A Lattice-Based MIMO Broadcast Precoder with Block Diagonalization for Multi-Stream Transmission

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***Abstract*— Precoding with block diagonalization is an attractive approach for approaching sum capacity in multiuser MIMO (multiple input multiple output) broadcast channels. This method though requires either global channel state information at every receiver or an additional training phase, which requires additional system planning. This paper proposes a lattice based multi-user precoder that uses block diagonalization combined with pre-equalization and perturbation for the multiuser MIMO broadcast channel. Achievable rates are computed and used to show that the proposed technique approaches the capacity with block diagonalization and water-filling but does not require the additional channel state information at the receiver. Monte Carlo simulations under the case of equal power allocation show that the proposed method provides better diversity and BER (bit error rate) performance than block diagonalization with a zero-forcing receiver.**

I. INTRODUCTION

Recent information theoretic work on multiple-input multiple-output (MIMO) communications has shown that the sum capacity, the maximum sum rate in the broadcast channel, is achieved by dirty paper coding (DPC) [1]–[3]. The key idea of DPC is to precancel interference at the transmitter using perfect channel state information (CSI) and complete knowledge of the transmitted signals. DPC, while theoretically optimal, is an information theoretic concept that has proven to be difficult to implement in practice. Several practical near-DPC techniques based on the concept of precoding have been proposed that offer different tradeoffs between complexity and performance [4]–[12]. All these approaches assume at a minimum complete and perfect CSI at the transmitter and we make this same assumption in this paper.

One of the simplest approaches for multiuser precoding is to premultiply the transmitted signal by a suitably normalized zero-forcing (ZF) or minimum mean squared error (MMSE) inverse of the multiuser matrix channel [4] and [5]. As shown in the analysis [4], however, the gap of the sum rate between DPC and this form of linear precoding is quite large due to the transmit power enhancement. A means of avoiding this problem is to use non-linear precoding, or lattice precoding [6]–[9], where a modulo operation is used to reduce transmit power enhancement. Tomlinson-Harashima MIMO precoding is one example of transmit precoding with a modulo operation [6]. Another example is vector perturbation where the transmit signal vector is perturbed by another vector to minimize transmit power from the extended constellation [7]. An alternative to implementing DPC is block diagonalization (BD), which supports multiple stream transmission [10]–[11]. The basic concept of BD is to use special transmit beamforming vectors that ensure zero interference between users. The resulting multiuser MIMO channel matrix has a block diagonal form, thus the origin of the name. Unlike the aforementioned inverse techniques, BD still requires equalization at the receiver but does not suffer from as much noise enhancement. The main challenge with BD is that unlike inverse or nonlinear methods, either global CSI is required at all the receivers (obtained through an iterative update for example [11]) or an additional training phase is needed so that each user can estimate their equivalent channel and perform detection.

In this paper, we present a lattice-based non-linear precoding scheme that supports multiple stream transmission in a multiuser broadcast channel. We exploit the BD linear precoding algorithm to transmit interference free groups of data to different users. To avoid the need for a complex receiver, however, we further use a ZF prefilter combined with a multi-stream vector perturbation to avoid the corresponding power enhancement. The main features of our approach is that we do not require global CSI or the additional receiver training phase and our approach has much lower receiver complexity, at the expense of additional transmit complexity over BD. We describe the proposed algorithm and present the achievable rate of our system under the assumption of an optimal perturbation. In our numerical results we show that the resulting rate is equivalent to that of BD combined with water-filling. We also compare the proposed algorithm with BD using equal power allocation [11]. We find that our approach has similar diversity performance as BD with an ML receiver and much better performance than BD with a ZF

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receiver. Thus from both a rate and diversity perspective, our approach achieves similar performance as BD with an optimal receiver but with much lower receiver complexity. This may be a particular advantage in multiuser systems with low-cost low-power mobile users.

This paper is organized as follows. In Section II we begin with the system model which includes multiple stream and multi-user. We also describe the BD algorithm and provide some discussion about the limitation of the BD in Section II. In Section III we propose and describe the lattice-based precoding with the BD algorithm to support multiple stream transmission. We present numerical results and conclude in Section IV and V.

II. Broadcast MIMO System with Block Diagonalization

In this section we discuss the narrow-band broadcast channel model and signal model under consideration. Then we discuss block diagonalization and its limitations.

