Multi-Layer Precoding: A Potential Solution for Full-Dimensional Massive MIMO Systems

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Outline

- Introduction
- System and Channel Models
- Multi-Layer Precoding Design
- Performance Results
MIMO is Big!

- **mmWave systems**
  - Large arrays are needed at transmitter & receiver
  - Leverage large available bandwidth at high frequency
  - Enable very high data rates

- **Massive MIMO gains**
  - Large numbers of users are simultaneously served
  - High sum-rates
  - Simplified multi-user processing
  - Small transmit power

Why not we keep scaling MIMO up?
Interference Management is Challenging

- Need high channel state information (CSI)
  - Large channel dimensions
  - Large amount of pilots
  - Basestation cooperation overhead

How to manage the interference with limited channel knowledge?
Interference Management is Challenging

- High precoding design complexity
  - Precoders need to be designed to manage different kinds of interference
  - Precoders of different cells need to be jointly designed (usually non-convex problems)
  - Large dimensions add more complexity

How to develop low-complexity precoders for large MIMO systems?
Multi-Layer Precoding: A Potential Solution

\[ F = F^{(1)} F^{(2)} F^{(3)} \]

- **Decoupling of Precoding Objectives**
  - Each precoding layer (matrix) is responsible of one objective

- **Dependence on large channel statistics**
  - Each precoding layer depends on channel statistics larger (slower) than next layers

- **Low-complexity design**
  - Requires limited CSI

- **Inter-cell interference management**

- **Desired signal optimization**

- **Multi-user interference management**

- **MU interference**

- **Inter-cell interference**
Connection to Prior Work

- **Multi-user hybrid analog/digital precoding [1]**
  - Motivated mainly by hardware constraints
  - Leverages the sparse nature of mmWave channels
  - Did not consider out-of-cell interference

- **Joint spatial-division multiplexing [2]**
  - Motivated by large channel feedback overhead in FDD
  - Groups the users based on their channel covariance
  - Did not consider out-of-cell interference

- **Pilot decontamination for massive MIMO [3]**
  - Leverages the low-dimensional interference subspace to get better desired channel estimate
  - Considered 1-D antenna arrays
  - Requires the knowledge of the interference covariance matrices

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**System Model**

- BS has a 2D antenna array of $N_V$ vertical elements x $N_H$ horizontal elements.
- $K$ single-antenna users are simultaneously served in each cell.
- All BS’s are assumed to be synchronized - TDD - Universal frequency reuse.
- In the downlink, precoder $F_c = [f_{c1}, f_{c2}, ..., f_{cK}]$ is used by BS $c$.
- Received signal by user $k$ in cell $c$:
  \[ y_{ck} = \sum_{\ell=1}^{L} h_{\ell ck}^* F_{\ell} s_{\ell} + n_{ck} \]
**Channel Model**

- **Kronecker product correlation [1]**
  \[
  R_{\ell ck} = E[h_{\ell ck} h^*_{\ell ck}] = R^A_{\ell ck} \otimes R^E_{\ell ck}
  \]
  
  **Azimuth correlation**
  **Elevation correlation**

- **Using Karhunen-Loeve representation [2]**
  \[
  h_{\ell ck} = \left[ U^A_{\ell ck} \Lambda^{A \frac{1}{2}}_{\ell ck} \otimes U^E_{\ell ck} \Lambda^{E \frac{1}{2}}_{\ell ck} \right] w_{\ell ck}
  \]

- **Assuming rank-1 elevation correlation [3]**
  \[
  h_{\ell ck} = h^A_{\ell ck} \otimes \lambda^{\frac{1}{2}}_{\ell ck} u_{\ell ck}
  \]

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Insights from Large Channel Characteristics

Channel covariance matrices have directional structure [1], [2]

Signal and interference are contained in low-dimensional subspaces

In the elevation direction, signal and interference may occupy different subspaces

Multi-Layer Precoding Design (1/3)

The SINR of user $k$ in cell $c$ is

$$
\text{SINR}_{c_k} = \frac{|h_{c_k}^* f_{c_k}|^2}{\sum_{m \neq k} |h_{c_k} f_{cm}|^2 + \sum_{\ell \neq c} |h_{\ell c_k} f_{\ell c}|^2 + \sigma^2}
$$

- **Desired signal power**
- **Multi-user interference**
- **Inter-cell interference**

In the design

- Leverage the Kronecker structure of the channel
- Focus on multi-layer precoding in the elevation direction

$$
F_c = \left( I \otimes F_c^{(1)} \right) \left( I \otimes F_c^{(2)} \right) \left( F^A \otimes F_c^{(3)} \right)
$$

Minimize inter-cell interference
Maximize effective signal power
Minimize multi-user interference

Requires large-scale channel statistics
Requires large-scale channel statistics
Requires instantaneous channel
Multi-Layer Precoding Design (2/3)

