Coverage in Dense Millimeter Wave Cellular Networks

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Why mmWave for Cellular?

Huge amount of spectrum available in mmWave bands*

- Cellular systems live with limited microwave spectrum ~ 600MHz
- 29GHz possibly available in 23GHz, LMDS, 38, 40, 46, 47, 49, and E-band

Technology advances make mmWave possible

- Silicon-based technology enables low-cost highly-packed mmWave RFIC**
- Commercial products already available (or soon) for PAN and LAN
- Already deployed for backhaul in commercial products

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Antenna Arrays are Important

Narrow beams are a new feature of mmWave

- Reduces fading, multi-path, and interference
- Implemented in analog due to hardware constraints

Arrays will change system design principles
Sensitivity to Blockages

- Signal and interference may be either LOS or NLOS
- Users may connect to a further unblocked base station
- Strong interferers may be blocked

LOS channel
Path loss exponent 2

NLOS channel
Path loss exponent 4
Additional loss

Need to include propagation models in the analysis

Contributions

- Incorporate beamforming & LOS/NLOS into coverage analysis
  - RX and TX communicate via steered directional beams
  - Steering directions at interfering BSs are random
  - Paths may be LOS or NLOS depending on blockage density

- Approach is to leverage stochastic geometric analysis of cellular
  - Extends work by [AndrewsGantiBaccelli2011] to include mmWave features
  - Extends our work [BaiHeath2013] with simplified expressions for dense networks
System Model
Stochastic Geometry for Cellular

- Stochastic geometry is a tool for analyzing microwave cellular
  - Reasonable fit with real deployments
  - Closed form solutions for coverage probability available
  - Provides a system-wide performance characterization

Need to incorporate LOS/non-LOS links and directional antennas

Accounting for Beamforming

- Each base station is marked with a directional antenna
  - Antenna directions of interferers are uniformly distributed
- Use “sector” pattern in analysis for simplicity
  - Antenna pattern fully characterized by $\theta$, $M$ and $m$
Incorporating Blockages

- Use random shape theory to model buildings
  - Model random buildings as a rectangular Boolean scheme
  - Buildings distributed as PPP with independent sizes & orientations
- Compute the LOS probability based on the building model
  - # of blockages on a link is a Poisson random variable
  - The LOS probability that no blockage on a link of length $R$ is $e^{-\beta R}$

Proposed mmWave Model

- Use stochastic geometry to model BSs as marked PPP
- Model the steering directions as independent marks of the BSs
- Use random shape theory to model buildings
  - Model the building as rectangle Boolean schemes
  - Different path loss exponents for LOS and NLOS paths

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System Parameters

Different path loss model for LOS and non-LOS links

- Line-of-sight with probability $e^{-\beta R}$: average LOS range is $1/\beta$
  
  The fraction of land covered by buildings
  
  $\beta = \frac{2\eta([L] + [W])}{\pi[W][L]}$
  
  Average building length and width

- LOS path Loss in dB: $PL_1 = C + 20 \log R(\text{m})$
- Non-LOS path loss in dB: $PL_2 = C + K + 40 \log R(\text{m})$
- 28GHz system: let $C=70$ dB, $K=10$ dB

General small scale fading $h$

- No fading case: small scaling fading is minor in mmWave [RapSun]

Link budget

- Tx antenna input power: 30dBm
- Signal bandwidth: 500 MHz (Noise: -87 dBm)
- Noise figure: 5dB
Coverage Results
SINR Expressions

\[ \text{SINR} = \frac{M_r M_t H_0 \ell(r_0)}{N_0/P_t + \sum_{k>0} A_k B_k H_k \ell(r_k)}, \]

where

\[ H_0 \ell(r_0) = \min_{k>0} \{ H_k \ell(r_k) \}, \]

\[ A_k = \begin{cases} M_t & \text{w. p. } \frac{\theta_t}{2\pi} \\ m_t & \text{w. p. } 1 - \frac{\theta_t}{2\pi} \end{cases}, \]

\[ B_k = \begin{cases} M_r & \text{w. p. } \frac{\theta_r}{2\pi} \\ m_r & \text{w. p. } 1 - \frac{\theta_r}{2\pi} \end{cases}, \]

\[ \ell(x) = \begin{cases} C x^{-2} & \text{w. p. } e^{-\beta x} \\ C' K x^{-4} & \text{w. p. } 1 - e^{-\beta x} \end{cases}. \]

- Serving BS and User connect via main lobe
- General small-scale fading
- Connecting to the strongest signal before BF
- Array gain of the TX antenna
- Array gain of the RX antenna
- Path Loss of LOS or non-LOS

Use stochastic geometry to compute SINR distribution
Theorem 1 [mmWave Coverage probability]

The coverage probability $\mathbb{P}[\text{SINR} > T]$ can be computed as

$$
\mathbb{P}(\text{SINR} > T) = \int_{-\infty}^{\infty} \int_{0}^{\infty} U(x, t) f_{L^*}(x) \frac{e^{j 2\pi t/T} - 1}{j 2\pi t} \, dx \, dt
$$

where

$$
U(x, s) = \exp \left[ -\frac{sx}{M_r M_t \rho} + \int_{x}^{\infty} \left( p_t p_r e^{-\frac{sx}{u}} + (1 - p_t) p_r e^{-\frac{s x m_t}{u M_t}} + p_t (1 - p_r) e^{-\frac{s x m_r}{u M_t}} \right) \Lambda(du) \right],
$$

$$
\Lambda(x) = 2\pi \lambda E_h \left[ \int_{0}^{(\frac{x h}{K})^{0.25}} t \left( 1 - e^{-\beta t} \right) dt + \int_{0}^{\sqrt{x h}} t e^{-\beta t} dt \right],
$$

$$
f_{L^*}(x) = -\frac{d}{dx} e^{-\Lambda(x)}.
$$

Good coverage requires dense BS deployments

- LOS BSs exist with high probability in dense networks
- Noise and NLOS interference become much weaker than LOS interf.

