Coverage and Capacity Analysis of mmWave Cellular Systems

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WNCG Enhances UT Visibility

- Large multi-PI grants, e.g.
  - $1m Intel / Cisco grand challenge on video networks (PI: Heath)
  - $1.3m on cross-layer delay-tolerant nets (PI: Shakkottai)
- Seven major best paper awards in last four years

WNCG Impacts Industry

- Definitive textbooks on wireless
- Widely-cited magazine articles on hot topics
- Developed key features of wireless standards
- Software packages and toolkits
- Host popular annual conference. www.twsummit.com

WNCG Affiliates

Affiliates champion large federal proposals, provide technical input/feedback, unrestricted gift funds

WNCG provides pre-prints, pre-competitive research ideas, vast expertise, first access to students

WNCG: Premier Wireless Research Center

20 faculty from 4 departments, all actively involved in center activities

Recent additions:

- Andrea Alù, Metamaterials
- Joydeep Ghosh, Data mining
- François Baccelli *, Stochastic geometry
- Alex Dimakis *, Information theory

* New faculty at UT

Students receive perks, special awards and travel funds for help with affiliates
Staff, space & other resources shared efficiently amongst all faculty/students

120 PhD students in pooled space
Many co-advised students
64%/yr intern at affiliates in last 4 yrs

WNCG Faculty

WNCG Students

WNCG Affiliates

Wireless Networking and Communications Group
Wireless Communications Lab

- Undergrad/grad lab course
  - QAM & OFDM experiments
  - Complete lab manual & software
  - Uses USRP equipment
  - LabVIEW programming
- Complete lab manual available

Conclusions
mmWave is Great for Cellular

There are many theoretical challenges
Introduction
Why mmWave for Cellular?

• Huge amount of spectrum available in mmWave bands*
  - Cellular systems live with limited microwave spectrum ~ 600MHz
  - 29GHz possibly available in 23GHz, LMDS, 38, 40, 46, 47, 49, and E-band

• Technology advances make mmWave possible
  - Silicon-based technology enables low-cost highly-packed mmWave RFIC**
  - Commercial products already available (or soon) for PAN and LAN
  - Already deployed for backhaul in commercial products

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The Need for Gain

Smaller wavelength means **smaller captured energy** at antenna

- 3GHz->30GHz gives 20dB extra path loss due to aperture

Larger bandwidth means **higher noise power** and lower SNR

- 50MHz -> 500MHz bandwidth gives 10dB extra noise power

\[ A_{\text{eff}} = \frac{\lambda^2}{4\pi} \]

\[ P_r = A_{\text{eff}} \frac{P_t}{4\pi R^2} = \left( \frac{\lambda}{4\pi R} \right)^2 \]

Solution: Exploit array gain from large antenna arrays
Antenna Arrays are Important

Narrow beams are a new feature of mmWave
- Reduces fading, multi-path, and interference
- Implemented in analog due to hardware constraints

Arrays will change system design principles
Traditional Beamforming Limitations

Possible solution  Precoding in the analog domain

Power consumption limits the # of RF & ADC/DACs

Analog beamforming has additional constraints
- Constant gains: Only phases is typically adjusted
- Quantized phases: Fixed set of steering directions are allowed

Need beamforming strategies suitable for mmWave hardware
Hybrid Beamforming for mmWave

- Combine both digital and analog beamforming
- Small number of digital basebands (2 or 4)
- Allows more advanced MIMO strategies to be exploited
- Spatial multiplexing or multiuser MIMO

Hybrid approach allows more advanced beam design

The mmWave Channel for Cellular

mmWave cellular channel measurement at UT campus*


mmWave cellular channel already measured in various environment

- Many characteristics of mmWave cellular channels are known
- Measurement results validate the feasibility of mmWave cellular networks

**Image Caption:*** Overhead image of a walkway area surrounded by buildings of 1 to 12 stories. The 7.3 p polarized horn antennas with gain 400 Mcps for 38 GHz, bandwidth is 400 MHz, transmission power at amplifier 30 dBm, and horn antenna gain 24.5 dBi for both transmitter and receiver. Since these measurement environments are dense urban, pathloss exponents are 1.68 in LOS and 4.58 in NLOS links.

**Image Caption:** Left: Measurement sites in UT Austin campus, Right: Pathloss and RMS delay spread results.
**Example Measurement Insights**

- **LOS (line-of-sight) signals propagate as in free space**
  - Path loss exponent of LOS is 2
  - Diffraction is weaker in higher frequency, thus less loss when LOS

- **NLOS (non-line-of-sight) is possible thanks to reflections**
  - Path loss exponent of NLOS (non-LOS) is about 4
  - Best reflected signal is still 20dB weaker than LOS
Impact of Propagation

- Users may connect to a further unblocked base station
- Strong interferers may be blocked
- Signal and interference may be either LOS or NLOS