**First layer:**

\[
y_{ck} = h_{ck}^A F_c^A \otimes \lambda_{ck}^{1/2} u_{ck}^* F_{c}^{(1)} F_{c}^{(2)} F_{c}^{(3)} E_c s_c \\
+ \sum_{\ell \neq c} h_{ck}^A F_{\ell}^A \otimes \lambda_{ck}^{1/2} u_{ck}^* F_{\ell}^{(1)} F_{\ell}^{(2)} F_{\ell}^{(3)} E_{\ell} s_{\ell} + n_{ck}
\]

Inter-cell interference

**To avoid inter-cell interference,** \( F_c^{(1)} \) is designed such that \( u_{ck}^* F_{c}^{(1)} = 0 \) \( \forall k, \forall \ell \neq c \)

\[
R_c^I = \mathbb{E}_{\mathcal{K}_c} \left[ \sum_{k \in \mathcal{K}_c} \sum_{\ell \neq c} \lambda_{ck} u_{ck} u_{ck}^* \right]
\]

Expectation over different scheduled users

\[
F_c^{(1)} = U_c^{NI}
\]

Interference null-space

**After the first layer**

\[
y_{ck} \approx h_{ck}^A F_c^A \otimes \lambda_{ck}^{1/2} u_{ck}^* U_c^{NI} F_c^{(2)} F_c^{(3)} E_c s_c + n_{ck}
\]

Due to the expectation
Multi-Layer Precoding Design (3/3)

**Second layer**
- Training: Acquires effective channel elevation covariance eigenvectors (reduced rank)

\[
\overline{U}_c = U_c^{NI*} [u_{lc1}, u_{lc2}, \ldots, u_{lcK}]
\]

\[
F_c^{(2)} = \overline{U}_c \quad \text{Conjugate beamforming of the effective channel covariance}
\]

- Remark: No uplink inter-cell interference during training phase due to first layer
- After the second layer

\[
y_{ck} \approx h_{cck}^A F_c^A \otimes \frac{1}{\sqrt{\gamma_{ck}}} \overline{U}_c^* F_c^{(3)} U_c E_c s_c + n_{ck}
\]

**Third layer**
- Training: Instantaneous effective channels (reduced-rank channel)

\[
H_{c,\text{eff}}^E = D_c^{1/2} \overline{U}_c U_c^*
\]

Effective channel is of $K \times K$ dimensions

\[
F_c^{(3)} = \text{ZF} \left( H_{c,\text{eff}}^E \right)
\]

\[
F_c^A = H_{c,\text{eff}}^E \quad \text{Effective channel is of } K \times K \text{ dimensions}
\]

- After the third layer

\[
y_{ck} \approx \frac{1}{\sqrt{\gamma_{ck}}} s_{ck} + n_{ck}
\]

Penalty of interference management
Achievable Rates

- Assume $\mathbf{u}_{ck} \in \mathcal{R}(\mathbf{U}_c^{NI})$, $\forall k$
- The achievable rate of user $k$ in cell $c$
  \[ R_{ck} \geq \log_2 \left(1 + \frac{P\|\mathbf{h}_{ck}^A\|^2\lambda_{ck}}{\sigma^2} G(\mathbf{U}_c)\right) \]
  with \[ G(\mathbf{U}_c) = 4 \left(\frac{\sigma_{\text{max}}^2(\mathbf{U}_c)}{\sigma_{\text{min}}^2(\mathbf{U}_c)} + \frac{\sigma_{\text{min}}^2(\mathbf{U}_c)}{\sigma_{\text{max}}^2(\mathbf{U}_c)} + 2\right)^{-1} \]
- Asymptotic optimality
  \[ \lim_{N_V \to \infty} R_{ck} = \hat{R}_{ck} \quad \text{and} \quad G(\mathbf{U}_d) \to 1 \]
- Performance approaches single-user rate
  \[ R_{ck} \xrightarrow{a.s.} \hat{R}_{ck} \]
**Simulation Results**

- Considerable gains compared with interference-limited massive MIMO systems
- Gains are mainly due to inter-cell interference management
- Gains increase with the number of antennas
- Very close performance to the case with exact interference covariance matrix
- Cell edge users may be blocked if the number of antennas is not large enough

**Setup:**
- Poisson layout of BS’s and MS’s
- MS poisson density is 30 times higher than BS densities
- Inter-site distance=200 m, antenna height=50 m
- MS’s are associated to the nearest BS
- BS randomly selects K=4 MS’s to be served
- Performance is averaged over 100 realizations
- Interference covariance is averaged over 20 realizations
- Pathloss exponent=3.5
Conclusion and Future Work

**Multi-layer precoding**
- Manages inter-cell and multi-user interference
- Requires limited channel knowledge
- Enables low-complexity designs (precoding objectives decoupling)
- Approaches the single-user rates in some special cases

**Future extensions**
- Performance analysis for more general channel settings, e.g., including angle spread
- Investigating solutions to improve cell-edge users in the non-asymptotic regime
- Evaluating the impact of channel estimation errors
- Approximation using hybrid analog/digital architecture

Thank You

Questions?

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