Theorem 1 sometimes inefficient to compute

- The underlying reason is that LOS region is very irregular
- Need to simplify expressions in Theorem 1

Approximate LOS region & neglect NLOS contributions
Dense Network Analysis (2/3)

Theorem 2 [Coverage probability with dense BSs]
In dense networks with a **LOS path loss exponent of 2**

\[
\mathbb{P}(\text{SINR} > T) \approx \rho e^{-\rho} \sum_{n=1}^{N} (-1)^{n+1} \binom{N}{\ell} \int_{0}^{1} \prod_{k=1}^{4} e^{\rho a_k (e^{-n b_k t} - te^{-n b_k})} \left( \frac{1 - e^{-n \eta b_k t}}{1 - e^{-n \eta b_k}} \right)^{n \rho a_k b_k t} dt,
\]

where \(a_k\) and \(b_k\) are constants determined by RX and TX antenna patterns.

- Derive approximate bounds on SINR distribution in dense network
  
  - Approximate LOS region as a ball with equal average size
  - Ignore noise, approximate LOS fading with high order gamma distribution
  - Use Alzer’s inequality to further simplify to single-finite integral
  - Provides approximate upper and lower bounds on SINR
  - Can be extended to other LOS exponents

How accurate is the approximations?
Dense Network Analysis (3/3)

- Larger $N$ (more terms) increases accuracy
- $N=5$ terms generally provides acceptable accuracy

What is the design insight from coverage analysis?
Insights for Dense Networks

\[ P(\text{SINR} > T) \approx \rho e^{-\rho} \sum_{n=1}^{N} (-1)^{n+1} \left( \frac{N}{\ell} \right) \int_{0}^{1} \prod_{k=1}^{4} e^{\rho a_k (e^{-n b_k t} - te^{-n b_k})} \left( \frac{1 - e^{-n \eta b_k t}}{1 - e^{-n \eta b_k}} \right)^{n a_k b_k t} \, dt \]

- Given antenna patterns, SINR only depends on \( \rho \)
  \[ \rho = \frac{\text{Average size of LOS region}}{\text{Average cell size}} \]

- The larger \( \rho \), the denser the BSs (and closer)

- Increasing BS density need not improve SINR
  - SINR goes to zero in a infinitely dense network
    \[ \lim_{\rho \to \infty} \text{SINR} \stackrel{p.}{=} 0 \]
  - Optimal BS density is finite

What is the optimal BS density in dense networks?
Finding Optimal BS Density

- Exhaustive search optimal BS density using Theorem 2
  - Maximize the coverage probability given a target SINR
  - Much efficient than simulations with minor errors
  - Optimal cell radius is approximately 2/3 of the avg. LOS range.

Tx directivity gain: 20 dB
Tx beamwidth: 30 degree
Rx directivity gain: 10 dB
Rx beamwidth: 10 dB
Avg. LOS range: $\frac{1}{\beta} = 141$ m
Target SINR: $T = 10$ dB

Average cell radius

Increasing BS density need not improve SINR

Optimal BS density is finite
Finding Optimal BS Density

Exhaustive search optimal BS density using Theorem 2

- Maximize the coverage probability given a target SINR
- Much efficient than simulations with minor errors
- Optimal cell radius is approximately 2/3 of the avg. LOS range.
Achievable Rate Analysis

Given coverage probability, the achievable rate is

\[ R = \frac{1}{\ln(2)} \int_0^C \frac{P_c(T)}{1 + T} dT \]

Microwave network 4X4 SU MIMO with bandwidth 50MHz:
- Spectrum efficiency is 4.56 bps/ Hz
- Data rate is 228 Mbps (invariant with the cell size \( R_c \))

mmWave network with bandwidth 500MHz:

<table>
<thead>
<tr>
<th>( M )</th>
<th>( R_c )</th>
<th>100m</th>
<th>200m</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 dB</td>
<td>1.61 Gbps</td>
<td>1.61 Gbps</td>
<td></td>
</tr>
<tr>
<td>20 dB</td>
<td>2.91 Gbps</td>
<td>1.88 Gbps</td>
<td></td>
</tr>
</tbody>
</table>

Average cell radius

Tx beamforming Gain

mmWave achieves high gain in average rate
Conclusions
Going Forward with mmWave

- mmWave coverage probability and rate
  - Need to include both LOS and Non-LOS conditions
  - Interference is reduced by directional antennas and blockages
  - Good rates and coverage can be achieved

- Theoretical challenges abound
  - Analog beamforming algorithms & hybrid beamforming
  - Channel estimation, exploiting sparsity, incorporating robustness
  - Multi-user beamforming algorithms and analysis
  - Microwave-overlaid mmWave system a.k.a. phantom cells
  - Going away from cells to a more ad hoc configuration