A. MIMO Broadcast Signal Model

Consider the MIMO broadcast signal model with K users each employing N_R receive antennas and each receiving their own data stream manipulated by a precoder at the base station with N_T antennas. We assume that the channel is flat fading and for the purpose of simulations the elements of each user channel matrix are independent complex Gaussian random variables with zero mean and unit variance. Such a narrow-band flat fading model is reasonable in future MIMO systems, for example, orthogonal frequency division multiplexing (OFDM) modulations, however, we defer detail discussion of OFDM to future works. Let $\mathbf{x}_k$, $\mathbf{H}_k$, and $\mathbf{n}_k$ denote the k^{th} transmit signal vector, the channel from base station to user k, and thermal noise, respectively. The noise $\mathbf{n}_k$ also represents additive white Gaussian noise with the variance of σ_n^2. In a broadcast channel, since the interference of the other users propagate in the same channel as the desired user does, the received signal at the k^{th} mobile station is thus

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{M}_k\mathbf{x}_k + \mathbf{H}_k\sum_{l=1,l\neq k}^{K}\mathbf{M}_l\mathbf{x}_l + \mathbf{n}_k, \qquad (1)$$

where $\mathbf{M}_l$ denotes the precoder for the l^{th} user [10]–[11].

Assuming all mobile stations have the same number of antenna N_R and transmit $L_k(\leq N_R)$ numbers of streams, the base station requires that the number of transmit antenna, N_T, is greater than and equal to $\sum_{k=1}^{K} L_k$ to satisfy dimensionality constraints [11].

B. Block Diagonalization

In [11], the authors choose $\mathbf{M}_k$ to be in the null space of $\mathbf{H}_l$ $(\forall l \neq k)$, that is, $\mathbf{H}_l\mathbf{M}_k = \mathbf{0}$ for $l = 1, \cdots, k-1, k+1, \cdots, K$. If we define $\mathbf{H}_{\tilde{k}}$ as

$$\tilde{\mathbf{H}}_k = \left[\mathbf{H}_1^T \cdots \mathbf{H}_{k-1}^T \quad \mathbf{H}_{k+1}^T \cdots \mathbf{H}_K^T\right]^T, \qquad (2)$$

then $\mathbf{M}_k$ can be obtained by calculating the null space of $\tilde{\mathbf{H}}_k$. Let us define the SVD of $\tilde{\mathbf{H}}_k$ as

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k\tilde{\mathbf{\Lambda}}_k\left[\tilde{\mathbf{V}}_k^{(1)} \quad \tilde{\mathbf{V}}_k^{(0)}\right]^H, \qquad (3)$$

where $\tilde{\mathbf{U}}_k$ and $\tilde{\mathbf{\Lambda}}_k$ denote the left singular matrix and the singular matrix of $\tilde{\mathbf{H}}_k$, respectively. Also, $\tilde{\mathbf{V}}_k^{(1)}$ and $\tilde{\mathbf{V}}_k^{(0)}$ denote the right singular matrices each consisting of the singular vectors corresponding to non-zero singular values and zero singular values, respectively. Especially for $\tilde{\mathbf{V}}_k^{(0)}$, $\mathbf{H}_l\tilde{\mathbf{V}}_k^{(0)} = \mathbf{0}$ $(\forall l \neq k)$. Here, $(\cdot)^H$ means the Hermitian transpose of $(\cdot)$. Assuming the dimensionality constraint is satisfied for all users, when we construct $\mathbf{M}_k$ with the columns of $\tilde{\mathbf{V}}_k^{(0)}$ which satisfies that span$(\mathbf{M}_k) \subset$ span$(\tilde{\mathbf{V}}_k^{(0)})$, the k^{th} user transmit one's own data without interfering any other users' data [11].

After removing the interference from other users by using the precoder $\mathbf{M}_k$, we obtain the received signal of the k^{th} mobile station, $\mathbf{y}_k$ given by

$$\mathbf{y}_k = \mathbf{H}_{eff,k}\mathbf{x}_k + \mathbf{n}_k, \qquad (4)$$

where $\mathbf{H}_{eff,k}$ denotes the effective channel of the k^{th} user and $\mathbf{H}_{eff,k} = \mathbf{H}_k\mathbf{M}_k$. Since the k^{th} user receive their own data stream without the interference from any other users, the methodology for designing the decoder can follow the same strategies like single user MIMO cases after channel estimation [10]. Note that we cannot use a common pilot for estimating $\mathbf{H}_{eff,k}$ since each user uses a different precoding filter $\mathbf{M}_k$ and thus $\mathbf{H}_{eff,k}$ consists of the precoding filter as well as the raw channel $\mathbf{H}_k$.