Need to include propagation models in the analysis

mmWave Performance Analysis

Directions Beamforming (BF) + LOS & non-LOS links

- Need to incorporate directional beamforming:
  - RX and TX communicate via main lobes to achieve array gain
  - Steering directions at interfering BSs are random

- Need to distinguish LOS and NLOS paths:
  - Incorporate different characteristics in LOS & NLOS channels
  - Better characterize blockages

How to including beamforming + blockages in mmWave cellular analysis?
Stochastic Geometry for mmWave Cellular System Analysis
Stochastic Geometry for Cellular

Stochastic geometry is a tool for analyzing microwave cellular

- Reasonable fit with real deployments
- Closed form solutions for coverage probability available
- Provides a system-wide performance characterization

Need to incorporate LOS/non-LOS links and directional antennas

Poisson Point Processes

- Poisson point process (PPP): the simplest point process
  - # of points is a Poisson variable with mean $\lambda S$
  - Given $N$ points in certain area, locations independent
  - Useful results like Campbell’s Theorem & Displacement Theorem apply
  - Assigning each point an i.i.d. random variable forms a marked PPP

Antenna steering orientations as marks of the BS PPP
Blockages in mmWave

- Use random shape theory to model buildings
  - Model random buildings as a rectangular Boolean scheme
  - Buildings distributed as PPP with independent sizes & orientations

- Compute the LOS probability based on the building model
  - # of blockages on a link is a Poisson random variable
  - The LOS probability that no blockage on a link of length $R$ is $e^{-\beta R}$

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Directional Transmission at the BS

Each base station is marked with a directional antenna

- Antenna directions of interferers are uniformly distributed

- Use “sector” pattern in analysis for simplicity
  - Equivalent to uniform linear array of $N_t$ antennas with spacing $\lambda/2$

Main lobe beamwidth:
$$\theta = 2 \arcsin \left( \frac{2.782}{\pi N_t} \right)$$

Main lobe array gain:
$$M = N_t \quad \# \text{antennas}$$

Front-back ratio:
$$\text{FBR} = \sin \left( \frac{3\pi}{2N_t} \right)$$

Back lobe gain:
$$m = N_t \times \text{FBR}$$
Proposed mmWave Model

- Use stochastic geometry to model BSs as marked PPP
  - Model the steering directions as independent marks of the BSs
- Use random shape theory to model buildings
  - Model the building as rectangle Boolean schemes
  - Different path loss exponents for LOS and non-LOS paths
System Parameters

- Different path loss model for LOS and nonLOS links
  - Line-of-sight with probability \( e^{-\beta R} \)
  - LOS path Loss in dB: \( PL_1 = C + 20 \log R(\text{m}) \)
  - Non-LOS path loss in dB: \( PL_2 = C + K + 40 \log R(\text{m}) \)
  - 28Ghz system: let \( C=50 \text{ dB}, K=10 \text{ dB} \)

- General small scale fading \( h \)
  - No fading case: small scaling fading is minor in mmWave [RapSun]

- Link budget
  - Tx antenna input power: 30dBm
  - Signal bandwidth: 500 MHz (Noise: -87 dBm)
  - Noise figure: 5dB
Results on Coverage
Coverage Analysis

SINR = \frac{N_r N_t H_0 \ell(r_0)}{N_0/P_t + \sum_{k>0} A_k B_k H_k \ell(r_k)},

where

H_0 \ell(r_0) = \min_{k>0} \{H_k \ell(r_k)\},

A_k = \begin{cases} N_t & \text{w. p. } \frac{\theta_t}{2\pi} \\ N_t \cdot \text{FBR}_t & \text{w. p. } 1 - \frac{\theta_t}{2\pi} \end{cases},

B_k = \begin{cases} N_r & \text{w. p. } \frac{\theta_r}{2\pi} \\ N_r \cdot \text{FBR}_r & \text{w. p. } 1 - \frac{\theta_r}{2\pi} \end{cases},

\ell(x) = \begin{cases} Cx^{-2} & \text{w. p. } e^{-\beta x} \\ CKx^{-4} & \text{w. p. } 1 - e^{-\beta x} \end{cases}.

Use stochastic geometry to compute SINR distribution.

- Serving BS and User connect via main lobe
- General small-scale fading
- Connecting to the strongest signal before BF
- Array gain of the TX antenna
- Array gain of the RX antenna
- Path Loss of LOS or non-LOS
Coverage Probability of mmWave

Main Theorem [mmWave Coverage probability]

The coverage probability \( \mathbb{P}[\text{SINR} > T] \) can be computed as

\[
\mathbb{P}(\text{SINR} > T) = \int_{-\infty}^{\infty} \int_{0}^{\infty} U(x, t) f_{L^*}(x) \frac{e^{j2\pi t/T} - 1}{j2\pi t} \, dx \, dt
\]

where

\[
U(x, s) = \exp \left[ -\frac{sx}{M_t M_r \rho} + \int_{x}^{\infty} \left( p_t p_r e^{-sx/u} + (1-p_t)p_r e^{-sx M_t/u M_r} + p_t (1-p_r) e^{-sx M_r/u M_t} \right) \Lambda(du) \right],
\]

\[
\Lambda(x) = 2\pi \lambda \mathbb{E}_h \left[ \int_{0}^{(\frac{x}{K})^{0.25}} t \left( 1 - e^{-\beta t} \right) \, dt + \int_{0}^{\sqrt{x}h} te^{-\beta t} \, dt \right],
\]

\[
f_{L^*}(x) = -\frac{d}{dx} e^{-\Lambda(x)}.
\]
Coverage Gain from Large Arrays

