{tr}] = e^{j2\pi \epsilon (n + N_{tr})} \sum_{\ell=0}^{L} h[\ell] s[n + N_{tr} - \ell] + v[n + N_{tr}] \]
Exploiting the periodicity

- **Note that**
  \[ s[n + N_{tr}] = s[n] = t[n] \text{ for } n = 0, 1, \ldots, N_{tr} - 1 \]

\[
y[n + N_{tr}] = e^{j2\pi n} e^{j2\pi n} \sum_{\ell=0}^{L} h[\ell] t[n - \ell] + v[n + N_{tr}]
\approx e^{j2\pi n} y[n].
\]

It does not depend on the unknown channel coefficients

- Least squares is one solution but \( \epsilon \) in the exponent requires a nonlinear least squares solution
Relaxing the offset problem

- A solution is to consider a relaxed problem

\[ y[n + N_{tr}] = ay[n] \]

- Solving for the LS estimate of \( a \) and find the offset from the phase

- This problem is similar to flat fading channel estimation
Frequency offset estimate from relaxed solution

- Simple frequency offset estimator

\[
\hat{\epsilon}_{LS} = \frac{1}{2\pi N_{tr}} \text{phase} \left( \sum_{n=0}^{N_{tr}-1} y^*[n]y[n + N_{tr}] \right) / \left( \sum_{n=0}^{N_{tr}-1} y^*[n]y[n] \right)
\]

where phase denotes the principle phase of the argument

- This is also the maximum likelihood estimate

Neglect the denominator since it does not contribute to the phase
Estimator range depends on the training length

- The estimate of $\epsilon$ will only be accurate for $|\epsilon N_{tr}| \leq \frac{1}{2}$ or equivalently
  $$|\epsilon| \leq \frac{1}{2N_{tr}}, \quad \text{or} \quad |f_e| \leq \frac{1}{2TN_{tr}}$$

  - Choosing larger $N_{tr}$ improves the estimate but reduces the range of offsets that can be corrected
  - Choosing smaller $N_{tr}$ improves the range but is more susceptible to noise

- Can use multiple shorter periods to achieve high range
  - Use multiple repetitions of a short training sequence

- IEEE 802.11a/n/ac and related standards use a combination of both repeated short training sequences and long training sequences

- LTE / 5G use a single sequence
**Example**

- **Compute the maximum allowable offset for a 1Ms/s QAM signal, with** \( f_c = 2 \text{ GHz} \), and \( N_{tr} = 10 \)

\[
\max |f_e| = \frac{1}{2TN_t} = \frac{1}{2} \times 10^6 \times \frac{1}{10} = \frac{1}{2} \times 10^5 = 50\text{kHz}.
\]

In terms of parts per million, we need an oscillator that can generate a 2 GHz carrier with an accuracy of \( 50e3/2e9 = 25\text{ppm} \).
Example

Consider a wireless system where each data frame is preceded by two training blocks each consisting of $N_{tr} = 12$ training symbols. Let the symbol period be $T = 4 \mu s$. What is the maximum frequency offset that can be corrected using training?
Frame synchronization

- A side benefit of the Moose algorithm is that it also provides a nice way of performing frame synchronization.
- The correlation peak should occur when the pair of training sequences is encountered at the receiver.
- Solve for the offset $d$ such that

$$\hat{d} = \arg \max \left| \frac{\sum_{n=L}^{N_{\text{tr}}-1} y[n + \Delta + N_{\text{tr}}] y^*[n + \Delta]}{\left( \sum_{n=L}^{N_{\text{tr}}-1} |y[n + \Delta]|^2 \right)^2} \right|^2$$

Example 18: Compute the maximum allowable offset for a $1$-MsQAM signal, with $f_c = 2$ GHz, and $N_{\text{tr}} = 10$.

Answer: The maximum frequency offset that can be corrected using training is

$$\max |f_e| \leq \frac{1}{2} \sqrt{10^6 \times 10^{10}} \approx 10^{5.5} = 50 kHz.$$
Complete QAM receiver

No matter which algorithm is used, both result in the same thing: a joint solution to the frame synchronization and frequency offset estimation and correction problem in an intersymbol interference channel.