To achieve the sum rate capacity, after nulling the effect of interfering users' stream out, BD requires another procedure for parallelizing the streams of each user and maximizing the data throughput with the well-know water-filling (WF) algorithm [11].

Define the SVD of $\mathbf{H}_{eff,k}$

$$\mathbf{H}_{eff,k} = \mathbf{U}_k\left[\begin{array}{cc}\mathbf{\Lambda}_k & \mathbf{0}\end{array}\right]\left[\begin{array}{cc}\mathbf{V}_k^{(1)} & \mathbf{V}_k^{(0)}\end{array}\right]^H, \qquad (5)$$

where $\mathbf{V}_k^{(1)}$ denotes the set of the right singular vectors corresponding to non-zero singular values and $\mathbf{U}_k$ is the left singular matrix, respectively. Therefore, when the precoder of the k^{th} user, $\mathbf{M}_k$, has $\mathbf{V}_{\tilde{k}}^{(0)}\mathbf{V}_k^{(1)}$ at the transmitter and the decoder of the k^{th} user also has $\mathbf{U}_k$ at the receiver, the data stream of the k^{th} user is received without the effect of multi-user interference and additionally the optimal capacity can be achieved by the WF algorithm. Therefore, the maximum achievable sum rate of the BD algorithm, C_{BD}, is given by

$$\begin{aligned} C_{BD} &= \sum_{k=1}^{K}\max\log\det\left(\mathbf{I} + \frac{\mathbf{\Lambda}_k^2\mathbf{\Sigma}_k}{\sigma_n^2}\right) \\ &= \sum_{k=1}^{K} C_{WF,k}, \end{aligned} \qquad (6)$$

where $\mathbf{\Sigma}_k$ and $C_{WF,k}$ denote the optimal power loading subject to total power constraint P and the optimal capacity

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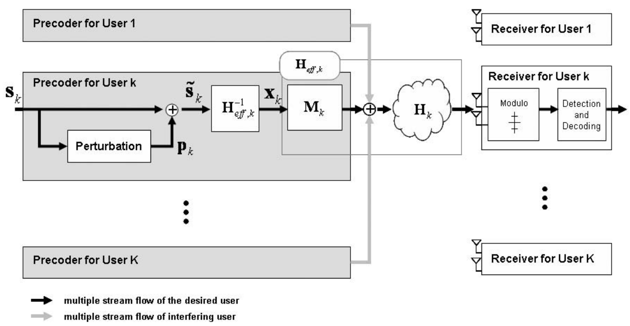

Fig. 1. The structure of a lattice-based broadcast SM-MIMO precoding system using the block diagonalization algorithm.

which the k^{th} user can achieve when using its optimal power allocation strategy, respectively [11]. Note that to implement the WF algorithm, we require the decoding filter $\mathbf{U}_k$ at each receiver. Therefore, if global CSI is not available at the receivers, then coordination information is required between the transmitter and the receivers [13].

III. Lattice-Based Broadcast SM-MIMO Precoding System

We introduce a lattice-based (LB) multi-user (MU) spatial multiplexing (SM) MIMO precoding system to support multiple stream transmission under broadcast channel without any coordination information in this section. Combining BD and perturbation algorithms, we provide a smart solution to avoid coordination information as well as to transmit multi-stream per each user. In addition, the proposed scheme requires a simple decoding filter at each user receiver including the modulo operation.