- Mobile user 16 antennas
- BS density $R_c=200$ m
- Buildings cover 5% of the land
- Average building size 15 m by 15 m

Large arrays provide better coverage probability

- the more antennas, the smaller beamwidth, the larger array gain
- Smaller beamwidth provides better coverage
- mmWave coverage probability comparable to microwave

Gain from directional antenna array

$N_t=64, \theta=1.6^\circ$
$N_t=32, \theta=3.2^\circ$
$N_t=16, \theta=6.5^\circ$
Microwave: SU MIMO 4X4
Coverage Gain from Higher Density

Higher density can also increase coverage probability

- Coverage probability no longer invariant with BS density
- Become interference-limited when coverage probability is good
Coverage probability differs in LOS and non-LOS region

- Need to incorporate blockage model & differentiate LOS and nonLOS
- Non-LOS coverage probability generally provides a lower bound
- Buildings may improve coverage by blocking more interference
Corollary 1 [Coverage probability by LOS BSs]

The coverage probability provided by LOS BSs is

\[ P(\text{SINR} > \tau) = \int_{-\infty}^{\infty} \int_{0}^{\infty} U_1(x, t) U_2(t) f_{L^*}(x) \frac{e^{j2\pi t/T} - 1}{j2\pi t} \, dx \, dt, \]

where

\[ U_1(x, s) = \exp \left[ -\frac{sx}{M_t M_s \rho} + \int_{x}^{\infty} \left( p_t p_r e^{-\frac{sx}{u}} + (1 - p_t) p_r e^{-\frac{sxm_t}{uM_t}} + p_t (1 - p_r) e^{-\frac{sxm_r}{uM_r}} \right) \Lambda_1(du) \right], \]

\[ U_2(s) = \exp \left[ \int_{0}^{\infty} \left( p_t p_r e^{-\frac{sx}{u}} + (1 - p_t) p_r e^{-\frac{sxm_t}{uM_t}} + p_t (1 - p_r) e^{-\frac{sxm_r}{uM_r}} \right) \Lambda_2(du) \right], \]

\[ \Lambda_1(x) = 2\pi \lambda \int_{0}^{\sqrt{x}} t e^{-\beta t} \, dt, \]

\[ \Lambda_2(x) = 2\pi \lambda \int_{0}^{\frac{x}{\kappa}} (\frac{x}{\kappa})^{1/4} t (1 - e^{-\beta t}) \, dt, \]

\[ f_{L^*}(x) = -\frac{d}{dx} e^{-\Lambda_1(x)}. \]
Reflections Improve Coverage

128 antennas at BSs
Blockages covers 30% land
(Heavy shadowing case)
Rc=200 m

Reflections can establish links in the shadowed areas

- With dense blockages, most users are served by reflected links
- Non-LOS links improve the coverage probability of mmWave
Results on Rate
Data Rate Comparison

Given coverage probability, the achievable rate is

\[ \eta = \int_0^\infty \frac{P_c(T)}{1 + T} dT \]

Microwave network 4X4 MU MIMO with bandwidth 50MHz:
- Spectrum efficiency is 4.95 bps/ Hz
- Data rate is 248 Mbps (invariant with the cell size \( R_c \))

mmWave network with bandwidth 500MHz:

<table>
<thead>
<tr>
<th>( N_t )</th>
<th>( R_c )</th>
<th>100m</th>
<th>200m</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td></td>
<td>3.4Gbps</td>
<td>3.25Gbps</td>
</tr>
<tr>
<td>64</td>
<td></td>
<td>3.8Gbps</td>
<td>3.45Gbps</td>
</tr>
</tbody>
</table>

I6 antenna at MS
Blockages covers 10% land

\# of antenna at BS

Average cell radius

mmWave achieves high gain in average rate
Cell Throughput Comparison

Gain from larger bandwidth

Gain from serving multiple users

mmWave can support much higher data rate

Conclusions
Going Forward with mmWave

- mmWave coverage probability and rate
  - Need to include both LOS and Non-LOS conditions
  - Interference is reduced by directional antennas and blockages
  - Good rates and coverage can be achieved

- Theoretical challenges abound
  - Analog beamforming algorithms & hybrid beamforming
  - Channel estimation, exploiting sparsity, incorporating robustness
  - Multi-user beamforming algorithms and analysis
  - Microwave-overlaid mmWave system a.k.a. phantom cells
  - More advanced stochastic geometry models including multi-tier
Questions?