Channel estimation is also facilitated using the Moose algorithm. Once the frequency offset is estimated and corrected, and the frame is synchronized, the pair of training sequences can be combined together for channel estimation. As a result, periodic training provides a flexible approach for solving key receiver functions in frequency-selective channels.

An illustration of the complete receiver can be found in Fig. 26. As expected, the frequency offset estimation and correction is performed prior to channel estimation and equalization but after the downsampling operation.

We conclude this section with a detailed example which describes the structure in the preamble for IEEE 802.15.3c single carrier mode.

Example 20
Consider the IEEE 802.15.3c preamble structure as described in Example 16 for the high rate mode of the SC-PHY. In this example we describe the role of the SYNC and SFD fields on carrier frequency offset estimation and frame synchronization.
Example

Consider the IEEE 802.15.3c preamble structure for the high rate mode of the SC-PHY. Explain how to use the SYNC and SFD fields for carrier frequency offset estimation and frame synchronization.

\[ \{a[n]\}_{n=0}^{N_g-1} \text{ and } \{b[n]\}_{n=0}^{N_g-1} \]

Golay complementary sequences
Using the SYNC field for synchronization

**Frame Detection (SYNC)**

14 repetitions of $a_{128}$.

- The maximum supported channel length is 128.
- Frame detection algorithm

$$
\hat{d} = \arg \max \sum_{p=1}^{13} \frac{\sum_{n=0}^{127} y[n + \Delta + 128p + 128] y^*[n + \Delta + 128p]}{\left( \sum_{n=0}^{127} |y[n + \Delta + 128 + 128p]|^2 \right)^2}
$$

First repetition of the SYNC acts as cyclic prefix.

- Frequency offset estimator

$$
\hat{\epsilon} = \frac{1}{2\pi 128} \sum_{p=1}^{13} \text{phase} \left( \sum_{n=0}^{127} y[n + \hat{d} + 128p + 128] y^*[n + \hat{d} + 128p] \right)
$$
Frequency offset estimation using periodic training

Learning objectives

- Use the Moose method to perform joint frequency offset estimation and frame synchronization
UT SAVES: Situation-Aware Vehicular Engineering Systems

COMMUNICATION
Communicating high rate sensor data, using sensing to make communication more efficient

SENSING
Fusing sensor data, discovering the relevance of data to safety, traffic, management

MACHINE LEARNING
Exchanging processed data, make decisions, enhance automated driving
Back to the main event
Frequency offset estimation in OFDM

Learning objectives

- Describe and implement carrier frequency offset estimation for an OFDM system
OFDM transmitter

The SC-FDE receiver illustrated in (20) performs a DFT on a portion of the received signal, equalizes with the DFT of the channel, and takes the IDFT to form the equalized sequence \( \hat{s}_r^n \). This offloads the primary equalization operations to the receiver.

In some cases, however, it is of interest to have a more balanced load between transmitter and receiver. A solution is to shift the IDFT to the transmitter. This results in a framework known as multicarrier modulation or OFDM modulation [35] [36].

Several wireless standards have adopted OFDM modulation including wireless LAN standards like IEEE 802.11a/b/n/ac/ad [37, 38], fourth generation cellular systems like 3GPP LTE [39–42], digital audio broadcasting (DAB) [43], and digital video broadcasting (DVB) [44, 45].

In this section, we describe the key operations of OFDM at the transmitter as illustrated in Fig. 21 and receiver as illustrated in Fig. 22. We present OFDM from the perspective of having already derived SC-FDE, though historically OFDM was developed several decades prior to SC-FDE. We conclude with a discussion on OFDM versus SC-FDE versus linear equalization techniques.
OFDM receiver

\[ r(t) \rightarrow \text{C/D} \rightarrow g_{rx}[n] \]

\[ \frac{T}{M} \]

\[ z^\hat{m} \]

\[ \hat{m} \]

\[ \text{Symbol Sync} \]

\[ z^\hat{d} \]

\[ \hat{d} \]

\[ \text{Frame Sync} \]

\[ \text{Serial to Parallel} \]