A. LB MU SM-MIMO Precoder Using BD

Fig. 1 represents the structure of a lattice-based multi-user spatial multiplexing MIMO precoding system using the block diagonalization algorithm in the broadcast channel. The transmitter encodes each user's data stream independently between users and the k^{th} precoder consists of the cascade of two filters $\mathbf{H}_{eff,k}^{-1}$ and $\mathbf{M}_k$. The nulling matrix $\mathbf{M}_k$ and the effective channel $\mathbf{H}_{eff,k}$ are calculated by the same way with (3) and (4), respectively. The main difference between the proposed system and the block diagonalization of [11] is the usage of the inversion of $\mathbf{H}_{eff,k}$ to parallelize each user's stream. Recall that the parallelization of the streams is implemented by the SVD and using an SVD requires additional coordination information between the transmitter and the receiver in [11]. We use the inversion of $\mathbf{H}_{eff,k}$ to parallelize streams instead of SVD, which *does not* require any supplementary implementation of decoding filter. Note that implementing decoding filters means requiring some additional coordination information since the effective channel used for SVD mainly consists of the nulling matrix and such nulling matrix is obtained from using CSIs of other users as mentioned in Section II-B.

The inversion of $\mathbf{H}_{eff,k}$ can be implemented by various methods such as ZF, MMSE, and successive cancellation algorithms. The use of channel inversion, however, brings about a transmit power enhancement due to the normalization of transmit signal power [4]. Therefore, we use a perturbation algorithm to prevent the transmit signal power from increasing [7]. The proposed perturbation operates in the domain of streams, not in the domain of users, given by

$$\begin{aligned}\mathbf{p}_k &= \arg \min_{\mathbf{p}'_k \in AC\mathbb{Z}^{L_k}} \left\| \mathbf{H}_{eff,k}^{-1} \tilde{\mathbf{s}}_k \right\|^2 \\ &= \arg \min_{\mathbf{p}'_k \in AC\mathbb{Z}^{L_k}} \left\| \mathbf{H}_{eff,k}^{-1} \left(\mathbf{s}_k + \mathbf{p}'_k\right) \right\|^2\end{aligned} \tag{7}$$

where $\tilde{\mathbf{s}}_k$ denotes the perturbed symbol of the k^{th} user and the sum of $\mathbf{s}_k$ and $\mathbf{p}_k$ which are the transmit signal vector of the k^{th} user before perturbing and the perturbing vector of the k^{th} user, respectively. The problem of finding the nearest points from the extended constellation is a complex version of the L_K-dimensional integer-lattice least-squares problem [7].

Since the transmitter sends the pre-distorted symbol with perturbation, the receiver has only to map the perturbed symbol ($\tilde{\mathbf{s}}_k$) back to the original symbol ($\mathbf{s}_k$) in the fundamental region using modulo operations [14], which results in a simple decoding filter at receiver. The modulo operation uses an extended constellation that has multiple equivalent points in the extended constellation region with original points in the fundamental constellation boundary and finds the proper points in fundamental boundary when the original points drop into the extended region distorted by power normalization.

B. Achievable Rate Analysis of the Proposed Scheme

We find that we can focus the problem of achievable rate analysis to the single-user MIMO case after applying nulling matrix $\mathbf{M}_k$ because the effect of multiuser interference is removed by $\mathbf{M}_k$. Recall that we have already obtained the received signal model that uses the effective channel in (4). Define $\mathbf{H}_{eff,k} = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{V}_k^H$ where $\mathbf{U}_k = [\mathbf{u}_1 \ \cdots \ \mathbf{u}_{L_k}]$, $\mathbf{V}_k = [\mathbf{v}_1 \ \cdots \ \mathbf{v}_{L_k}]$ and $\mathbf{\Lambda}_k = \mathrm{diag}\left(\lambda_1 \ \cdots \ \lambda_{L_k}\right)$. Then we can transform (4) into the equivalent signal modeling given by

$$\mathcal{Y}_k = \mathbf{\Lambda}_k \mathcal{X}_k + \mathcal{N}_k, \tag{8}$$

where $\mathcal{Y}_k = \mathbf{U}_k^H \mathbf{y}_k$, $\mathcal{X}_k = \mathbf{V}_k^H \mathbf{x}_k$ and $\mathcal{N}_k = \mathbf{U}_k^H \mathbf{n}_k$. Define the normalization factor of transmit signal γ as

$$\begin{aligned}\gamma &= \left\| \mathbf{H}_{eff,k}^{-1} \tilde{\mathbf{s}}_k \right\|^2 \\ &= \sum_{l=1}^{L_k} \mu_l^2 \xi_l^2,\end{aligned} \tag{9}$$

where μ_l and ξ_l denote $\frac{1}{\lambda_l}$ and $\left|\mathbf{u}_l^H \tilde{\mathbf{s}}_k\right|$, respectively.

Assume that $\mathrm{E}\{\|\mathbf{x}_k\|^2\} = P$ in (4). From (9), the received signal to noise ratio (SNR) of each stream, SNR_l, can be represented by

$$SNR_l = \frac{\rho \xi_l^2}{\sum_{m=1}^{L_k} \mu_m^2 \xi_m^2}, \tag{10}$$

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where $\rho = \frac{P}{\sigma_n^2}$. Note that the received SNR is degraded by the normalization factor. Therefore, the achievable rate of the k^{th} user, R_k, is given by

$$R_k = \sum_{l=1}^{L_k} \log\left(1 + \frac{\rho \xi_l^2}{\sum_{m=1}^{L_k} \mu_m^2 \xi_m^2}\right). \quad (11)$$

Perturbing means that we force the perturbing vector $\mathbf{p}_k$ to minimize γ and generate a $\tilde{\mathbf{s}}_k$ that can only be coarsely oriented in the coordinate systems defined by $\mathbf{u}_1, \cdots, \mathbf{u}_{L_k}$ [7]. If we find the proper perturbing vector and control ξ_m to minimize the normalized factor γ, then we can calculate the achievable sum rate of the proposed scheme as

$$R_{k,prop} = \sum_{l=1}^{L_k} \log\left(1 + \frac{\rho \xi_l^2}{\min\limits_{\xi_m}\left(\sum_{m=1}^{L_k} \mu_m^2 \xi_m^2\right)}\right). \quad (12)$$

We can find the achievable rate, $R_{k,prop}$ represented by

$$R_{k,prop} = \sum_{l=1}^{L_k} \log\left(1 + \frac{\rho \frac{\omega_0^2}{\mu_l^2}}{L_k \omega_0^2}\right) \quad (13)$$

$$= \sum_{l=1}^{L_k} \log\left(1 + \frac{\rho \lambda_l^2}{L_k}\right) \quad (14)$$

for an arbitrary constant $\omega_0^2 = \mu_1^2 \xi_1^2 = \cdots = \mu_{L_k}^2 \xi_{L_k}^2$. Eq. (14) is obtained by substituting μ_j for $\frac{1}{\lambda_j}$. Note that the solution of (14) is valid when the lattice size is infinite because ξ_j is the relative variable of the perturbed symbol vector $\tilde{\mathbf{s}}_k$ and we would find the proper $\tilde{\mathbf{s}}_k$ providing that the searching range of the lattice is infinite.

Note that (14) is the same expression as the capacity of the single user (SU) MIMO system with equal power allocation, C_{EQ} [15]. Assuming that the elements of each user channel matrix are identically and independently distributed (i.i.d.), C_{EQ} approaches the capacity of the MIMO system with WF optimal power allocation, C_{WF} for asymptotically high SNR [16]. Now we conclude that the achievable rate of any k^{th} user, $R_{k,prop}$ obtained by the LB MU SM-MIMO precoding system approaches the optimal capacity asymptotically for high SNR, and that the achievable sum rate, R_{sum}, is defined by

$$R_{sum} = \sum_{k=1}^{K} R_{k,prof} \Rightarrow \sum_{k=1}^{K} C_{WF,k} \text{ (for high SNR)}. \quad (15)$$

Recall that $C_{WF,k}$ is already defined by the optimal capacity which each user can achieve when using the optimal power allocation strategy of each user in Section II-B.

IV. Numerical Experiments and Results

In this section we compare sum rate and BER performance between the proposed scheme and other various schemes through Monte Carlo simulations. To verify the system performance of the proposed precoding system, we consider several special cases. We assume that the number of receive antenna for each user is equal to N_R and that each user receives the same number of streams (L_k) as the number of receive antenna, that is, $L_k = N_R$ and that $N_T = KN_R$. We use the notation $\{N_R, K\}$ to index the number of each user's antennas and the number of users. We assume that the elements of each user channel matrix are independent complex Gaussian random variables with zero mean and unit variance for all numerical results.