\[ 1: N + L_c \]

\[ L_c \]

\[ H^{-1}[0] \]

\[ H^{-1}[N-1] \]

\[ \text{DFT} \]

\[ \text{Channel Estimation} \]

\[ \text{DFT} \]

\[ \hat{s}[n] \]

\[ \text{QAM Detection} \]

\[ \text{Parallel to Serial} \]

\[ N:1 \]

\[ \text{Frame Sync} \]

\[ \text{Serial to Parallel} \]

\[ 1: N + L_c \]

\[ \text{DFT} \]

\[ \hat{d} \]

\[ \text{Symbol Sync} \]

\[ \hat{m} \]

\[ \frac{T}{M} \]

\[ g_{rx}[n] \]

\[ \text{C/D} \]

\[ r(t) \]

\[ \text{OFDM receiver} \]

\[ y_r[n] = h_{rs}[n]s[n] \]

\[ N \]

\[ L_c \]

\[ H^{-1}[0] \]

\[ H^{-1}[N-1] \]

\[ \text{DFT} \]

\[ \hat{s}[n] \]

\[ \text{QAM Detection} \]

\[ \text{Parallel to Serial} \]

\[ N:1 \]

\[ \text{Frame Sync} \]

\[ \text{Serial to Parallel} \]

\[ 1: N + L_c \]

\[ \text{DFT} \]

\[ \hat{d} \]

\[ \text{Symbol Sync} \]

\[ \hat{m} \]

\[ \frac{T}{M} \]

\[ g_{rx}[n] \]

\[ \text{C/D} \]

\[ r(t) \]
Motivation for the Schmidl and Cox algorithm

- Moose’s method can be used directly provided that two OFDM symbols are repeated

- It is of interest to develop methods that can work with only a single OFDM symbol

- Modification proposed by Schmidl and Cox
  - Create periodicity in one OFDM symbol instead of repeating multiple OFDM symbols
  - Use a second OFDM symbol to enhance correction range

Creating periodicity through zeros

- As an example consider turning off the odd subcarriers

\[ w[n] = \frac{1}{N} \sum_{m=0}^{N-1} s[2m]e^{j\frac{2\pi 2m(n-L_c)}{N}} = \frac{1}{N} \sum_{m=0}^{N-1} s[2m]e^{j\frac{2\pi m(n-L_c)}{N/2}} \]

\[ \Rightarrow \text{for } n \geq L_c \text{ and } n < L_c + \frac{N}{2}, \, w[n] = w\left[n + \frac{N}{2}\right] \]

- This means that the OFDM signal contains a portion which is periodic

\[ w[n + L_c] = w[n + L_c + N/2] \quad \text{for } n = 0, 1, \ldots, N/2 - 1 \]

Apply Moose algorithm

Extends to turn off \( K-1 \) carriers

periodicity of \( N/K \)
Exploiting periodicity for offset estimation

◆ Consider the received signal

\[ y[n] = e^{j 2 \pi \epsilon n} \sum_{\ell=0}^{L} h[\ell] w[n - \ell] + v[n] \]

◆ Let \( \bar{y}[n] = y[n + L_c] \). Because of the periodicity, in the absence of noise

\[ \bar{y}[n + N/2] = e^{j 2 \pi \epsilon N/2} \bar{y}[n] \]

◆ Simple frequency offset estimator follows as before

\[ \hat{\epsilon}_{LS} = \frac{1}{2 \pi N_{tr}} \text{phase} \left( \sum_{n=L}^{N/2-1} \bar{y}^*[n] \bar{y}[n + N] \right) \]

Performs OFDM symbol synchronization and frequency offset estimation jointly
Frequency offset range

- Maximum correctable offset is up to one subcarrier width
  \[ |\epsilon| \leq \frac{1}{2N/2} = \frac{1}{N} \]

  - Fine offset correction

  - Unfortunately, in an OFDM system \( N \) is quite large

- Decompose the offset as
  \[ \epsilon = m \frac{1}{N/2} + \epsilon_f \]

  - Integer offset
  - Fine offset

  - Estimate using correlation

- Can estimate the fine offset using the correlation as described
Selecting a second training symbol