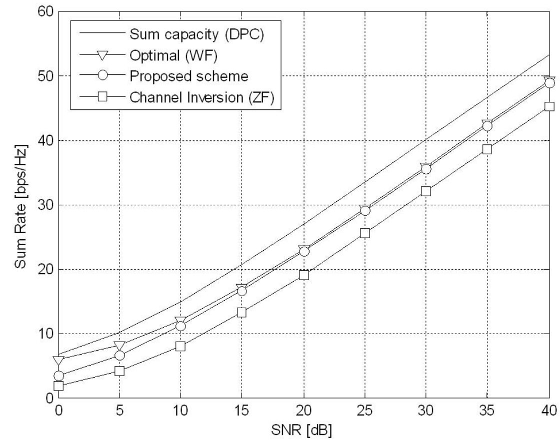

Fig. 2. The comparison of achievable sum rate between the sum-capacity [5], the optimal methods (C_{BD}, [11]), the channel inversion (ZF) and the proposed system (R_{sum}).

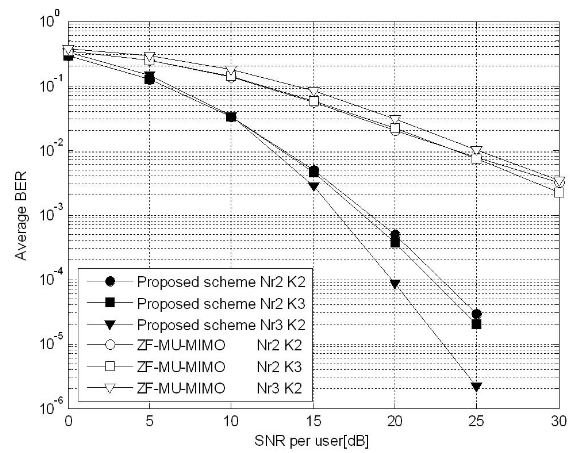

Fig. 3. BER performance comparison between the proposed scheme and the ZF precoding scheme without perturbing for 4-QAM.

Fig. 2 compares the maximum achievable sum rate of the proposed system with the other implementation systems in the case of $\{2, 2\}$. The sum capacity, C_{sum}, denotes the maximum sum rate that can be achieved by the DPC [5], [17]. The optimal sum rate means C_{BD} obtained from (6). The channel inversion is the system that each user exploits the ZF algorithm for precoding after using nulling matrix and block diagonalization with the constraint that equal power is

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transmitted to each user receive antenna without perturbation. To obtain the capacity with channel inversion, the transmit power should be normalized to satisfy the power constraint. From Fig. 2, we observe that the sum rate of the proposed scheme is better than that of the channel inversion scheme and also achieves the sum rate of the optimal WF schemes asymptotically for high SNR as what we expected in Section III-B without any additional coordination information and iterative update for implementing the precoding and decoding filters. The sum rate of the channel inversion scheme is degraded by the power normalization of the transmit precoding. On the contrary, the proposed scheme exploits the perturbation as a kind of power allocation to compensate the degraded effect of power normalization.

Fig. 3 shows BER performance comparing the proposed scheme with the ZF MU SM-MIMO for 4-QAM. ZF MU SM-MIMO is the same system as the channel inversion scheme mentioned above. We assume three $\{N_R, K\}$ scenarios to observe the BER performance: $\{2, 2\}$, $\{2, 3\}$, and $\{3, 2\}$. Overall BER performance of the proposed scheme is better than that of the ZF MU SM-MIMO. We have 10 dB or more SNR gain in the proposed system compared with the ZF MU SM-MIMO at 10^{-2} BER. From the viewpoint of diversity gain, we also observe that the proposed system has full diversity gain because the proposed system exploits optimal type of decoders such as maximum likelihood decoder as the transmit precoder basically and also find the perturbed symbol which has the optimal decision boundary in the Vornoi region to minimize the transmit power [14]. Therefore, among the BER plots of the proposed LB MU SM-MIMO schemes, the case that $N_R = 3$ show better diversity gain than $N_R = 2$.

V. Conclusions

We have proposed a lattice-based broadcast spatial multiplexing MIMO precoding scheme that supports multi-stream transmission without any coordination information. The proposed lattice-based multi-user precoder uses block diagonalization to remove multi-user interference and applies channel inversion algorithm for the calculated effective channel as the precoding algorithm to avoid additional coordination information. It also exploits a perturbation to reduce transmit power enhancement when the channel inversion algorithms are used for the precoding algorithm. The proposed scheme can achieve a sum rate approached by the block diagonalization scheme with the water-filling algorithm asymptotically for high signal to noise ratio without any coordination information. Through the Monte Carlo simulation, we verified that the proposed scheme has 10 dB or more SNR gain compared with zero-forcing multi-user MIMO precoding with the block diagonalization at 10^{-2} BER. Furthermore, we observed that the proposed system gets the full diversity gain as the optimum decoding system does and achieves the optimal sum rate asymptotically assuming that the elements of each user channel are identically and independently distributed.

Acknowledgement

This work was supported in part by Samsung Electronics. This work was also supported in part by the Korea Research Foundation Grant (KRF-2005-214-D00319) funded by the Korean Government (MOEHRD).

References

[1] G. Caire and S. Shamai (Shitz), "On the achievable throughput of a multi-antenna gaussian broadcast channel," *IEEE Trans. on Infomation Theory*, Vol. 43, No. 7, pp. 1691 - 1706, Jul. 2003.

[2] M. Costa, "Writing on dirty paper," *IEEE Trans. on Information Theory*, Vol. 29, No. 3, pp.439 - 441, May 1983.

[3] H. Weingarten, Y. Steinberg and S. Shamai (Shitz), "The Capacity Region of the Gaussian MIMO Broadcast channel," *submitted to IEEE Trans. on Information Theory*, Jul. 2004.

[4] C. B. Peel, B. M. Hochwald and A. L. Swindlehurst, "A vector-perturbation technique for near capacity multiantenna multiuser communication - part I: channel inversion and regularization," *IEEE Trans. on Communications*, Vol. 53, No. 1, pp. 195 - 202, Jan. 2005.

[5] D. Samardzija and N. Mandayam, "Multiple antenna transmitter optimization schemes for multiuser systems," *Proc. IEEE Veh. Technol. Conf.*, Vol. 1, pp. 399 - 403, Oct. 2003.

[6] C. Windpassinger, R. F. H. Fischer, T. Vencel and J. B. Huber, "Precoding in multiantenna and multiuser communications," *IEEE Trans. on Wireless Communications*, Vol. 3, No. 4, pp. 1305 - 1315, Jul. 2004.

[7] B. M. Hochwald, C. B. Peel and A. L. Swindlehurst, "A vector-perturbation technique for near capacity multiantenna multiuser communication - part II: perturbation," *IEEE Trans. on Communications*, Vol. 53, No. 3, pp. 537 - 544, Mar. 2005.

[8] C. Windpassinger, R. F. H. Fischer and J. B. Huber, "Lattice-reduction-aided broadcast precoding," *IEEE Trans. on Communications*, Vol. 52, No. 12, pp. 2057 - 2060, Dec. 2004.

[9] M. Airy, S. Bhadra, R. W. Heath, S. Shakkottai, "Transmit precoding for the multiple antenna broadcast channel," *to appear in Proc. IEEE Veh. Technol. Conf.*, May 2006.

[10] L.-U. Choi and R. D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Trans. on Wireless Communications*, Vol. 3, No. 1, pp. 20 - 24, Jan. 2004.

[11] Q. H. Spencer, A. L. Swindlehurst and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Signal Processing*, Vol. 52, No. 2, pp. 461 - 471, Feb. 2004.

[12] R. Chen, R. W. Heath, and J. G. Andrews, "Transmit selection diversity for unitary precoded multiuser spatial multiplexing systems with linear receivers," *accepted to IEEE Trans. on Signal Processing*.

[13] C. B. Chae, D. Mazzarese and R. W. Heath, "Coordinated beamforming MU-MIMO systems with Grassmannian codebook," *IEEE Communication Theory Workshop*, Available at http://www.ece.rice.edu/ctw2006/poster.html, Mar. 2006.

[14] H. Yao, *Efficient signal, code, and receiver designs for MIMO communication systems*, Ph.D dissertation, Massachusetts Institute of Technology, 2003.

[15] I. E. Telatar, "Capacity of multiple-antenna Gaussian channels," *European Trans. on Telecommunications*, Vol. 10, pp. 585 - 595, Nov. 1999.

[16] A. J. Paulraj, R. Nabar and D. Gore, *Introduction to space-time wireless communications*, Cambridge University Press, 2003.

[17] P. Ding, D. J. Love, and D. Zoltowski, "On the sum rate of channel subspace feedback for multi-antenna broadcast channels," *Proc. IEEE Glob. Telecom. Conf*, Vol. 5, pp. 2699-2703, Dec. 2005.

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