- Let $\{t_1[n]\}_{n=0}^{N/2-1}$ and $\{t_2[n]\}_{n=0}^{N/2-1}$ two sequences of QPSK coefficients

- $\{t_1[n]\}_{n=0}^{N/2-1}$ forms a pseudonoise (PN) sequence which is transmitted on the even subcarriers for the first training symbol

- **Zeros are transmitted on the odd subcarriers for the first training symbol**

- The two halves of the first training symbol are identical

- The even coefficients for the second training symbol are

  $$t_2[2n]t^*[2n] = t_3[2n]$$

- A PN sequence transmitted on the even subcarriers to help determine the integer frequency offset

- Odd subcarriers could contain other training data
Rewriting and leveraging the cyclic shift property

\[ \epsilon = \frac{2q}{N} + \epsilon_{\text{frac}} \]

\[ e^{j2\pi\epsilon N/2} = e^{j2\pi \left( \frac{2q}{N} + \epsilon_{\text{frac}} \right) \frac{N}{2}} = e^{j2\pi \left( \frac{qN^2}{N^2} + \epsilon_{\text{frac}} \right) \frac{N}{2}} = e^{j2\pi \epsilon_{\text{frac}} \frac{N}{2}}, \]

- For a given symbol, with a larger frequency offset, after fine offset correction
  \[ y[n] = e^{j2\pi \frac{2q}{N} n} \sum_{\ell=0}^{L} h[\ell] w[n - \ell] + v[n]. \]

- In the frequency domain
  \[ y[k] = e^{-j2\pi \frac{2q}{N} L} h[((k - 2q)_N)] s[((k - 2q)_N)] + \tilde{v}[k] \]
Estimating the integer ambiguity

- Let \( y_1[k] \) and \( y_2[k] \) the DFTs of the first and second received training symbols after fine offset correction

\[
y_1[k] = e^{-j2\pi \frac{2q}{N} L_c} h[ ((k - 2q)_N] t_1[(k - 2q)_N] + \tilde{v}[k]
\]

\[
y_2[k] = e^{-j2\pi \frac{2q}{N} L_c} h[ ((k - 2q)_N] t_2[(k - 2q)_N] + \tilde{v}[k + N + L_c]
\]

- Consider the product

\[
y_2[2k] y_1^*[2k] = |h[((2k - 2q)_N]|^2 t_2[((2k - 2q)_N] t_1^*[((2k - 2q)_N] + \tilde{v}'[k]
\]

exploit differential encoding

\[
= |h[((k - 2q)_N]|^2 t_3[((2k - 2q)_N] + \tilde{v}'[k]
\]

- Solve LS problem

\[
\hat{q} = \max_{p=0,1,...,N/2-1} \frac{\sum_{k=0}^{N/2-1} y_2[2k + 2p] y_1^*[2k + 2p] t_3^*[2k + 2p] \bigg|}{2 \bigg( \sum_{k=0}^{N/2-1} |y_2[2k]|^2 \bigg)^2}
\]
Schmidl-Cox algorithm

- Two-step carrier frequency offset estimation
- Fine correction stage estimates the fractional offset
- Coarse correction stage estimates the integer offset
- The final frequency offset estimate is
  \[ \epsilon = \frac{2\hat{q}}{N} + \hat{\epsilon}_{\text{frac}}. \]
- The main benefits over Moose’s method is that a large range of offsets can be corrected.
OFDM receiver with frequency offset correction

\[ \frac{T}{M_{\text{rx}}} \]

\[ g_{\text{RX}}[n] \]

\[ \downarrow M_{\text{rx}} \]

Sample Timing

Frequency Offset Correction

Symbol Timing Correction

Symbol Timing

Remove CP

Serial to Parallel 1:N

\[ \cdot \cdot \cdot \]

FFT

\[ \frac{N}{N} \]

EQ

QAM Detection

Demapping

Channel Estimation

Figure 27: OFDM receiver frequency offset estimation and correction, channel estimation, and linear equalization.
Frequency offset estimation in OFDM

Learning objectives

- Describe and implement carrier frequency offset estimation for an OFDM system